1. Proof of $x_n > x_{n+1}$ by induction

While $n=1$, $x_1 = 8$, $x_2 = \frac{1}{2}x_1 + 2 = 6$ so $x_1 > x_2$

Assume $x_n > x_{n+1}$

$x_{n+1} = \frac{1}{2}x_{n+2} > \frac{1}{2}x_{n+1} + 2 = x_{n+2}$ so $x_n > x_{n+1}$ holds.

Proof of $x_n$ is bounded.

$x_1 = 8$ because of 1. $x_n < x_1 = 8$ for $n > 1$.

So for new, $x_n \leq 8$

While $x_{n+1} = \frac{1}{2}x_{n+2}$, $x_1 = 8$ it is easy to see that $x_n > 0$

So $0 < x_n \leq 8$ so $x_n$ is bounded.

To prove $\lim_{n \to \infty} x_n = 4$ (OMIT)

10. To prove $S_n = \sup \{x_k : k \geq n\}$ is monotone.

$S_n = \sup \{x_k : k \geq n\} = \sup \{x_n, x_{n+1}, x_{n+2}, \ldots\}$

$S_{n+1} = \sup \{x_k : k \geq n+1\} = \sup \{x_{n+1}, x_{n+2}, \ldots\}$

Assume $b = \sup \{x_{n+1}, x_{n+2}, \ldots\}$

So $S_n = \sup \{x_n, b\}$

1. If $x_n > b$ $S_n = x_n > b = S_{n+1}$

2. $x_n \leq b$ $S_n = b = S_{n+1}$

Based on 0 and 2 $S_n > S_{n+1}$

So $S_n$ is monotone, decreasing.

The monotone of $t_n$ is same with $S_n$.