Home Work Assignment - 04 MATH 221.

Section 1.4 Pg 47: 14, 22, 23 (b, c, e), 34, 36.

Section 1.7 Pg 71: 6, 8, 14, 18, 20, 21, 33, 34, 37.

0.5 Points

Pg 47: 14, 22, 34, 36

Pg 71: 6, 8, 14, 18

01 Points

Pg 47: 23 (b, c, e)

Pg 71: 20, 21, 33, 34, 37.

Pg 47 Q14 Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is u in the subset of R^3 spanned by the columns of A? Why or why not?

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix}$$

Step 01 $R_3 = 5R_3 - R_1$

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 7 & -7 & 8 \end{bmatrix}$$

Step 02 $R_3 = 7R_2 - R_3$

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{bmatrix}$$

Here we have Row of form $[0\,0\,0\,x]$ where $x \neq 0$ which shows that system is inconsistent. ~~which~~

No u in the subset of R^3 is not spanned by the columns of A.

Pg 47 Q22 Let $v_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$. Does $[v_1, v_2, v_3]$ span R^3? Why or why not.

$$\begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -1 \\ -2 & 8 & -5 \end{bmatrix}$$

Step 01 → R_1 interchanging with R_3

$$\begin{bmatrix} -2 & 8 & -5 \\ 0 & -3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

We have pivot in each row. Therefore $[v_1 v_2 v_3]$ span R^3.

Pg 47 Q23 State True or False and Justify your answer.

23.b A vector b is a linear combination of the coloumns of a matrix A if and only if the equation $Ax = b$ has at least one solution $\boxed{\text{TRUE}}$ from pg 42 Existence of Solution.

23.c The equation $Ax = b$ is consistent if the augmented matrix $[A\ b]$ has a pivot position in every row

$\boxed{\text{FALSE}}$ from pg 44 read warning after theorem 4.

23.e If the coloumns of an $m \times n$ matrix A span R^m, then the equation $Ax = b$ is consistent for each b in R^m.

$\boxed{\text{TRUE}}$ from pg 43 Theorem 4 point a & c.

Pg 47 Q34 Suppose A is a 3×3 matrix and b is a vector in R^3 with the property that $Ax = b$ has a unique solution. Explain why the coloumns of A must span R^3?

Solution from theorem 4 it is clear that either all a to d are True or they are false.

Now since $Ax = b$ has a unique solution which means A has a pivot position in every row and thus coloumns of A span R^m.

Pg 47 Q36 Let A be a 5×3 matrix, let y be a vector in R^3, and let z be a vector in R^5. Suppose $Ay = z$. What fact allows you to conclude that the system $Ax = 4z$ is consistent?

Solution from theorem 4 it is evident that

if $Ay = z$ is true

then each vector z in R^5 is linear combinations of coloumns of A

Now $A(\frac{1}{4})x = \frac{1}{4}(Ax)$ i.e $Ax = 4z$ [used property $A(cu) = cAu$]

Therefore it is also true for $Ax = 4z$

means any vector z in R^5 is linear combination of coloumns of A.

Pg 71 Q6 Determine if the coloumns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$

Step 01 Interchanging R_1 & R_3

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ -4 & -3 & 0 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$

Step 02 $R_3 = 4R_1 + R_1$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & -3 & 12 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix}$$

Step 03 $R_4 = 5R_1 - R_4$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & -3 & 12 & 0 \\ 0 & -4 & 9 & 0 \end{bmatrix}$$

Step 04 $R_3 = 3R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & 9 & 0 \end{bmatrix}$$

Step 05 $R_4 = 4R_2 - R_4$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix}$$

Step 06 Interchanging R_3 & R_4

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

here we have 3 basic variable & no free variable. Thus $Ax=0$ has only the TRIVIAL SOLUTION.
Therefore $\boxed{\text{LINEARLY INDEPENDENT}}$

Pg 71 Q8 $\begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$

Number of vectors > Numbers of Entries in a vector

$\boxed{\text{LINEARLY DEPENDENT}}$

Pg 71 Q14. Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix}1\\-1\\-3\end{bmatrix} \begin{bmatrix}-5\\7\\8\end{bmatrix} \begin{bmatrix}1\\1\\h\end{bmatrix}, \quad \begin{bmatrix}1 & -5 & 1 & 0\\-1 & 7 & 1 & 0\\-3 & 8 & h & 0\end{bmatrix}$$

Step 01 $R_2 = R_1 + R_2$

$$\begin{bmatrix}1 & -5 & 1 & 0\\0 & 2 & 2 & 0\\-3 & 8 & h & 0\end{bmatrix}$$

Step 02 $R_3 = 3R_1 + R_3$

$$\begin{bmatrix}1 & -5 & 1 & 0\\0 & 2 & 2 & 0\\0 & -7 & 3+h & 0\end{bmatrix}$$

Step 03 $R_2 = 1/2 R_2$

$$\begin{bmatrix}1 & -5 & 1 & 0\\0 & 1 & 1 & 0\\0 & -7 & 3+h & 0\end{bmatrix}$$

Step 04 $R_3 = 7R_2 + R_3$

$$\begin{bmatrix}1 & -5 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & h+10 & 0\end{bmatrix}$$

Now $h+10=0$

$\boxed{h=-10}$

Pg 71 Q18&20 Determine by inspection whether the vectors are linearly independent. Justify your answer.

Q18 $\begin{bmatrix}4\\4\end{bmatrix}, \begin{bmatrix}-1\\3\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}8\\1\end{bmatrix}$

Number of vectors > Number of entries in a vector

∴ $\boxed{\text{LINEARLY DEPENDENT}}$

Q20 $\begin{bmatrix}1\\4\\-7\end{bmatrix}, \begin{bmatrix}-2\\5\\3\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}$

contains zero vector ∴ $\boxed{\text{LINEARLY DEPENDENT}}$

Q21 State TRUE or FALSE, justify your answer:

a) The columns of a matrix are linearly independent if the equation $Ax=0$ has the trivial solution

$\boxed{\text{FALSE}}$ It should have only the trivial solution

b) If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S.
FALSE Pg 68 Theorem 7 warning.

c) The columns of any 4x5 matrix are linearly dependent TRUE Pg 69 Theorem 8.

d) If x and y are linearly independent and if (x, y, z) is linearly dependent, then z is in span(x, y)
TRUE Pg 68 Example 04 specifically read 2nd last para

Pg 71 Q33 If $v_1 -- v_4$ are in R^4 and $v_3 = 2v_1 + v_2$, then $\{v_1, v_2, v_3, v_4\}$ is linearly dependent.
Solution: TRUE see Theorem 7 and also

$$v_3 = 2v_1 + v_2 + 0v_4.$$

Pg 71 Q34 If $v_1 --- v_4$ are in R^4 and $v_3 = 0$ then $\{v_1, v_2, v_3, v_4\}$ is linearly dependent.
Solution: TRUE see Theorem 9

Pg 71 Q37 If $v_1 -- v_4$ are in R^4 and $\{v_1, v_2, v_3\}$ is linearly dependent then $\{v_1, v_2, v_3, v_4\}$ is also linearly dependent.
TRUE (v_1, v_2, v_3) can be extended to (v_1, v_2, v_3, v_4) by simply adding v_4 with weight 0. Thus one of the vectors is linear combination of other three.