Forecasting, Naive Methods and Single Equation Models

Su, Chapter 10

What is a Forecast?

- Stating with some precision an opinion concerning unknown future events
- "Tomorrow will be cold" is a weather forecast that predicts the temperature tomorrow
- The accuracy of forecasts depends in part on how the forecasts were made
- Economic forecasts are generally measured in quantities, prices, percent changes, or by indexes - they are *quantitative predictions*
- Based in mathematical or statistical techniques and historical data
- Economic forecasts take into account economic theories and relevant data
- Making well-thought-out and carefully designed forecasts is costly and time consuming
- In some instances Naive methods are useful

Who Makes Economic Forecasts?

- Economists trained in forecasting make economic forecasts - these individuals

1. Must be familiar with collecting, handling and analyzing economic data
2. Understand how economic data are disseminated and revised
3. Have familiarity with computers and statistical software
4. Understand economic theory as it relates to functional relationships and causality
5. Understand statistical inference as it related to forecasting
6. Must be familiar with forecasting techniques
7. Can communicate with forecast users

How To Forecast

- Although the techniques vary considerably, the general procedure for making forecasts are similar:

1. Build a Forecasting Device
2. Put inputs into the Forecasting Device
3. Make a Forecast

- This process is called a “forecasting cycle”

Inputs	$\longrightarrow$	**Forecasting Device**	$\longrightarrow$	**Forecast**
Historical Data		Dynamics of historical data		
Judgement		Economic Theory		
		Statistical Inference		

Economic Forecasting Device

- Not a black box
- Typically a single or system of mathematical equations derived from economic theory or statistical models of the economy
- Can be simple or complex
- This device is typically put through some economic and statistical tests to assess its usefulness before being used to create forecasts
- Example: the Adelman and Adelman Study of the Klein-Goldberger Model

Forecast Inputs

- Include historical data and judgement
- Part art and part science, because of the role judgement plays in constructing the forecasting device
- A *general rule* of forecasting is that the more recent the data used, the more accurate the forecast, implying that one should obtain the most recent data available for inputs into a forecasting cycle
- **Jump-off Period**: the last period of available data, or the period just before the forecasting period
- When making a forecast of a variable for the fourth quarter of a given year and historical data are available only through the third quarter, the jump-off is the third quarter

Types of Predictions

- *Conditional Predictions*: Predictions made under the assumption that some other future events must simultaneously occur
- *Unconditional Predictions*: predictions made regardless of the outcomes of other relevant events
- The *conditions* usually refer to public policies or other noneconomic events, have to be set before before making forecasts and usually depend on the forecasters judgement
- The conditions are events that are beyond prediction and a good forecast typically includes alternative versions of these events
- *Example*: A forecast of short term interest rates would depend on the action taken by the FOMC at the next scheduled meeting , and a good forecast would include alternative versions depending on the decision of the FOMC (cut rates, raise rates, do nothing)

Multiperiod Forecasts

- Weather forecasts always predict the conditions of the next time period, because the current conditions are already known
- The same relationship exists in economic forecasts - values of historical data are already known, but unlike weather forecasts, single period forecasts not useful to decision makers
- We have studied the time path of many economic variables and clearly the behavior of these variables in a single period may not convey information about their behavior over many periods
- **Problem**: Forecasting over multiple periods is difficult because of the contingent nature of these forecasts - the forecast for period $t+2$ depends on the forecast for period $t+1$ and so on
- Since the first period forecast contains prediction errors, these errors will tend to be propagated to additional periods, reducing the precision of future forecasts
- The precision of a forecast deteriorates as the *forecast horizon* widens

Types of Economic Forecasts

1. **Noneconometric Forecasts**

   (a) *Simple Extrapolation*: Naive Forecasts, Moving Averages, Trend-smoothing, autoregressive schemes

   (b) *Judgemental Forecasts*

   (c) *Economic Indicators*: Leading-lagging indicators, composite indexes, Diffusion indexes

   (d) *Survey and Consensus Forecasts*: Public Opinion Surveys, Expert Surveys

2. **Econometric Forecasts**

   (a) *Simultaneous Equation Models*: Structural, Reduced Form

   (b) *Time-Series Models*: $MA(p,d), AR(d,q), ARIMA(p,d,q), VAR$

Classifications

- Many ways to classify forecasts
- Some methods are similar, others diverse
- All are viewed as complementary
- Two broad groups, based on the level of sophistication: *Noneconmetric* and *Econometric*
- The econometric forecasts we cover will not go beyond simple and multiple regression techniques

