QUIZ 01: MATH 221

Question 01 Solve following system of equations

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

Solution:

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

Step 01 by interchanging R_1, R_2

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

Step 02 by $1/2 R_1$

$$\begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

Step 03 by $R_3 = 5R_1 - R_3$

$$\begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & 1/2 & -2 & 3/2 \end{bmatrix}$$

Step 04 $2R_3$

$$\begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & 1 & -4 & 3 \end{bmatrix}$$

Step 05 $R_3 = R_2 - R_3$

$$\begin{bmatrix} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

In step 05 we get row of form $[0\ 0\ 0\ x]$ where $x \neq 0$

Therefore system is Inconsistent

Question 02: Determine the values of h for which system is consistent

$$2x_1 - 6x_2 = -3$$
$$-4x_2 + 12x_2 = -h$$

$$\begin{bmatrix} 2 & -6 & -3 \\ -4 & 12 & -h \end{bmatrix}$$

Step 01 by $R_2 = 2R_1 + R_2$

$$\begin{bmatrix} 2 & -6 & -3 \\ 0 & 0 & -6-h \end{bmatrix}$$

In order for system to be consistent, we need to have row $[0\ 0\ x]$ where x should be zero

$\therefore -6 - h = 0$

Question 03 : Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 10 \\ 3 \\ 0 \end{bmatrix}$

True or False? b is in the set of all linear combinations of the coloumn A.

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

step 01 : $R_1 = 1/2 R_1$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

step 02 : $R_2 = R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

Step03 $R_3 = R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 2 & 2 & 5 \end{bmatrix}$$

step04 : $R_3 = R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

In step 04 we get row of form $[0\ 0\ 0\ x]$ Where $x \neq 0$

∴ System is not consistent.

$\boxed{\text{FALSE}}$

Question 04 Let $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $V_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $V_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$

a) TRUE OR FALSE? There exist scalars c_1, c_2, c_3 so that $c_1 V_1 + c_2 V_2 + c_3 V_3 = b$

02

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 2 & 5 & 1 & 7 \\ 3 & 6 & 1 & 9 \end{bmatrix}$$

Step 01 by $R_2 = 2R_1 - R_2$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 3 & 1 & 3 \\ 3 & 6 & 1 & 9 \end{bmatrix}$$

Step 02 by $R_3 = 3R_3 - R_1$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 3 & 1 & 3 \\ 0 & 6 & 2 & 6 \end{bmatrix}$$

Step 03 $R_2 = 1/3 R_2$ & $R_3 = 1/6 R_3$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 1 & 1/3 & 1 \\ 0 & 1 & 1/3 & 1 \end{bmatrix}$$

Step 04 $R_3 = R_2 - R_3$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here
$x_1 = 1 + 1/3 x_3$
$x_2 = 1 - 1/3 x_3$
x_3 free variable

More over
At $c_1 = c_2 = 1$ & $c_3 = 0$

$$(1)\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (1)\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

proves $c_1 v_1 + c_2 v_2 + c_3 v_3 = b$

$\boxed{\text{TRUE}}$

b) True or false? The vector b is a linear combination of two vectors v.

Lets take $v_1 = v_2 = 1$

$\therefore \quad 1 v_1 + 1 v_2 = b$

i.e $(1)\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (1)\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$

$\boxed{\text{TRUE}}$

c) The above two statements are true with regard to any vector b

Let's take b as $\begin{bmatrix} 5 \\ 7 \\ 8 \end{bmatrix}$ and solve for vectors v_1, v_2, v_3

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 2 & 5 & 1 & 7 \\ 3 & 6 & 1 & 8 \end{bmatrix}$$

Step 01 $R_2 = 2R_1 - R_2$ & $R_3 = 3R_1 - R_3$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 3 & 1 & 3 \\ 0 & 6 & 2 & 7 \end{bmatrix}$$

Step 02 $R_2 = 1/3 R_2$ & $R_3 = 1/6 R_3$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 1 & 1/3 & 1 \\ 0 & 1 & 1/3 & 7/6 \end{bmatrix}$$

Step 03 $R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 4 & 1 & 5 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 0 & 1/6 \end{bmatrix}$$

we have row of form $[0\ 0\ 0\ x]$ where $x \neq 0$

$\therefore$ System is inconsistent.

Hence we can say that above two statements are not true with regard to any vector b.

$\boxed{\text{FALSE}}$

Questions 5: Let v_1 and v_2 be vectors with two entries so that for each vector with two entries b the equation $x_1 v_1 + x_2 v_2 = b$ has only one solution. True or False? The vectors v_1 and v_2 are linearly dependent.

Solution: Here if we take vectors

$$v_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 9 \end{bmatrix} \quad \& \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

by $R_2 = 3R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 9 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -9 & 0 \end{bmatrix}$$

$\therefore$ $\boxed{x_1 = x_2 = 0}$ only solution

Also we can write.

$$x_1 v_1 + x_2 v_2 = b$$

equals $x_1 v_1 + x_2 v_2 = 0$ for our eq

Now $x_1 = x_2 = 0$

$\therefore$ According to definition of linear independency

if $x_1 v_1 + x_2 v_2 + \dots + x_n v_n = 0$

has only trivial solution means $x_1 = x_2 \dots = x_n = 0$

$\therefore$ Hence v_1 & v_2 are linearly independent.

$\boxed{\text{FALSE}}$