MATH221-05

quiz #3, 12/04/18

Total 100

Solutions

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:______________________

1. (40) Let A be a $1 \times n$ matrix $[11 \ldots 1]$.

   (a) (10) Describe Null A.

   Solution. If $\mathbf{x} \in$ Null A, then

$$A\mathbf{x} = 0, \text{ and } x_1 = -x_2 - x_3 - \ldots - x_n, \text{ and } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} =$$

$$\begin{bmatrix} -x_2 - x_3 - \ldots - x_n \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ \ldots \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ \ldots \\ 0 \end{bmatrix} + \ldots + x_n \begin{bmatrix} -1 \\ 0 \\ 0 \\ \ldots \\ 1 \end{bmatrix}$$

   Finally $\text{Null } A = \text{span} \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ \ldots \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ \ldots \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} -1 \\ 0 \\ 0 \\ \ldots \\ 1 \end{bmatrix} \right\}$.

   Null A

   (b) (10) Compute rank A.

   Solution. rank $A = 1$

   rank $A =$

(c) (10) Describe Col A.

Solution. Col $A = \mathbf{R}^1$.
Col A is

(d) (10) Let $T : \mathbf{R}^n \to \mathbf{R}^1$ defined by $T(\mathbf{x}) = A\mathbf{x}$. Describe Range of T.

Solution. Range of T is Col $A = \mathbf{R}^1$.
Range of T

2. (20) Consider a linear transformation $T : \mathbf{P}_3 \to \mathbf{P}_2$ defined by

$$T(p(x)) = T(a_0 + a_1x + a_2x^2 + a_3x^3) = p'(x) = a_1 + 2a_2x + 3a_3x^2.$$

(a) (10) Describe kernel of T.

Solution. $T(p(x)) = p'(x) = a_1 + 2a_2x + 3a_3x^2 = 0$ yields $a_1 = a_2 = a_3 = 0$. Hence kernel of T contains all constant polynomials.
kernel of $T =$

(b) (10) Describe image of T.

Solution. If $q(x) = b_0 + b_1x + b_2x^2$, then $T(p(x)) = q(x)$ for $p(x) = b_0x + \frac{b_1}{2}x^2 + \frac{b_2}{3}x^3$. Hence image of T is $\mathbf{P}_2$.
image of $T =$

3. (20) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 1 \\ 7 & 5 & 5 & 5 \end{bmatrix}$. Compute det A.

Solution. Since Row 1 + Row 3 = Row 4 one has det $A = 0$.

det $A =$

4. (40) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$.

(a) (20) Compute det A.

Solution.

$$\det \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} = \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 2 & 12 \end{bmatrix} = 2.$$

det $A =$

(b) (20) If A^{-1} exists, then compute det A^{-1}.

Solution. Since det $A = 2$ the matrix A is invertible, and det $A^{-1} = (\det A)^{-1} = 1/2$.

Alternatively compute A^{-1}, and then compute det A^{-1}.