MATH-221 HOME-WORK ASSIGNMENT-03

Section 1.3 Page 37: 9, 10, 12, 14, 23 (c,d), 24 (c,d), 26

Section 1.4 Page 47: 2, 4, 6, 8, 9, 12, 13

Points Distribution

0.5 Points → Section 1.3 Page 37: 9, 10, 12, 14, 23 (c,d), 24 (c,d)

Section 1.4 Page 47: 2, 4

01 Points → Section 1.3 Page 37: 26

Section 1.4 Page 47: 6, 8, 9, 12, 13

Pg 37 Question 09 Write a vector equation that is equivalent to the given system of equations

$$x_2 + 5x_3 = 0$$
$$4x_1 + 6x_2 - x_3 = 0$$
$$-x_1 + 3x_2 - 8x_3 = 0$$

$$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Pg 37 Question 010

$$4x_1 + x_2 + 3x_3 = 9$$
$$x_1 - 7x_2 - 2x_3 = 2$$
$$8x_1 + 6x_2 - 5x_3 = 15$$

$$x_1 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}$$

Pg 37 Question 12 Determine if b is a linear combination of a_1, a_2 &

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \quad a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} \quad a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} \quad b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

System of equations

$x_1 + 0x_2 + 2x_3 = -5$
$-2x_1 + 5x_2 + 0x_3 = 11$
$2x_1 + 5x_2 + 8x_3 = -7$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix}$$

Step 01 $R_2 = 2R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 2 & 5 & 8 & -7 \end{bmatrix}$$

Step 02 $R_3 = 2R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & -5 & -4 & -3 \end{bmatrix}$$

Step 03 $R_3 = R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

In step 03, we have third row of form $[0\ 0\ 0\ x]$ where $x \neq 0$
$\therefore$ It is inconsistent.

NO, b is not linear combination of x_1, x_2 & x_3

Pg 37 Question 014 Determine if b is a linear combination of the vectors formed from the coloumns of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}$$

Step 01 $R_3 = R_1 - R_3$

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & -11 & 2 \end{bmatrix}$$

$\therefore \boxed{a_3 = -2/11}$

$3a_2 = -5 - 7a_3$
$= \dfrac{-5 + 14}{11} = \dfrac{-41}{11}$

$\therefore \boxed{a_2 = \dfrac{-41}{33}}$

$a_1 = 11 + 6a_3 + 2a_2 = 11 + 6\left(\dfrac{-2}{11}\right) + 2\left(\dfrac{-41}{33}\right) = \dfrac{(11 \times 33) - 82 - 36}{33}$ $\boxed{a_1 = \dfrac{243}{33}}$

$\therefore$ b is a linear combination of the vectors formed from coloums of

Matrix A.

Pg 37 Q23 (c,d) True or False. Justify your answer.

23-c An example of a linear combination of vectors v_1 and v_2 is the vector $\frac{1}{2}v_1$

$\boxed{\text{TRUE}}$ $\frac{1}{2}v_1 \left(= \frac{1}{2}v_1 + 0v_2\right)$ See Pg 32

23-d The solution set of the linear system whose augmented matrix is $[a_1\ a_2\ a_3\ b]$ is the same as the solution set of the equation $x_1a_1 + x_2a_2 + x_3a_3 = b$

$\boxed{\text{TRUE}}$ See Pg 34

Pg 37 Q24 (c,d) True or False. Justify your answer.

24-c The weights $c_1 \ldots c_p$ in a linear combination $c_1v_1 + \ldots + c_pv_p$ cannot all be zero.

$\boxed{\text{FALSE}}$ See Pg 32

24-d When u and v are non-zero vectors, Span(u,v) contains the line through u and the origin.

$\boxed{\text{TRUE}}$ See Pg 35

In particular, Span (u,v) contains the line in R^3 through u and 0 and the line through v and 0.

Pg 37 Q26 Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $b = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$ and let W be the set of all linear combination of the coloumn of A.

a.) Is b in W?

$$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$

Step 01 $R_1 = 1/2 R_1$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$

Step 02! $R_2 = R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$

Step 03 $R_3 = R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

Step 04 $R_3 = R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} a_3 &= \text{free} \\ a_2 &= 1 - a_3 \\ a_1 &= 5 - 3a_3 \end{aligned}}$$

$\therefore$ $\boxed{\text{b is in W}}$

b.) Show that the third coloumn of A is in W.

From a) we say that W is span(a_1, a_2, a_3) since it is set of all linear combinations of vectors a_1, a_2, a_3

$\therefore$ Vectors of form

$a_3 = 0a_1 + 0a_2 + (1)a_3$ will be contained in span of W.

Pg47 Q2 Compute the products, using.
A) The definition as in Example 01
B) The row-vector for computing Ax.

$$\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

Cannot compute product as coloumns of 1^{st} matrix $\neq$ rows of 2^{nd} matrix.

Q4
$$\begin{bmatrix} 8 & \cancel{7}\,3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a)
$$1\begin{bmatrix} 8 \\ 5 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 1\begin{bmatrix} -4 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

b)
$$\begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8(1) + 3(1) - 4(1) \\ 5(1) + 1(1) + 2(1) \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Pg47 Q6 Use the definition of Ax to write the matrix equation as a vector equation or vice versa.

$$\begin{bmatrix} 7 & -3 \\ 2 & 1 \\ 9 & -6 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$$

$$(-2)\begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} + (-5)\begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$$

Pg47 Q8 $z_1\begin{bmatrix} 4 \\ -2 \end{bmatrix} + z_2\begin{bmatrix} -4 \\ 5 \end{bmatrix} + z_3\begin{bmatrix} -5 \\ 4 \end{bmatrix} + z_4\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$

$$\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

Pg 47 Q9: Write the system first as a vector equation and then as a matrix equation

$$3x_1 + x_2 - 5x_3 = 9$$
$$x_2 + 4x_3 = 0$$

a) Vector Equation

$$x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ +4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

b) Matrix Equation

$$\begin{bmatrix} 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

Pg 47 Q12 Write the augmented matrix for the linear system that corresponds to the matrix equation $Ax=b$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}$$

Step 01 $R_2 = 3R_1 + R_2$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}$$

Step 02 $R_3 = R_2 - R_3$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$\therefore a_3 = 1$

& $5a_2 = 1 - 5(a_3)$

$a_2 = -4/5$

& $a_1 = -a_3 - 2a_2$

$= 3/5$

$$\therefore \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \\ 01 \end{bmatrix}$$

Pg 47 Q13 Let $u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is u in the plane in R^3 spanned by the coloumns of A? Why or why not.

$$\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Step 01) Interchanging R_1 & R_3

$$\begin{bmatrix} 1 & 1 & 4 \\ -2 & 6 & 4 \\ 3 & -5 & 0 \end{bmatrix}$$

Step 02: $R_2 = 2R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 3 & -5 & 0 \end{bmatrix}$$

Step 03 $R_3 = 3R_1 - R_3$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & 8 & 12 \end{bmatrix}$$

Step 04 $R_3 = R_2 - R_3$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore x_2 = 3/2$

& $x_1 = 4 - 3/2$

$= 5/2$

$x_1 = 5/2$

$x_2 = 3/2$

Therefore v is a linear combination of coloumn of A and thus v is in the plane in R^3 spanned by the coloumns of A.