Simulation Methods

Su, Chapter 9

Chapter Goals

1. Understand how to use simulation techniques in macroeconomic models
2. Understand the terminology used in economic simulation analysis
3. Understand how Samuelson's accelerator-multiplier model generates cycles
4. Understand the Adelman and Adelman approach to simulating the Klein-Glodberger model

Why are Simulations Important?

- We have studied formal macroeconomic models which explain the behavior of the economy
- These models have been used to explain many important macroeconomic phenomena like the use of monetary and fiscal policy
- Business cycles are important economic events
- Clearly need to know more about the dynamic properties of these models to asses them

Dynamic Models

- Recall that the IS/LM model of chapter 5 was static without the Phillips Curve
- Static models cannot be used to simulate business cycles because they do not change over time. Must modify static models to generate dynamics
- Must add lagged endogenous variables somewhere (Y_{t-1} or something similar)

Impulse vs. Propagation

- When working with dynamic models, must understand this distinction
  - **Impulse**: A robust disturbance that is external to the model
  - **Propagation**: The reverberation through the model that amplifies any internal or external imbalances and causes fluctuations
- "Ringing a bell"

General Procedure

Step 1: Formulate or identify an appropriate dynamic economic model for the question

Step 2: Find/solve for the reduced form equations for that model

Step 3: Code the reduced forms into a spreadsheet

Step 4: Conduct experiments

Step 5: Observe the effects of the experiments on the endogenous variables

Endogenous and Exogenous Variables

- **Endogenous Variables**: Determined inside the model
- **Exogenous Variables**: Determined outside the model
- **Structural Equations**: Have endogenous variables on the right hand side. Most economic models are presented as structural equations
- **Reduced Form Equations**: Have only exogenous and lagged endogenous variables on the right hand side

Simulation 1: Samuelson (1939)

- Based on a paper by Paul Samuelson - "Interactions Between the Multiplier Analysis and the Principle of Acceleration," *Review of Economics and Statistics*," vol. 21, May 1939, pp. 75-78

- Goals:

  1. To use a multiplier/accelerator process to explain business cycles using a macroeconomic model
  2. To see if the cyclical nature of such a model responded to structural changes

- Model

$$\begin{aligned} C &= \alpha_1 + \beta_1 Y \\ I &= \alpha_2 + \beta_2 \Delta Y + \gamma_2 R \\ Y &\equiv C + I + G \end{aligned}$$

where β_1 is the Marginal Propensity to Consume, β_2 is the Accelerator and note that the model is dynamic, as it contains lagged Y

Simplifications

- To model:

$$\begin{aligned} C &= \beta_1 Y_{t-1} \\ I &= \beta_2 \Delta C_t = \beta_2 (C_t - C_{t-1}) \\ Y &\equiv C + I + G \end{aligned}$$

- For simplicity: $\alpha_1 = \alpha_2 = 0$
- No Money Market: $\alpha_2 = 0$
- Investment function changed to focus on accelerator and simplify solution

Solution: Reduced Form Equation

- $Y_t = G_t + \beta_1[1 + \beta_2]Y_{t-1} - \beta_1\beta_2 Y_{t-2}$
- Note that RHS has only parameters, exogenous variables and lagged endogenous variables
- Second-order linear difference equation, contains Y_{t-1}, Y_{t-2}
- **Exogenous Variable**: G_t
- **Endogenous Variables**: Y, I, C

Reproduce Table 9.1

- Spreadsheet with 5 columns, 15 rows
- *Column 1*: Time
- *Column 2*: G
- *Column 3*: C
- *Column 4*: I
- *Column 5*: Y

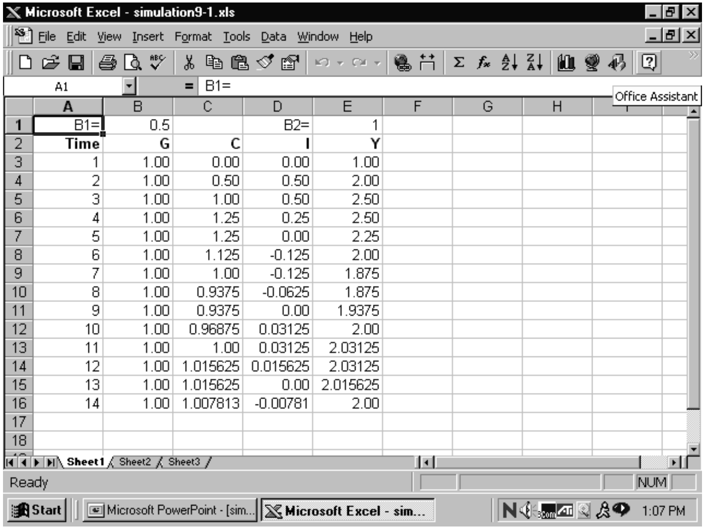
Spreadsheet Set-up

b1=		b2=			
Time	G	C	I	Y	
1	1.00	0.00	1.00	1.00	
2	1.00				
3	1.00				
4	1.00				
5	1.00				
6	1.00				
7	1.00				
8	1.00				
9	1.00				
10	1.00				
11	1.00				
12	1.00				
13	1.00				

