ECON 423 - Simulation Lab I

Structural Model

- Four equation model: Income Identity, Consumption Function, Investment Function, Tax Function

$$
\begin{aligned}
Y &\equiv C + I + G && \text{(8.2a)}\\
C &= \alpha_1 + \beta_1(Y - T) && \text{(8.2b)}\\
I &= \alpha_2 + \beta_2 Y_{-1} + \gamma_2 R && \text{(8.2c)}\\
T &= \alpha_3 + \beta_3 Y && \text{(8.2d)}
\end{aligned}
$$

- **Exogenous Variables**: G, R
- **Endogenous Variables**: Y, C, I, T
- **Parameters**: α_1, α_2, α_3, β_1, β_2, β_3, γ_2
- No money market, R exogenous

Parameterized Structural Model

- Obtained by statistical techniques - data were obtained and these parameters were estimated by regression methods

$$
\begin{aligned}
Y &\equiv C + I + G && \text{(8.3a)}\\
C &= 16 + 0.7(Y - T) && \text{(8.3b)}\\
I &= 6 + 0.1Y_{-1} - 0.3R && \text{(8.3c)}\\
T &= 0.0 + 0.2Y && \text{(8.3d)}
\end{aligned}
$$

$$
\begin{array}{llll}
(C) & \alpha_1 = 16 & \beta_1 = 0.7 & \\
(I) & \alpha_2 = 6 & \beta_2 = 0.1 & \gamma_2 = -0.3\\
(T) & \alpha_3 = 0.0 & \beta_3 = 0.2 &
\end{array}
$$

Reduced Form Equations

- The "solution" to a structural model is called "reduced form equations"

$$
\begin{aligned}
Y &= 50 + 0.2273Y_{-1} - 0.6818R + 2.2727G && \text{(8.4a)}\\
C &= 44 + 0.1273Y_{-1} - 0.3818R + 1.2727G && \text{(8.4b)}\\
I &= 6 + 0.01Y_{-1} - 0.03R && \text{(8.4c)}\\
T &= 10 + 0.0455Y_{-1} - 0.1364R + 0.4545G && \text{(8.4d)}
\end{aligned}
$$

- The numbers are reduced form parameters
- Note that an explicit reduced form equation for Y has been solved for

- First-order linear difference equations
- Endogenous on Right Hand Side, Exogenous on Left Hand Side
- Note that the general form of a reduced form model is

$$\begin{aligned}
Y &= a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G \\
C &= a_{20} + a_{21}Y_{-1} + a_{22}R + a_{23}G \\
I &= a_{30} + a_{31}Y_{-1} + a_{32}R + a_{33}G \\
T &= a_{40} + a_{41}Y_{-1} + a_{42}R + a_{43}G
\end{aligned}$$

- One equation for each endogenous variable
- A constant parameter for each reduced form equation
- One reduced form parameter for each endogenous variable

Reduced Form Equations

$$Y = \tfrac{\alpha_1+\alpha_2-\beta_1\alpha_3}{1-\beta_1+\beta_1\beta_3} + \tfrac{\beta_2}{1-\beta_1+\beta_1\beta_3}Y_{-1} + \tfrac{\gamma_2}{1-\beta_1+\beta_1\beta_3}R + \tfrac{1}{1-\beta_1+\beta_1\beta_3}G$$

$$\begin{array}{lllll}
Y &= a_{10} & +a_{11}Y_{-1} & +a_{12}R & +a_{13}G \\
T &= \alpha_3 + \beta_3 a_{10} & +\beta_3 a_{11}Y_{-1} & +\beta_3 a_{12}R & +\beta_3 a_{13}G \\
I &= \alpha_2 & +\beta_2 Y_{-1} & +\gamma_2 R & \\
C &= \alpha_1 + (\beta_1-\beta_1\beta_3)(a_{10}) & +(\beta_1-\beta_1\beta_3)a_{11}Y_{-1} & +(\beta_1-\beta_1\beta_3)a_{12}R & +(\beta_1-\beta_1\beta_3)a_{13}G
\end{array}$$

$$\begin{array}{lllll}
(Y) & a_{10} = \tfrac{\alpha_1+\alpha_2-\beta_1\alpha_3}{1-\beta_1+\beta_1\beta_3} & a_{11} = \tfrac{\beta_2}{1-\beta_1-\beta_1\beta_3} & a_{12} = \tfrac{\gamma_2}{1-\beta_1+\beta_1\beta_3} & a_{13} = \tfrac{1}{1-\beta_1+\beta_1\beta_3} \\
(C) & a_{20} = \alpha_1 + (\beta_1-\beta_1\beta_3)(a_{10}) & a_{21} = (\beta_1-\beta_1\beta_3)a_{11} & a_{22} = (\beta_1-\beta_1\beta_3)a_{12} & a_{23} = (\beta_1-\beta_1\beta_3)a_{13} \\
(I) & a_{30} = \alpha_2 & a_{31} = \beta_2 & a_{32} = \gamma_2 & a_{33} = 0 \\
(T) & a_{40} = \alpha_3 + \beta_3 a_{10} & a_{41} = \beta_3 a_{11} & a_{42} = \beta_3 a_{12} & a_{43} = \beta_3 a_{13}
\end{array}$$

Multipliers

- The model is dynamic which complicates the analysis of the effects of changes in the exogenous variables, must distinguish between short-term impact of changes in the exogenous variables and long-term impact
- **Short-term multiplier**: Reflects the one-period effect of a change in an exogenous variable on some endogenous variable
- The *Short-term multiplier* is simply the reduced form parameter on each exogenous variable
- **Long-term multiplier**: Cumulative impact of a change in an exogenous variable on some endogenous variable; if m_S is the short-term multiplier on an exogenous variable, then

$$\textit{long term multiplier} = m_L = 1 + m_S + m_S^2 + m_S^3 + \ldots = \frac{1}{1-m_S}$$