Extrapolations and Naive Methods

- *Rationale*: Some past tendency or trend in the variable being forecast reflects what will happen in the future
- Extrapolation quantifies this trend and uses this quantification to predict the future behavior
- The underling economic relationship between a single variable and other factors is ignored in this procedure
- **Only** the past history of a single variable is used to make these forecasts

Naive Forecasts

- Application of simple statistical techniques to historical observations
- Mechanical and Objective, no subjective opinion required
- What will happen in the future can be related to what happened in the past
- **No Change Model**: The anticipated level of the variable in the current period is the same as the observed level in the previous period $X_t^* = X_{t-1}$ where a $*$ refers to a forecast value and $t-1$ is the jump-off period
- Often used by individuals without realizing it
- **Same Change Model**: No Change model in first differences $\Delta X_t^* = \Delta X_{t-1}$
- Alternatively $X_t^* - X_{t-1} = X_{t-1} - X_{t-2}$
- Change from $t-2$ to $t-1$ will be the same as the change from $t-1$ to t, requires knowledge of two periods of historical data

Naive Forecasts

- **Same Change Model in Percentages**: Multiplicative form

$$\frac{X_t^*}{X_{t-1}} = \frac{X_{t-1}}{X_{t-2}}$$

- Alternatively

$$X_t^* = X_{t-1} \times \frac{X_{t-1}}{X_{t-2}}$$

- This is also called the **Same Ratio Model**
- **Partial Adjustment Model**: Refined same change model, includes only a portion of the past change $\Delta X_t^* = \rho \Delta X_{t-1}$
- Where $1 > \rho > 0$ = reflects caution about the expected permanence of past changes

Pros and Cons

- Useful as benchmarks for comparing more complicated methods
- Simple and require minimal data
- Useful for very short-run forecasts of variables that are relatively stable or changing in a uniform way
- Tend to be quite inaccurate at predicting variables that fluctuate a lot or are subject to changes in irregular factors
- Do not generate oscillations, so cannot be used to forecast turning points

Moving Average methods

- More efficient mechanical projection of short-term movements
- Has the advantage of flexibility and presenting more realistic pictures of long-run movements
- Data are not forced into any particular patters because moving averages simply smooth the data series - a moving average of order n is

$$X_t^* = \frac{1}{n}\sum_{i=1}^{n} X_{t-i}$$

- Note that this is not a *centered* moving average - the centered moving average of order n for variable X_t is the average of $n/2$ observations before the tth observations and $n/2$ observations after this observation

Choosing n

- Only choice for the forecaster making a moving average forecast is n
- $n = 1$ is the no change forecast
- $n = 2$ is average of past two periods
- The larger n the more smooth the forecast and the less the forecast is affected by one-time random fluctuations
- The larger n the more slowly the forecast reacts to changes in the series
- n can be chosen optimally using goodness of fit tests if a long historical series is available

Other Moving Average Techniques

- **First Difference / Percent Change Model**

$$Y_t^* = Y_{t-1} + \frac{(Y_{t-1} - Y_{t-2}) + \ldots + (Y_{t-n} - Y_{t-n-1})}{n}$$

- or

$$Y_t^* = Y_{t-1} \times \left[1 + \frac{(Y_{t-1} - Y_{t-2})/(Y_{t-2} + \ldots + (Y_{t-n} - Y_{t-n-1}/(Y_{t-n-1})}{n} \right]$$

- **Weighted Moving Average**: Unlike others that give equal weights to past changes, this one allows for unequal weights

$$X_t^* = \frac{nX_{t-1} + (n-1)X_{t-2} + \ldots + 1X_{t-n}}{1 + 2 + \ldots + n}$$

- Cannot generate a meaningful multi-period forecast and may become less reliable as the forecasting horizon gets longer

- Cannot generate significant cycles, tends to underpredict growing variables

Single-Equation Regression Models

- Represent functional relationships between economic variables and usually estimated by OLS
- General Form

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots + \beta_k X_{1k} + u_t$$