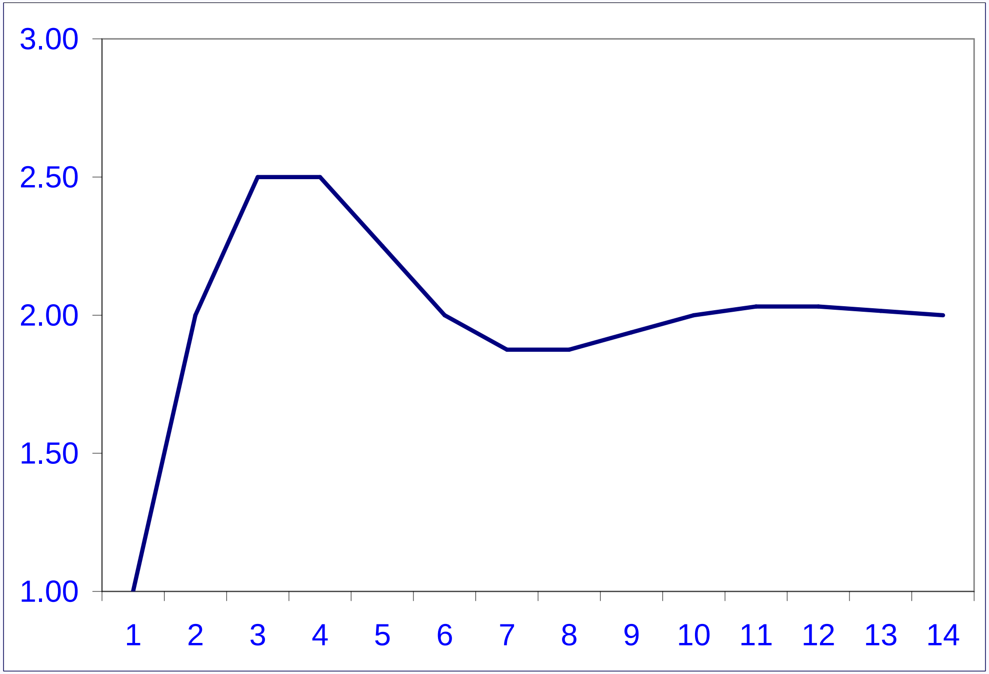
Formulas for Period 2

- C_2: Depends on β_1 and Y_1
- I_2: Depends on β_2, C_1 and C_2
- Y_2: Depends on β_1, β_2, Y_1 and Y_0
- *Hint*: Set $Y_0 = 0$
- If set up correctly, can just copy formulas to next row

Final Set-up



	A	B	C	D	E
1	B1=	0.5		B2=	1
2	Time	G	C	I	Y
3	1	1.00	0.00	0.00	1.00
4	2	1.00	0.50	0.50	2.00
5	3	1.00	1.00	0.50	2.50
6	4	1.00	1.25	0.25	2.50
7	5	1.00	1.25	0.00	2.25
8	6	1.00	1.125	-0.125	2.00
9	7	1.00	1.00	-0.125	1.875
10	8	1.00	0.9375	-0.0625	1.875
11	9	1.00	0.9375	0.00	1.9375
12	10	1.00	0.96875	0.03125	2.00
13	11	1.00	1.00	0.03125	2.03125
14	12	1.00	1.015625	0.015625	2.03125
15	13	1.00	1.015625	0.00	2.015625
16	14	1.00	1.007813	-0.00781	2.00


