ECON 423 - Simulation Lab III

Definitions and Procedures

- **Endogenous Variables**: Determined inside the model
- **Exogenous Variables**: Determined outside the model
- **Structural Equations**: Have endogenous variables on the right hand side. Most economic models are presented as structural equations
- **Reduced Form Equations**: Have only exogenous and lagged endogenous variables on the right hand side
- **Impulse**: A robust disturbance that is external to the model
- **Propagation**: The reverberation through the model that amplifies any internal or external imbalances and causes fluctuations

General Procedure

Step 1: Formulate or identify an appropriate dynamic economic model for the question

Step 2: Find/solve for the reduced form equations for that model

Step 3: Code the reduced forms into a spreadsheet

Step 4: Conduct experiments

Step 5: Observe the effects of the experiments on the endogenous variables

Structural Model

$$\begin{aligned} C &= \alpha_1 + \beta_1 Y \\ I &= \alpha_2 + \beta_2 \Delta Y + \gamma_2 R \\ Y &\equiv C + I + G \end{aligned} \tag{1}$$

- **Exogenous Variables**: G, R
- **Endogenous Variables**: Y, C, I
- **Parameters**: α_1, α_2, β_1, β_2, γ_2

Reduced Form Solution

First, make some simplifications to the model, add subscripts to keep track of the time path of the variables

$$\begin{aligned} C_t &= \beta_1 Y_{t-1} \\ I_t &= \beta_2 \Delta C_t = \beta_2 (C_t - C_{t-1}) \\ Y_t &\equiv C + I + G \end{aligned}$$

Model Changes:

- For simplicity: $\alpha_1 = \alpha_2 = 0$
- No Money Market: $\gamma_2 = 0$
- Investment function changed to focus on accelerator and simplify solution

Model Solution

Reduced form equation for Y can be found algebraically

$$Y_t = G_t + \beta_1[1 + \beta_2]Y_{t-1} - \beta_1\beta_2 Y_{t-2} \tag{2}$$

Note that RHS has only parameters, exogenous variables and lagged endogenous variables. Lagged endogenous variables are "predetermined." Reduced form equation is a second-order linear difference equation, because it contains Y_{t-1}, Y_{t-2}

Reduced form equations for C_t and I_t can be found recursively using the solution for Y_t

$$C_t = \beta_1 Y_{t-1} \tag{3}$$

$$I_t = \beta_2(C_t - C_{t-1}) \tag{4}$$

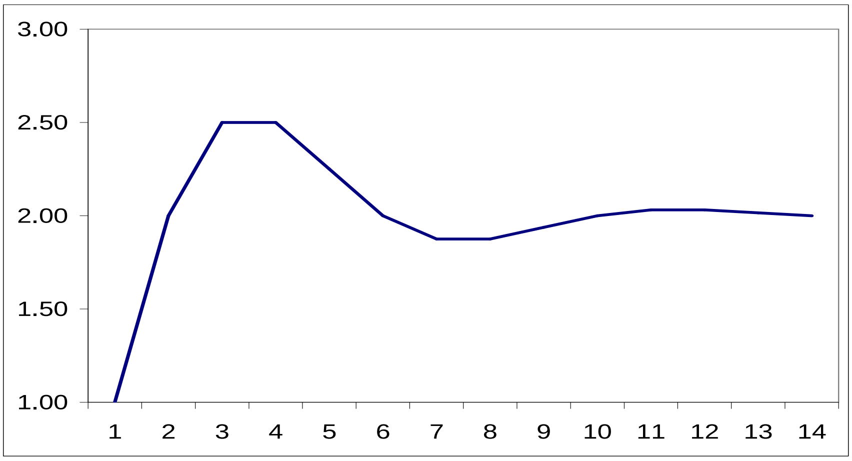
LAB EXERCISES

1. Set up spreadsheet. Code equations (2), (3), and (4) into spreadsheet beginning in period 2. Note that the value for Y_0 is required for this. Simply set $Y_0 = 0$.
2. Obtain time path for endogenous variables. Replicate Table 9.1. Correctly written formulas for C_t, I_t, and Y_t will replicate the table when copied down through period 14.