- Causal relationship where Y_t is the Dependent Variable, X_{it}s are Explanatory Variables β_is Unknown Parameters and u_t a Stochastic Term
- To forecast, apply OLS to model for a sample of data to get estimates of the parameters and use

$$Y_t = \hat{\beta}_0 + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \ldots + \hat{\beta}_k X_{1k} + e_t$$

  where $\hat{}$s represent estimated parameters and e_t the OLS residual
- After model is estimated, tests can be performed to evaluate the forecasting properties of the model

Regression: Forecasting Ability

- Depends on the structure of the regression equation, including:
  1. Degrees of Freedom: Should be > 30
  2. Statistical Significance and sign of parameters
  3. High Goodness of Fit
  4. Low Standard Error of Estimate
  5. High R^2

Forecasting with Regression Models

- Depends on choice of Xs, which is generally guided by economic theory
- *Example*: According to the IS/LM model, what variables would be useful for forecasting GDP?
- Generally speaking, more data should be preferred when using this approach

Evaluation of Ex Post Forecasts

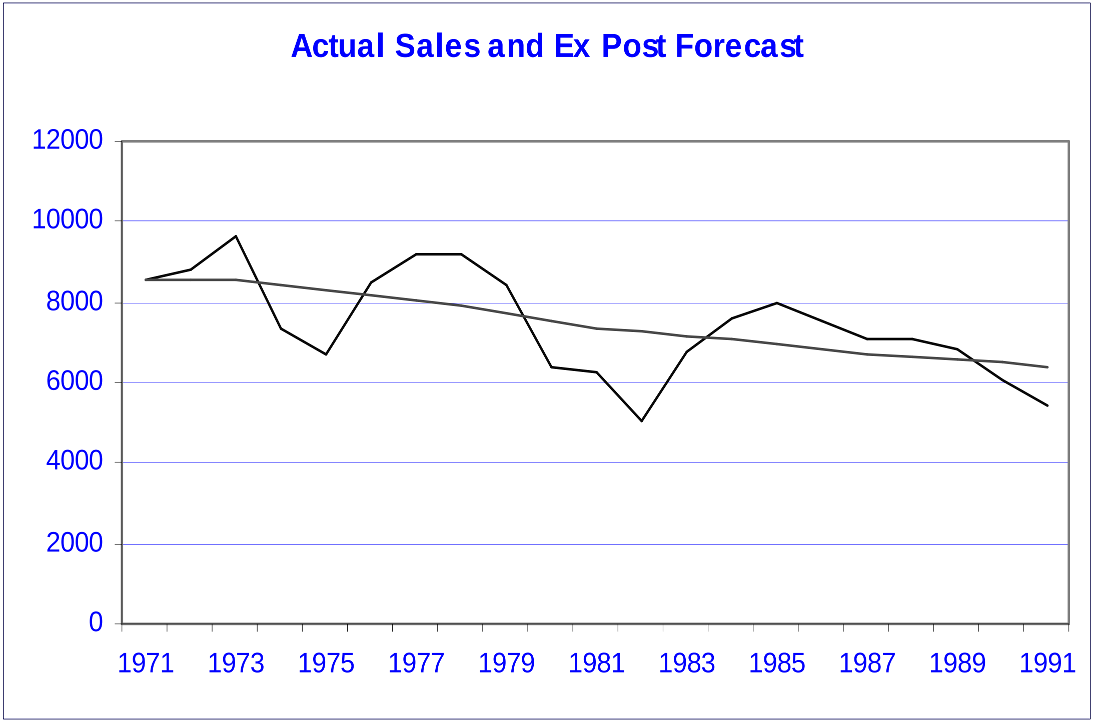
- Can evaluate forecasts generated by regression models by looking at within sample predictions
- Where in the typical regression output can one find an *ex post* forecast?
- *Ex post* forecasts are based on the regression parameters and the explanatory variable
- Predicted values from a regression model:

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_{1t} + \hat{\beta}_2 X_{2t} + \ldots + \hat{\beta}_k X_{1k}$$

- Parameters with a $\hat{}$ are the point estimates obtained from OLS estimation
- *OLS Residuals*: $\hat{u}_t = Y_t - \hat{Y}_t$

