MATH221 HOMEWORK ASSIGNMENT 02

Sec 1.1 pg 11 : 16, 21, 23 (a, b, c) [Each question is of 01 points]
Sec 1.2 pg 25 : 10, 12 ← [both questions 02 points] & 14, 20, 31 [1 points each]

Pg 11 Q16 Determine if the systems are consistent. Do not completely solve the systems.

$$\begin{aligned} x_1 \quad\quad\quad - 2x_4 &= -3 \\ 2x_2 + 2x_3 \quad\quad &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

Given Step 01

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \xrightarrow{R_4 = 2R_1 + R_4} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix}$$

Step 02
by $1/2 R_2$ Step 03

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix} \xrightarrow{\text{by } R_4 = R_4 - 3R_2} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{bmatrix}$$

Step 04
by $R_4 = R_3 + R_4$ ~~Here~~ Step 05

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now here if we further perform elementary row operation it appear that we will not get row of for [0 0 0 0 x] where x is some non zero value ($x \neq 0$) which will makes the system inconsistent.
Hence we can conclude that system is $\boxed{\text{CONSISTENT}}$

Pg11 Q21 Determine value of h such that the matrix is the augmented value of a consistent Linear system

Given $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \xrightarrow{R_2 = 4R_1 + R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & 12+h & 0 \end{bmatrix}$

Here we are getting a row of form [0 X 0] where X is 12+h.

Now irrespective of X be non zero or zero the system is consistent in either cases.

$\therefore$ h can be any value

Pg11 Q23 ~~e)~~ State True / False with reason.

a) Every elementary row operation is reversible.

$\boxed{\text{TRUE}}$ Refer: 2nd last paragraph 1st Line on Pg 07

b) A 5X6 matrix has six rows.

$\boxed{\text{FALSE}}$ A 5X6 has 5 rows & 6 coloumns.

c) The solution set of a linear system involving variables $x_1 ... x_n$ is a list of numbers $(s_1 ... s_n)$ that makes each equation in the system a true statement when the values $s_1 ... s_n$ are substituted for $x_1 ... x_n$ respectively

$\boxed{\text{FALSE}}$

Reason: The solution set contains all solutions of a system hence in case when a system has many solutions a single list $(s_1 -- s_n)$ is a solution set, rather than "the solution set".

Pg25 Q10 Find the general solutions of the system whose augmented matrices are

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \xrightarrow{R_2 = 3R_1 - R_2} \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & -1 & 7 \end{bmatrix}$$

$\therefore\ x_3 = -7$

& $x_1 = 2x_2 + x_3 + 3$

$= 2x_2 - 4$

Solution
$$\boxed{\begin{aligned} x_1 &\Rightarrow -4 + 2x_2 \\ x_2 &\Rightarrow \text{free} \\ x_3 &\Rightarrow -7 \end{aligned}}$$

Pg25 Q12
$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \xrightarrow[\text{Step 01}]{R_3 = R_1 + R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix}$$

Step 02 $R_3 = 4R_2 + R_3$

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here x_1 & x_3 are basic
x_2 & x_4 are free.

$x_1 - 7x_2 + 6x_4 = 5$ & $x_3 - 2x_4 = -3$

$x_1 = 5 + 7x_2 - 6x_4$ $\quad x_3 = -3 + 2x_4$

Solution:
$$\boxed{\begin{aligned} x_1 &= 5 + 7x_2 - 6x_4 \\ x_2 &\ \text{free} \\ x_3 &= -3 + 2x_4 \\ x_4 &\ \text{free} \end{aligned}}$$

Pg 25 Q14

$$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{here}} \begin{array}{l} x_1, x_2 \ \& \ x_5 \text{ are basic} \\ x_3 \ \& \ x_4 \text{ are free} \end{array}$$

Now $x_2 = 6x_3 + 3x_4 + 2$

& $x_1 = \underbrace{-12x_3 - 6x_4 - 4}_{2(x_2)} + 5x_4 + 6x_4 - 5$

$= -7x_3 - 9$

Solution:

$$\boxed{\begin{array}{l} x_1 = -7x_3 - 9 \\ x_2 = 6x_3 + 3x_4 + 2 \\ x_3 \text{ free} \\ x_4 \text{ free} \\ x_5 = 0 \end{array}}$$

Pg 25 Q20 Choose h & k such that system is

a) No Solution $\boxed{h = 9 \ \& \ k \neq 6}$

b) A unique solution $\boxed{h \neq 9}$

c) Many solution $\boxed{h = 9 \ \& \ k = 6}$

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix} \xrightarrow{R_2 = 3R_1 - R_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 9-h & 6-k \end{bmatrix}$$

Pg 25 Q31 A system of linear equations with more equations than unknowns is sometimes called an overdetermined system. Can such a system be consistent? Illustrate your answer with a specific system of three equation in two unknowns?

Solution: Yes, it can be consistent.

$x_1 - 4x_2 = 1$

$2x_1 - x_2 = -3$

$-x_1 - 3x_2 = 4$

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix} \xrightarrow{R_3 = R_1 + R_3} \overset{\text{step 01}}{\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ 0 & -7 & 5 \end{bmatrix}}$$

Step 02 by $R_2 = 2R_1 - R_2$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \\ 0 & -7 & 5 \end{bmatrix} \xrightarrow{R_3 = R_2 - R_3}$$

Step 04

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

here we rows of form [0 0 x] where x is zero
hence system is consistent
& moreover $x_2 = -\frac{5}{7}$ & $x_1 = -\frac{13}{7}$