Table 9.1 Interactions of Multiplier-Accelerator

Time	G_t	C_t	I_t	Y_t
1	1.00	0.00	0.00	1.00
2	1.00	0.50	0.50	2.00
3	1.00	1.00	0.50	2.50
4	1.00	1.25	0.25	2.50
5	1.00	1.25	0.00	2.25
6	1.00	1.125	-0.125	2.00
7	1.00	1.00	-0.125	1.875
8	1.00	0.9375	-0.0625	1.875
9	1.00	0.9375	0.00	1.9375
10	1.00	0.96875	0.03125	2.00
11	1.00	1.00	0.03125	2.03125
12	1.00	1.015625	0.015625	2.03125
13	1.00	1.015625	0.00	2.015625
14	1.00	1.0078125	-0.0078125	2.00

3. Replicate Figure 9.1, the time path of Y_t.



Note the interesting cyclical behavior in this time path. Y_t increases, decreases, and then increases again before settling down. Keep in mind that there is no *impulse* in this case at all. The observed behavior of Y_t is pure *propagation*.

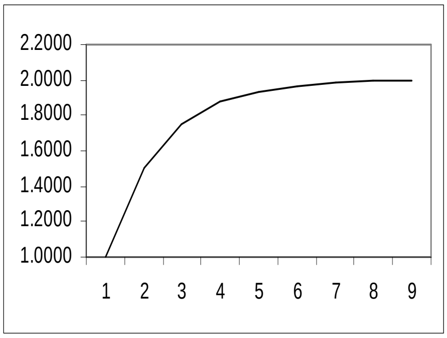
4. Replicate Table 9.2 by systematically varying the values of β_1 and β_2.

Table 9.2 Simulation Results

Time	(1) $\beta_1 = 0.5$ $\beta_2 = 0.0$	(2) $\beta_1 = 0.5$ $\beta_2 = 2.0$	(3) $\beta_1 = 0.6$ $\beta_2 = 2.0$	(4) $\beta_1 = 0.8$ $\beta_2 = 4.0$
1	1.0000	1.0000	1.0000	1.0000
2	1.5000	2.5000	2.8000	5.0000
3	1.7500	3.7500	4.8400	17.8000
4	1.8750	4.1250	6.3520	56.2000
5	1.9375	3.4375	6.6256	168.84
6	1.9688	2.0313	5.3037	496.52
7	1.9844	0.6094	2.5959	1446.79
8	1.9922	-0.1172	-0.6918	4199.30
9	1.9961	0.2148	-3.3603	12168.48

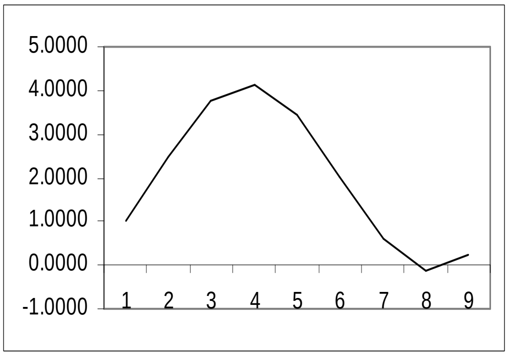
5. Plot the time path of Y_t for each set of parameter values. Note that there are three general types of behavior shown on the plots of these time paths of Y.

