1.3 – What is the value of the restoring constant (k) in each case for the original mass (m)?

1.1 – Determine the frequency (Hz) and the period of oscillation T in each case.

1.2 – If the mass of the oscillating object is doubled in each of the systems in figures a), b) and c), determine the new period (Tnew) of oscillation in each case.

A cross section of the base of the cylinder:

where m = 1.00kg; k = 80.00N/m; \( \rho_{\text{fluid}} = 0.90 \text{kg/m}^3 \); \( r_{\text{cylinder}} = 0.20 \text{m} \).

\begin{align*}
0 &= \Delta \phi + \frac{zIp}{\lambda z} w \quad (c) \\
0 &= x - \frac{l}{Bw} + \frac{zIp}{x_i p} w \quad (q) \\
0 &= x \gamma + \frac{zIp}{x_i p} w \quad (a)
\end{align*}

The equations of motion for the three systems below can be written as:

MID-TERM EXAM I

PHYS 224 - Introductory Physics III

Name:
Consider the following plots, the corresponding equations, and the additional parameters below:

a) \[
\phi = \frac{\omega}{2}\pi
\]

b) \[
\left( \begin{array} {c} \frac{\partial^2 z}{\partial t^2} \\ \frac{\partial z}{\partial t} \end{array} \right) + \omega^2 \left( \begin{array} {c} z - \omega \phi \\ \phi \end{array} \right) = \omega^2 V
\]

where: \[\omega^2 = 1.1\]

c) \[
\left( \begin{array} {c} \frac{\partial^2 z}{\partial t^2} \\ \frac{\partial z}{\partial t} \end{array} \right) + \gamma \left( \begin{array} {c} z - \omega \phi \\ \phi \end{array} \right) = \omega^2 V
\]

where: \[\omega = \frac{\omega_0}{\sqrt{m/k - b^2/(4m^2)}}\]
and \[\gamma = \frac{b}{m}\]

Other system’s parameters:

\[\phi = \frac{\pi}{2}\ rad\]
\[b = 0.30\ kg/s\]
\[k = 30.00\ N/m\]
\[m = 0.20\ kg\]
2.1 - Based on the topics discussed in class, what is the appropriate name for each one of the oscillatory motions represented in the plots above?

2.2 – The maximum amplitude \( A_0 \) and the angular frequency of undamped oscillations \( \omega_0 \) are the same in all the systems above. Based on the information presented in the plots plus your actual calculations, determine the numerical values of \( A_0 \) and \( \omega_0 \).

2.3 – For the case plotted in figure b), determine the two frequencies and the two periods of the resulting motion.

2.4 – For the case plotted in figure c), and a value of \( b = 0.30 \text{ Kg/s} \), determine the frequency and the period of the resulting motion.

2.5 – What value of \( b \) would make this system critically damped? Does the system oscillate in this condition?

2.6 - Which values of \( b \) would make the system over-damped? Using complex exponential arguments and the general solution 
\[
A(t) = A e^{\gamma t} e^{i \omega t}
\]
demonstrate analytically if this over-damped system will oscillate or not.

3.5 points) P3 - Consider a mass-spring system with periodic forcing and damping, described by the following equation of motion:
\[
f(t) = f_0 \cos(2\pi t)
\]
where:
- \( f_0 = 2 \text{ N} \)
- The other constants of the system are specified in problem P1-a) above.

1.1 – Using complex exponentials for the case of NO DAMPING, try a solution \( y(t) = A e^{\alpha t} \) in the equation of motion and determine the analytical equation for the amplitude and the phase of the system as a function of the forcing frequency \( \omega \).

1.2 – Make a sketch of \( A(\omega) \) and \( \phi(\omega) \) for the case of NO DAMPING.

1.3 – Determine the amplitude of the system with NO DAMPING, when the forcing angular frequency is \( \omega = \frac{\pi}{2} \text{ rad/s} \).

1.4 – What is the main change when DAMPING is added to the system? Make a sketch of \( A(\omega) \) in this case.

1.6. Which values of \( b \) would make the system over-damped? Using complex exponentials show analytically if this over-damped system will oscillate or not.

2.6 – Which values of \( b \) would make the system critically damped? Does the system oscillate in this condition?

2.5 – What value of \( b \) would make this system over-damped? Make a sketch of \( A(\omega) \) in this case.

2.4 – What is the main change when DAMPING is added to the system? Make a sketch of \( A(\omega) \) in this case.