3.00
2.50
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14

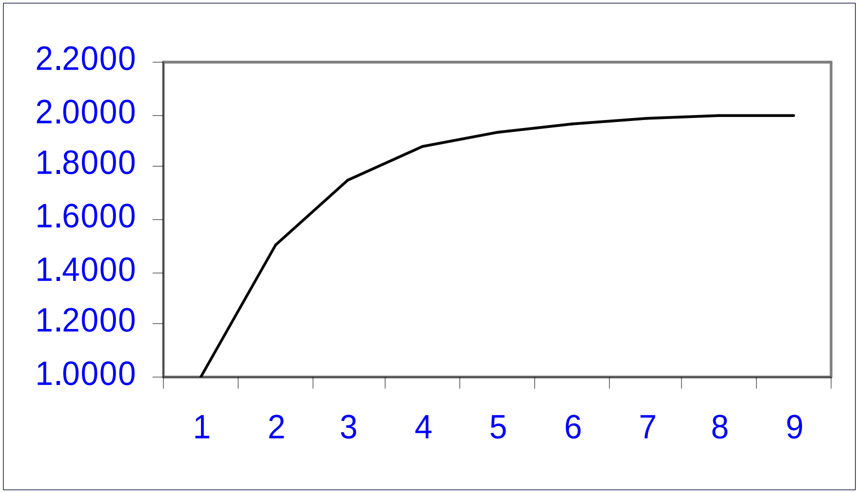
Implications of Simulation

- Time path for Y_t is cyclical even though there is no impulse
- Starts at 1.0 and converges to 2.0 without any external intervention - Propagation is stable in this model
- What mechanism drives Y_t to 2.0?

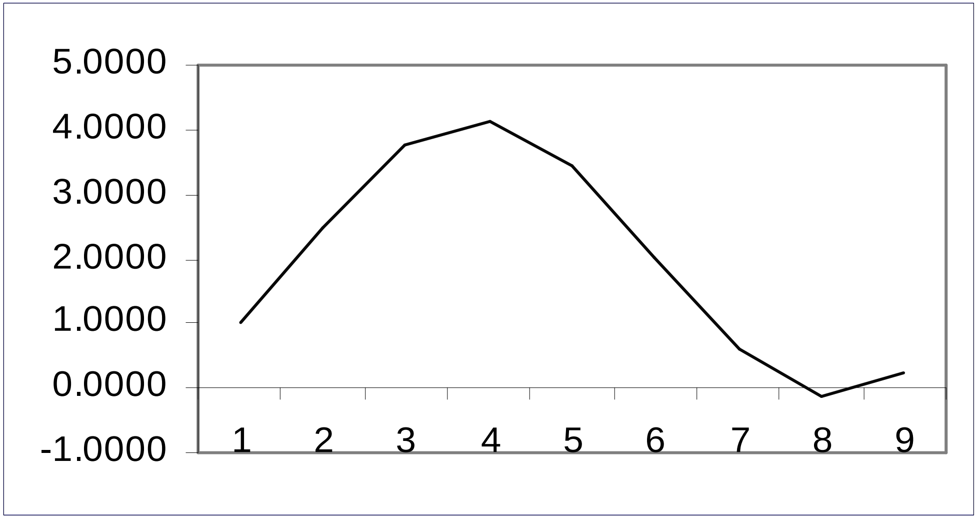
Reproducing Table 9.2

- Lets see what happens when the parameters are changed
- Can be done simply by changing the values in cell $C1$ and $E1$
- After changing the parameter values, copy the first 9 values for Y to another location.
- Use “Copy Special …” $< Values >$