New Car Sales Example

- Recall the regression forecasting example in Chapter 10
- Model: $Y_t = \alpha + \beta X_t + u_t$
- Y: Automobile Sales, X: New Car Price (Linear Demand Curve)
- Estimation Results: $Y_t = 10,200.23 - 30.275X_t(10.20)$
- Can use this equation to get predicted values for new car sales, using the new car price from in the sample or outside it



Actual Sales and Ex Post Forecast
12000
10000
8000
6000
4000
2000
0
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991

Residuals and Forecast Errors

- In the terminology of econometrics, *ex post* forecast errors are called residuals
- The OLS estimator is designed to minimize the sum of the squared residuals - OLS estimates by definition minimize MSE and $RMSE$
- To find value of MSE, look on the ANOVA table, for the row labeled Residual and under the column labeled SS
- Analysis of Variance Tables

Source of Variation	**Degrees of Freedom**	**Sum of Squares**	**Mean Square**	**F**
Explained	$K-1$	$SSR=\sum(\hat{y}_t-\overline{y})^2$	$\frac{SSR}{K-1}$	$\frac{\mathbf{b_s}'[c\hat{o}v(\mathbf{b_s})]^{-1}\mathbf{b_s}}{K-1}$
Unexplained	$T-K$	$SSE=\sum\hat{u}_t^2$	$\frac{SSE}{T-K}$	
Total	$T-1$	$SST=\sum(y_t-\overline{y})^2$	$\frac{SST}{T-1}$	

Regression Output

SUMMARY OUTPUT			
Regression Statistics			
Multiple R	0.59		
R Square	0.35		
Adjusted R Square	0.31		
Standard Error	1013.7142		
Observations	20		
ANOVA			
	df	*SS*	*MS*
Regression	1	9794932.261	9794932
Residual	18	18497098.29	1027617
Total	19	28292030.55	
	Coefficients	*Standard Error*	*t Stat*
Intercept	10200.23	887.95	11.49
X Variable 1	-30.2750	9.8062	-3.09

Interval Forecasts

- Note that the point forecasts based on the regression results do not reflect that fact that these estimates are random variables - can also use information about the statistical precision of these estimates in forecasts
- *Interval forecasts*: Indicates a numerical interval in which we expect the actual value will fall, along with some probability of this event occurring
- Analogous to confidence intervals
- Suppose that the true relationship between X, the explanatory variable and Y the dependent variable to be forecast is

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

- This is a *population* relationship and is not observable - instead use OLS to estimate

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_t + e_t$$

- Note the change from u_t to e_t - equation error to residual

Regression Forecasts

- Suppose the estimated parameters ($\hat{\beta}_0$ and $\hat{\beta}_1$) are used to forecast Y k periods in the future (where $k > t$)

$$Y_k^* = \hat{\beta}_0 + \hat{\beta}_1 X_k$$

- And the actual value of Y in period k is

$$Y_k = \beta_0 + \beta_1 X_k + u_k$$

- The forecast error is

$$FE_k = Y_k^* - Y_k = \hat{\beta}_0 + \hat{\beta}_1 X_k - \beta_0 + \beta_1 X_k$$
$$FE_k = (\hat{\beta}_0 - \beta_0) + (\hat{\beta}_1 - \beta_1) X_k - u_k$$

- The variance of the forecast is then

$$var(Y_k^*) = E[(Y_k^* - Y_k)^2] = var(\hat{\beta}_0) + var(\hat{\beta}_1) X_k^2 + 2cov(\hat{\beta}_0, \hat{\beta}_1) + var(u_k)$$

Prediction Interval

- Can manipulate the expression for the variance to get

$$var(Y_k^*) = \sigma_u^2 \left[1 + \frac{1}{n} + \frac{(X_k - \overline{X})^2}{\sum x^2}\right]$$

- Can also make a probability statement

$$P[Y_k^* + t_{\epsilon/2} \times \sqrt{var(Y_k^*)} > Y_k > Y_k^* - t_{\epsilon/2} \times \sqrt{var(Y_k^*)} = 1 - \epsilon$$

- ϵ is the "Critical Value" - the probability of Y_k falling in the interval
- Can simply manipulate this to get the confidence interval for prediction

$$(\hat{\beta}_0 + \hat{\beta}_1 X_k) \pm t_{\epsilon/2}\sigma_e \sqrt{\left[1 + \frac{1}{n} + \frac{(X_k - \overline{X})^2}{\sum x^2}\right]}$$