2.3 – For the case plotted in figure c), and a value of \( b = 0.30 \text{ Kg/s} \), determine the frequency and the period of the resulting motion.

2.2 – The maximum amplitude of oscillations \( A(\omega) \) and the angular frequency of undamped oscillations determine the numerical values of \( A_0 \) and \( \omega_0 \).

2.1 - Based on the topics discussed in class, what is the appropriate name for each one of the oscillatory motions represented in the plots above?
PHYS 2344 - Introductory Physics III

Mid-Term Exam 1

Name: ____________________

Date: ____________________

SOLUTION

1. a) $w = \sqrt{\frac{E}{\mu}} = \sqrt{\frac{80}{1}} = 8.94 \text{ rad/s}$
   $\omega = 8.94 \text{ rad/s}$
   $T = \frac{2\pi}{\omega} = 0.71 \text{ s}$
   $f = \frac{1}{T} = 1.42 \text{ Hz}$
   $p = \frac{1}{f} = 0.71 \text{ Hz}$

b) $\omega = 4.43 \text{ rad/s}$
   $T = 1.42 \text{ s}$
   $f = 0.71 \text{ Hz}$
(2.1) \( \text{of simple harmonic motion} \)

2.2) \( A_0 = 0.30 \text{m}, \Omega = \sqrt{\frac{k}{m}} = 12.25 \text{ rad/s} \)

2.3) \( \omega_0 = \sqrt{\frac{k}{m}} = \frac{12.25 + \sqrt{12.25^2 + 4 \times 0.5 \times 12.25}}{2} \)

2.4) \( b = 0.3 \text{ kg/s} \)

2.5) \( \omega = \frac{1}{2} \sqrt{\frac{k}{m}} = 0.25 \text{ rad/s} \)

2.6) \( T = 2\pi \sqrt{\frac{m}{k}} = 0.514 \text{ s} \)
2.6) Over-damped system: \( w^2 = \omega_0^2 - \frac{b^2}{4K} < 0 \Rightarrow \omega < 0 \)

\[
\omega^2 = \omega_0^2 - \frac{b^2}{4K} < 0 \quad \Rightarrow \quad \omega < 0
\]

\[ w = \omega_0 e^{-\frac{bt}{2K}} \]

Solution: \( z = A e^{-\frac{bt}{2K}} (\cos \omega t + \phi) \)

\[ \phi = \frac{\delta - \omega_0^2 + \frac{b^2}{4K}}{2\omega_0} \]

Substituting back to the solution:

\[ z = A e^{-\frac{bt}{2K}} (\cos \omega t + \phi) \]

\[ x = \mathbf{R}(t) \]

\[ x = A e^{-(\alpha + \beta)t} \]

\[ \alpha = -\frac{K}{2m} - \frac{b}{2m} \]

\[ \beta = \frac{b}{2m} \]

Which leads to the solution:

\[ x = A e^{-\frac{bt}{2K}} (\cos \omega t + \phi) \]

Only exponential decay.
\[ \rho \infty \frac{(\rho^m - \rho^m')}{\rho_f^m} = 0 \]

\[ \rho \rho = 0 \quad 0 = 0 = 0 = \rho \infty \frac{m}{\rho_f} \]

\[ \rho \rho - \frac{m}{\rho_f} = \frac{(\rho^m + m) \rho_f^m}{\rho_f^m} = 0 \]

\[ \rho \rho - \frac{m}{\rho_f} = (\rho + m) \rho_f^m \left( \frac{m}{\rho_f} + \rho^m \rho_f \right) \]

\[ (\rho + m) \rho_f^m \rho_f^m = \frac{\rho_f^m}{\rho_f} \left( \rho + m \right) \]

\[ (\rho + m) \rho_f^m \rho_f^m = 2 \]

\[ (\rho + m) \rho_f^m \rho_f^m = \frac{\rho_f^m}{\rho_f} \left( \rho + m \right) \]

\[ (\rho + m) \rho_f^m \rho_f^m = 2 \]
3.9) The amplitude at the resonance frequency, $f_0$, is given by:

$$A = 0.0455 \text{ m}$$

and:

$$40 = \frac{8.94^2 \omega^2 - \left( \frac{8}{\sqrt{W}} \right)^2}{2}$$

Solving for $\omega$ gives:

$$\omega = \sqrt{\frac{80}{8}} \text{ rad/s}$$

where $W = 5000 \text{ kg}$.

$$f_0 = \frac{2\pi}{\omega}$$