The Appraisal of Forecasts
Su, Chapter 16

Defining and Measuring Accuracy

- The criteria that should be used to measure forecast accuracy are open to debate
- Well look at the main competing methods used to evaluate forecast accuracy
- Assessment of forecast accuracy is a very important component of forecast evaluation

Forecast Errors

- If the forecast of an economic variable does not agree with the actual observed value, there is a *forecasting error*
- The **Forecast Error** in the tth period is simply

$$FE_t \text{ in level } = F_t - A_t$$

where FE_t is the forecast error, F_t is the forecast value and A_t is the observed or "actual" value

- This is called the *Forecast error in levels* to distinguish it from other types of forecast errors
- It is in the same units as the forecasted variable
- $FE_t > 0$: The forecast overestimated the actual value, the forecast error is positive
- $FE_t < 0$: The forecast underestimated the actual value, the forecast error is negative

Forecast Errors in Percentages

- Forecasters often predict changes over time,and such forecasts are based on the jump-off data
- $F_t - A_{t-1}$: Forecast change
- $A_t - A_{t-1}$: Actual change
- The forecast error in changes is

$$FE_t \text{ in change } = (F_t - A_{t-1}) - (A_t - A_{t-1}) = F_t - A_t$$

- This is identical to the forecast error in level
- The forecast error in percent changes is

$$FE_t \text{ in \% change } = \left(\frac{F_t - A_{t-1}}{A_{t-1}}\right) - \left(\frac{A_t - A_{t-1}}{A_{t-1}}\right)$$
$$= \left(\frac{F_t}{A_{t-1}}\right) - \left(\frac{A_t}{A_{t-1}}\right) = \frac{F_t - A_t}{A_{t-1}}$$

- Unitless measure

Log Transformations and Forecast Errors

- These forecast errors are easier to calculate in logs
- In levels

$$FE_t \text{ in level } = \ln F_t - \ln A_t = \ln\left(\frac{F_t}{A_t}\right)$$

- In % change

$$FE_t \text{ in \% change } = \ln\left(\frac{F_t - A_{t-1}}{A_{t-1}}\right) - \ln\left(\frac{A_t - A_{t-1}}{A_{t-1}}\right) = \ln\left(\frac{F_t}{A_t}\right)$$

- They are equivalent in logs

“Jump-off Period”

- Forecasting jargon
- Refers to the last period of realization, or the period just before the forecasting period
- Suppose we were forecasting some variable for the second quarter of the year at a point in time when data from the first quarter of the year are available, then the first quarter is the *jump-off period*

Multi-period Forecasts

- Most economic forecasts are made for more than one period ahead:
- The period 1 forecast depends on jump-off data
- The period 2 forecast depends on the period 1 forecast, etc.

- Allows forecast errors to accumulate
- In general, errors in later periods will be larger than those in earlier periods, unless errors tend to be of opposite sign and offset each other

Forecast Errors and Data Revisions

- Data revisions complicate the calculation of forecast errors, as they lead to changes in the realized values that are in turn the basis of forecasts
- Let F_t be a forecast value, A_t^P be a preliminary actual value and A_t^r be a revised actual value - so errors due to a revision in a series is $A^r - t - A_t^p$ and actual changes in a revised series $A^r - t - A_{t-1}^r$
- Using this notation, forecast errors can be expressed

$$FE_t \text{ in change } = (F_t - A_{t-1}^p) - (A_t^r - A_{t-1}^r) \neq F_t - A_t$$

- And in percents

$$FE_t \text{ in \% change } = \left(\frac{F_t - A_{t-1}^p}{A_{t-1}^p}\right) - \left(\frac{A_t - A_{t-1}^r}{A_{t-1}^r}\right)$$
$$= \left(\frac{F_t}{A_{t-1}^p}\right) - \left(\frac{A_t^r}{A_{t-1}^r}\right) \neq \frac{F_t - A_t}{A_{t-1}}$$

Cumulative Changes

- Total error made during forecasting horizon

$$FE_{t+h} = (F_{t+k} - A^p_{t-1}) - (A^r_{t+k} - A^r_{t-1})$$

- where k is the *Forecast Horizon* and $h = 1, 2, \ldots, k$
- Cumulative error in percentage change

$$FE_{t+h} = \left(\frac{F_{t+k}}{Ar_{t-1}}\right) - \left(\frac{Ar_{t+k}}{Ar_{t-1}}\right)$$

- These measures allow the period-to-period errors to reinforce or offset
- Note the problem associated with signs of forecast errors - cant simply add them up!
- Two ways to correct for this: Absolute Values and Squaring

Summary Statistics

1. Mean Absolute Error (MAE)

$$MAE = \sum_1^n |FE_t|/n = \sum_1^n |F_t - A_t|/n$$

2. Mean Absolute Percentage Error $MAPE$)

$$MAPE = \sum_1^n |FE_t|/n = \sum_1^n \left|\frac{F_t - A_t}{A_{t-1}}\right|$$

3. Mean Square error (MSE)

$$MSE = \sum_1^n (FE_t)^2/n = \sum_1^n (F_t - A_t)^2/n$$

4. Root Mean Square Error ($RMSE$)

$$RMSE = \sqrt{\sum_1^n (FE_t)^2/n} = \sqrt{\sum_1^n (F_t - A_t)^2/n}$$

Summary Statistics

5. Theils *Inequality Coefficient*

$$U = \frac{\sqrt{\frac{1}{n}\sum(F_t - A_t)^2}}{\sqrt{\frac{1}{n}\sum A_t^2}}$$

- Numerator is Root Mean Square Error
- Denominator is Root Mean Square Error of a*No Change Forecast*
- If $U = 0 \rightarrow F_t = A_t \forall t$, a “Perfect Forecast”
- If $U = 1$ then $RMSE$ same as a no change forecast so U must be less than 1 for the forecast to be useful

Other Evaluations

- **Subjective Evaluations**:
- Did the forecast adequately serve its intended purpose?
- Did the forecast help its users make better decisions?
- Were the forecasting errors limited to the level one would expect?
- Were there means available to improve accuracy?
- **Absolute Evaluations**:
- Examines a single set of forecasts
- Determines the extent to which the forecast deviates from actual values
- Determines whether the deviations are persistent in any particular direction or in any particular variable or sector