$\beta_1 = 0.5$ $\beta_2 = 0.0$



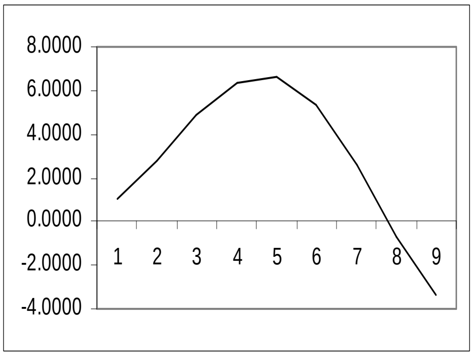
Monotonic Convergent

$\beta_1 = 0.5$ $\beta_2 = 2.0$



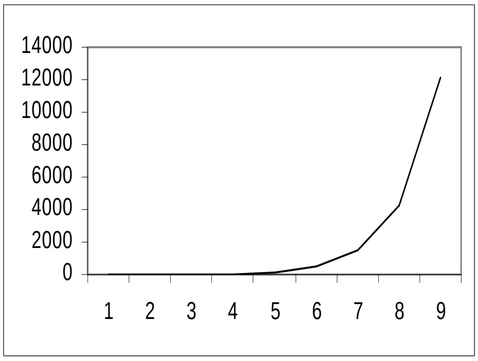
Damped Oscillation

$\beta_1 = 0.6$ $\beta_2 = 2.0$



Explosive Oscillation

$\beta_1 = 0.8$ $\beta_2 = 4.0$



Monotonic Explosive

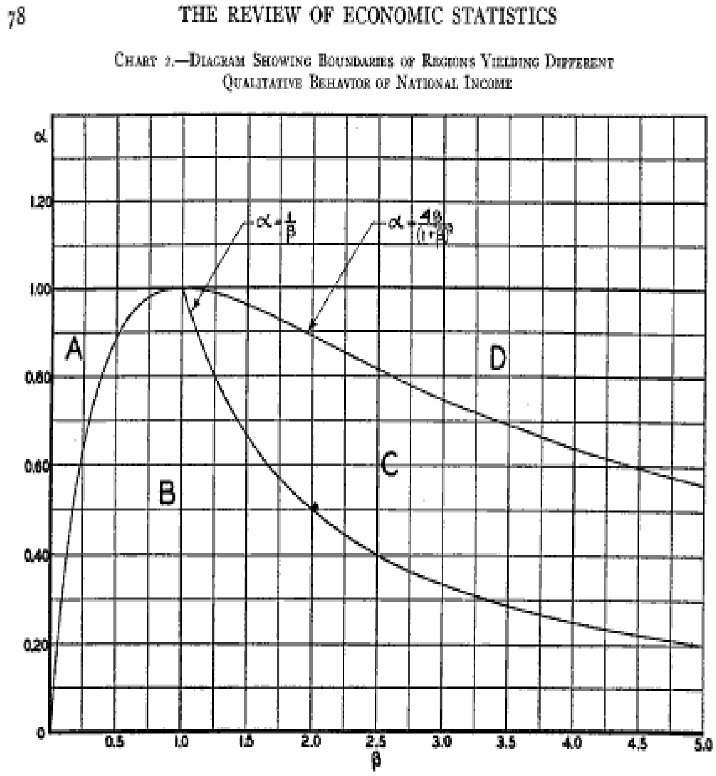
A Bit on Characteristic Roots

Consider again the reduced form equation for Y, with the assumption that $G = 1$ substituted into the equation

$$Y_t = 1 + \beta_1[1 + \beta_2]Y_{t-1} - \beta_1\beta_2 Y_{t-2}.$$

Equations of this form are called second order difference equations. It is second order because both Y_{t-1} and Y_{t-2} appear in the equation. In general, a second order difference equation can be factored into two parts, called characteristic roots. This procedure is similar to finding the roots of a quadratic equation in algebra. The time path of Y depends on the value of these characteristic roots, and the characteristic roots depend on the parameters β_1 and β_2.

Samuelson derived a chart which shows how the behavior of the time path of Y_t varies by the values of the parameters β_1 and β_2. By locating the parameter combinations used on Table 9.2 on this chart, the behavior of Y_t can be explained.



Note that the notation differs slightly. In Samuelson's notation, $\alpha = \beta_1$ and $\beta = \beta_2$.

- **Region** A: Values of any of these combinations of β_1 and β_2 lead to *monotonic convergent* time paths for Y_t
- **Region** B: Values of any of these combinations of β_1 and β_2 lead to *damped oscillation* time paths for Y_t
- **Region** C: Values of any of these combinations of β_1 and β_2 lead to *explosive oscillation* time paths for Y_t
- **Region** D: Values of any of these combinations of β_1 and β_2 lead to *monotonic explosive* time paths for Y_t