Name:

MATH221

quiz #4, 11/29/07

Sections 4.4–4.6, 3.1–3.2

Total 100

Solutions

Show all work legibly.

1. (20) Find the change–of–coordinate matrix from $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \end{bmatrix} \right\}$ to the standard basis in $\mathbf{R}^2$.

   Solution:

$$P_{\mathcal{B}} = \begin{bmatrix} 2 & 1 \\ -9 & 8 \end{bmatrix}.$$

2. (20) Find the dimension of the subspace spanned by the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}$.

   Solution:

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 4 \\ 0 & -5 & -20 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Only two vectors are linearly independent, and the dimension of the subspace is 2.

3. (20) If the null space of a 5×6 matrix is 4−dimensional, what is the dimension of the row space of A?

   Solution:

$$\text{rank } A + \dim \text{ Null } A = n, \text{ hence rank } A + 4 = 6, \text{ and rank } A = 2.$$

   The dimension of the row space of A is 2.

4. (20) Let $A = \begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 3 \\ 0 & 5 & -1 \end{bmatrix}$.

   - (10) Compute det A.

     Solution:

$$\det A = 3 \begin{vmatrix} 3 & 3 \\ 5 & -1 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = 3 \cdot (-18) + 4 \cdot 10 = -14.$$

- (10) If A^{-1} exists, then compute det A^{-1}.

  Solution: $\det A^{-1} = \dfrac{1}{\det A} = -\dfrac{1}{14}$.

5. (20) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

   - (10) Compute det A.

     Solution:

     $$\det A = 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \cdot (-3) + 1 \cdot 1 = -2.$$

   - (10) Compute det A^{10}.

     Solution:
     $\det A^{10} = (-2)^{10} = 1024$.

6. (20) (**extra credit**) Let A and P be square matrices. True or False? If P^{-1} exists, then $det \left(PAP^{-1}\right) = det\ A$.

   Solution:

   $$det \left(PAP^{-1}\right) = (det\ P)\,(det\ A)\, det \left(P^{-1}\right) = (det\ P)\,(det\ A)\,(det\ P)^{-1} = det\ A.$$