

HOMEWORK ASSIGNMENT 01

MATH 221

Question: Pg-11 1, 3, 12, 17 [each question is worth 2.5 points]

Q1. $x_1 + 5x_2 = 7$
 $-2x_1 - 7x_2 = -5$

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array} \right]$$

By $2R_1 + R_2$ on R_2

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array} \right]$$

$\therefore 3x_2 = 9$

$x_2 = 9/3 \Rightarrow 3$

Now $x_1 + 5x_2 = 7$

$x_1 = 7 - 5(3)$

$= -8$

Solution $(x_1, x_2) = (-8, 3)$

Checking

$x_1 + 5x_2 = 7$

$-8 + 5(3) = 7$

$7 = 7$

$-2x_1 - 7x_2 = -5$

$-2(-8) - 7(3) = -5$

$16 - 21 = -5$

Q3. $x_1 + 5x_2 = 7$ Find the point (x_1, x_2) that lies
 $x_1 - 2x_2 = -2$ on the given lines

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 1 & -2 & -2 \end{array} \right]$$

by $R_1 - R_2$ on R_2

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 7 & 9 \end{array} \right]$$

$\therefore 7x_2 = 9$

$x_2 = 9/7$

& $x_1 + 5x_2 = 7$

$x_1 = 7 - 5\left(\frac{9}{7}\right)$

$= \frac{4}{7}$

Solution $(x_1, x_2) = \left(\frac{4}{7}, \frac{9}{7}\right)$

Checking.

$$x_1 + 5x_2 = 7$$

$$\frac{4}{7} + 5\left(\frac{9}{7}\right) = 7$$

$$\frac{49}{7} = 7$$

$$\boxed{7 = 7}$$

$$x_1 - 2x_2 = -2$$

$$\frac{4}{7} - 2\left(\frac{9}{7}\right) = -2$$

$$\frac{4-18}{7} = -2$$

$$\boxed{-2 = -2}$$

Q12 Solve the systems

$$x_1 - 3x_2 + 4x_3 = -4$$

$$3x_1 - 7x_2 + 7x_3 = -8$$

$$-4x_1 + 6x_2 - x_3 = 7$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{array} \right]$$

by $3R_1 - R_2$ on R_2

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -2 & 5 & -4 \\ -4 & 6 & -1 & 7 \end{array} \right]$$

by $4R_1 + R_3$ on R_3

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -2 & 5 & -4 \\ 0 & -6 & 15 & -9 \end{array} \right]$$

by $3R_2 - R_3$ on R_3

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -2 & 5 & -4 \\ 0 & 0 & 0 & -3 \end{array} \right]$$

Now Row 3 is $0x_1 + 0x_2 + 0x_3 = -3$ i.e. $0 \neq -3$

which ^{shows system} is inconsistent.

Q17. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$ and $-x_1 - 3x_2 = 4$ have a common point of intersection? Explain.

Let's consider 2 lines

$$x_1 - 4x_2 = 1$$

$$-x_1 - 3x_2 = 4$$

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ -1 & -3 & 4 \end{array} \right]$$

By $R_1 + R_2$ on R_2

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & -7 & 5 \end{array} \right]$$

$$\therefore -7x_2 = 5$$

$$x_2 = -5/7$$

$$\& x_1 - 4x_2 = 1$$

$$x_1 = 1 + 4\left(-\frac{5}{7}\right)$$

$$= \frac{7-20}{7} = -\frac{13}{7}$$

$$\text{We have } (x_1, x_2) = \left(-\frac{13}{7}, -\frac{5}{7}\right)$$

Let's put these values of (x_1, x_2) (point ~~is~~ ~~per~~ common in both lines) in line 3

$$2x_1 - x_2 = -3$$

$$\text{Here } x_1 = -13/7 \& x_2 = -5/7$$

$$2\left(-\frac{13}{7}\right) - \left(-\frac{5}{7}\right) = -3$$

$$\text{i.e. } -\frac{21}{7} = -3$$

$$\boxed{-3 = -3}$$

Thus $(x_1, x_2) = (-13/7, -5/7)$ is also at line 3

Thus they have a common point which is $(x_1, x_2) = (-13/7, -5/7)$