- Given the same ϵ, a better forecast gives a smaller interval
- Given the same interval, a better forecast has a higher confidence level

Some Useful Concepts

- **Ex Post Forecast**: Extrapolation goes beyond sample period but not into future
- *Example*: Sample period for regression is 1970-1997, forecast through 2000
- **Ex Ante Forecast**: Extrapolation extends into future
- *Example*: Sample period is 1990:1-2001:1, forecast through 2002:1
- **Conditional Forecasts**: Some contemporaneous explanatory variables appear on RHS, must also predict values for these contemporaneous explanatory variables
- **Unconditional Forecasts**: Only lagged explanatory variables appear on RHS
- **Point Forecast**: Predicts a single number
- *Example*: The Dow will be 1100 on July 1
- **Interval Forecast**: Shows a numerical interval in which the actual value can be expected to fall
- *Example*: The Dow will be between 1000 and 2000 on July 1 with 99% probability

Autoregressive Models

- Even though they use sophisticated statistical techniques, these models are extrapolations just like the naive and moving average methods
- Regression models where the explanatory variables (Xs) are lagged values of the dependent variable
- Assumes that the time path of a variable is self-generating, also called the "Chain Principle"
- Functional forms

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-3}, \ldots, \beta_1, \beta_2, \beta_3, \ldots, u_t)$$

  where u_t: residual term, captures random components
- Must specify form and lag length
- Linear form, lag length k

$$X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ + \beta_k X_{t-k} + u_t$$

- Note that both No Change and Same Change naive forecasts are special cases of this model

AR Models: Determining Lag Length

- The general form has an infinite number of parameters, but we never have this much data - model must be restricted to be useful for forecasting
- Assume that the impact of some distant X_{t-j} are trivial and insignificant so they can be ignored
- *Rule of thumb*: don't use a $k > 4$ because of econometric problems
- *Pro*: Easy to implement, needs no advanced theories or statistical techniques
- *Pro*: Easy to do on a computer, can generate fluctuations
- *Con*: Ignores theory and causality and interrelations - does not describe X, only reproduces it

Forecasting: Regressions with Dummy Variables

- Requires no additional economic data to implement in a regression model
- Ad hoc in that there is no theory underlying these factors, but it can be used to improve forecasting ability
- Topic was discussed in chapter 2 in a different context
- Two Types:

1. *Trend*: Assumes constant period to period change
2. *Seasonal / annual*: Assumes constant effects by month/quarter/year

Dummy Variables: Trends

- Uses a time variable T_t $(= 1, 2, 3, \ldots)$ as an explanatory variable and extrapolates X along its time path
- Several different possible functional forms for trend variables
- Linear: $X_t = \alpha + \beta T_t$
- Exponential: $X_t = e^{\alpha + \beta T_t}$
- Reciprocal: $X_t = \frac{1}{\alpha + \beta T_t}$
- Parabolic: $X_{=}\beta_0 + \beta_1 T_t, + \beta_2 T_t^2$

Dummy Variables: Seasonal

- "Intercept shifters" - they allow the intercept term b_0 to vary systematically in a regression model
- Single Equation Model with Quarterly Dummies:

$$Y_t = \gamma_1 Q_1 + \gamma_2 Q_2 + \gamma_3 Q_3 + \gamma_4 Q_4 + \beta_1 X_{1t} + \ldots + \beta_k X_{1k} + u_t$$

- Allows for the intercept to differ for each quarter
- Note no constant term in model - must avoid "dummy variable trap"
- Can also use monthly dummies if Y is monthly
- Xs can include trends, explanatory variables, etc.
- Model permits a different forecast value for each quarter

Other Dummy Variables

- Dummy variables can be useful tools in forecasting
- Recall from the earlier section that the single equation forecast for new car sales was high for 1991 because it was a recessionary year
- Can use a dummy variable for recessions to improve this forecast
- *Example*: Recession Dummy
- Model: $Yt = \alpha + \beta X_t + \gamma D_R + u_t$
- Y: Automobile Sales, X: New Car Price, $_R$: Recession Dummy, $= 1$ in years with troughs