

Name:

MATH221
quiz #1, 09/25/08
Sections 1.1–1.5
Total 100
Solutions

Show all work legibly.

1. (20) Solve the system:

$$\begin{array}{rcrcrcrcrcrl} x_1 & - & 2x_2 & + & x_3 & = & 0 \\ & & 2x_2 & - & 8x_3 & = & 8 \\ -4x_1 & + & 5x_2 & + & 9x_3 & = & 0 \end{array}$$

Solution.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 12 \end{bmatrix} \rightarrow$$
$$\begin{bmatrix} 1 & -2 & 0 & -12 \\ 0 & 1 & 0 & 52 \\ 0 & 0 & 1 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 92 \\ 0 & 1 & 0 & 52 \\ 0 & 0 & 1 & 12 \end{bmatrix}$$

$$x_1 = 92$$

$$x_2 = 52$$

$$x_3 = 12$$

2. (20) Write a system of equations that is equivalent to the vector equation

$$x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \\ -5 \end{bmatrix}.$$

Solution. The system of equations is:

$$6x_1 = 0, \quad -x_1 + 4x_2 = -7, \quad 5x_1 = -5.$$

3. (20) Determine the values of h for which the system

$$2x_1 - 6x_2 = -3, \quad -4x_1 + 12x_2 = h$$

is consistent.

Solution.

$$\begin{bmatrix} 2 & -6 & -3 \\ -4 & 12 & h \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -6 & -3 \\ 0 & 0 & h - 6 \end{bmatrix}$$

$$h = 6$$

4. (20) Let

$$A = \begin{bmatrix} 2 & 0 & 5 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 0 \end{bmatrix}.$$

True or False? $\mathbf{b}$ is in the set of all linear combinations of the columns of A .

Solution.

$$\begin{bmatrix} 2 & 0 & 5 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ -1 & 8 & 5 & 3 \\ 2 & 0 & 5 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 6 & 6 & 3 \\ 0 & 4 & 3 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1/2 \\ 0 & 4 & 3 & 10 \end{bmatrix} \rightarrow$$
$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 8 \\ 0 & 1 & 0 & 17/2 \\ 0 & 0 & 1 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 25 \\ 0 & 1 & 0 & 17/2 \\ 0 & 0 & 1 & -8 \end{bmatrix}$$

Mark one and explain.

☐ True ☐ False

5. (20) True or False? If A is 5×3 matrix, $\mathbf{y} \in \mathbf{R}^3$, and $\mathbf{b} \in \mathbf{R}^5$ such that $A\mathbf{y} = \mathbf{b}$, then the equation $A\mathbf{x} = 2\mathbf{b}$ is consistent.

Solution.

If $A\mathbf{y} = \mathbf{b}$, then $A(2\mathbf{y}) = 2\mathbf{b}$. Hence the equation $A\mathbf{x} = 2\mathbf{b}$ is consistent.

Mark one and explain.

- ☐ True ☐ False

6. (20) **Bonus Problem.**

True or False? If A is a 3×2 matrix such that $A\mathbf{x} = \mathbf{0}$ has a unique solution, then for each $\mathbf{b}$ the system $A\mathbf{x} = \mathbf{b}$ has a unique solution.

Solution.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The equation $A\mathbf{x} = \mathbf{b}$ has no solution.

Mark one and explain.

☐ True ☐ False