MATH221-05
quiz #1, 09/20/18
Total 100
Solutions

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Show all work legibly. **Name**:_____

1. (20) Solve the system

$$\begin{array}{rrrcr} x_1 & -5x_2 & +4x_3 & = & -3 \\ 2x_1 & -7x_2 & +3x_3 & = & -2 \\ -2x_1 & +x_2 & +7x_3 & = & -1 \end{array}$$

Solution

$$\left[\begin{array}{rrrr} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{array}\right] \to \left[\begin{array}{rrrr} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{array}\right] \to \left[\begin{array}{rrrr} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right].$$

Mark one:

- ▫ The solutions are:
  $x_1 =$ $x_2 =$ $x_3 =$
- ▫ The system has no solutions.

2. (20) Determine the values of h for which the system

$$2x_1 - 6x_2 = 1,\ hx_1 + x_2 = 1$$

is consistent.

Solution.

$$\left[\begin{array}{rrr} 2 & -6 & 1 \\ h & 1 & 1 \end{array}\right] \to \left[\begin{array}{rrr} 2 & -6 & 1 \\ 0 & (1+3h)x_2 & 1-h/2 \end{array}\right]$$

$h \neq -1/3$

3. (20) Let

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}.$$

True or False? $\mathbf{b}$ is in the set of all linear combinations of the columns of A.

Solution

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ 0 & -4 & 2 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -4 & 2 & 6 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 6 & 10 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 5/3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 4/3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 5/3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 5/3 \end{bmatrix}$$

Finally

$$\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix} = -\frac{2}{3} \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + \frac{5}{3} \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

Mark one and explain.

▫ True ▫ False

4. (20) True or False? For each pair of vectors $\mathbf{v}$ and $\mathbf{u}$ such that $|\mathbf{v}| > 1$, and $|\mathbf{u}| > 2$ one always has $|\mathbf{v} + \mathbf{u}| > 3$.

Solution. If $\mathbf{v} = 2$, and $\mathbf{u} = -3$, then $|\mathbf{v} + \mathbf{u}| = 1$.

Mark one and explain.

▫ True ▫ False

5. (20) Let $A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \end{bmatrix}$. True or False? If $A\mathbf{x} = 0$, and c_1 and c_2 are scalars, then $(c_1\mathbf{a}_1 + c_2\mathbf{a}_2)^T \mathbf{x} = 0$.

Solution.

$$A\mathbf{x} = \begin{bmatrix} \mathbf{a}_1^T\mathbf{x} \\ \mathbf{a}_2^T\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } (c_1\mathbf{a}_1 + c_2\mathbf{a}_2)^T \mathbf{x} = c_1\left(\mathbf{a}_1^T\mathbf{x}\right) + c_2\left(\mathbf{a}_2^T\mathbf{x}\right) = 0.$$

Mark one and explain.

▫ True ▫ False

6. (20) True or False? If the linear system $A\mathbf{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is consistent, then the system $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is also consistent.

Solution. If $A\mathbf{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, then $A\left(\frac{1}{2}\mathbf{v}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.