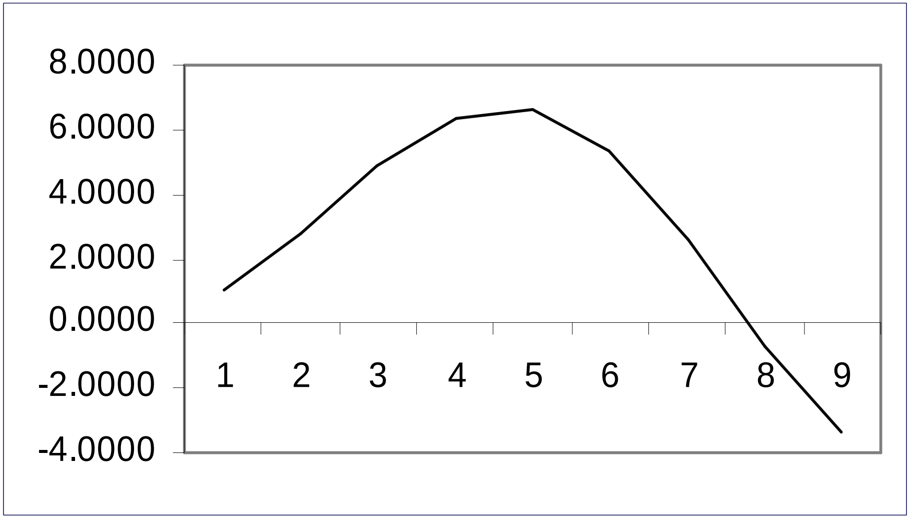
$\beta_1 = 0.5 \ \beta_2 = 0.0$



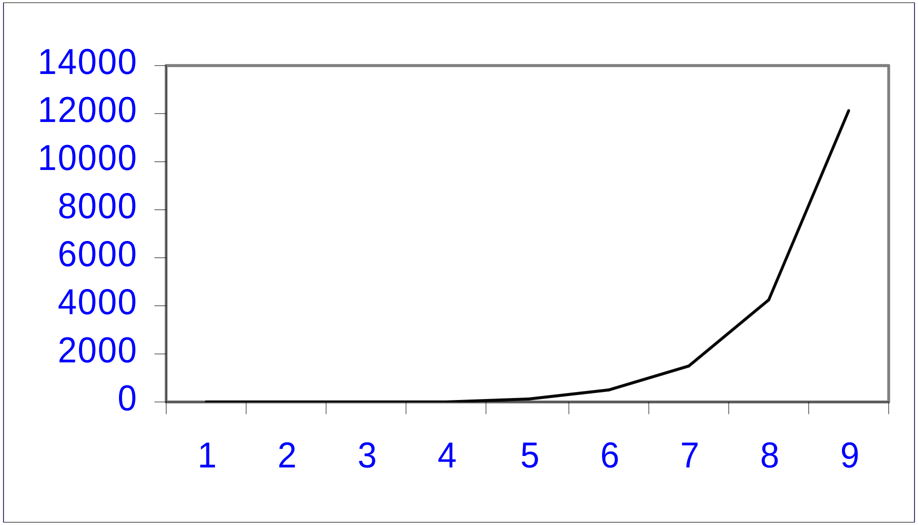
$\beta_1 = 0.5 \ \beta_2 = 2.0$



$\beta_1 = 0.6 \ \beta_2 = 2.0$



$$\beta_1 = 0.8 \; \beta_2 = 4.0$$



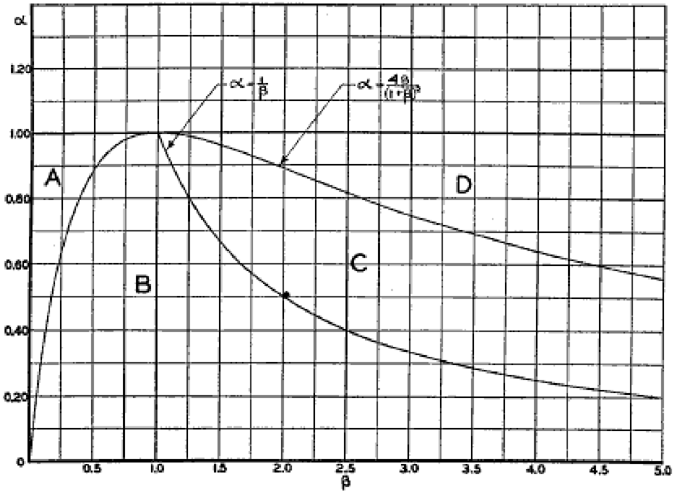
Conclusions from Parameter Variation Exercise

- The behavior of the time path for Y depends on the values for the parameters in the model
- Four cases emerge:
  1. Monotonic Convergence
  2. Damped Oscillation
  3. Explosive Oscillation
  4. Monotonic Explosive
- Which is closest to the behavior of GDP?
- This sort of behavior is common to all second order difference equations, which can be used to describe cyclical behavior

Second-order Linear Difference Equations

- Has two *characteristic roots* (the solution to the difference equation) and the behavior of the time path of the variable is governed by the relative size of these roots
- Samuelson worked this relationship out, summarized on Figure 9.3 (which has a typo - should read $\beta_1 = \frac{1}{\beta_2}$)
- The cyclical behavior of Y in the simulations depends on the values of β_1 and β_2,
- **Region** A: Values of any of these combinations of β_1 and β_2 lead to *monotonic convergent* time paths for Y_t
- **Region** B: Values of any of these combinations of β_1 and β_2 lead to *damped oscillation* time paths for Y_t
- **Region** C: Values of any of these combinations of β_1 and β_2 lead to *explosive oscillation* time paths for Y_t
- **Region** D: Values of any of these combinations of β_1 and β_2 lead to *monotonic explosive* time paths for Y_t

Chart 2.—Diagram Showing Boundaries of Regions Yielding Different Qualitative Behavior of National Income



Dynamic Models and Dynamic Economies

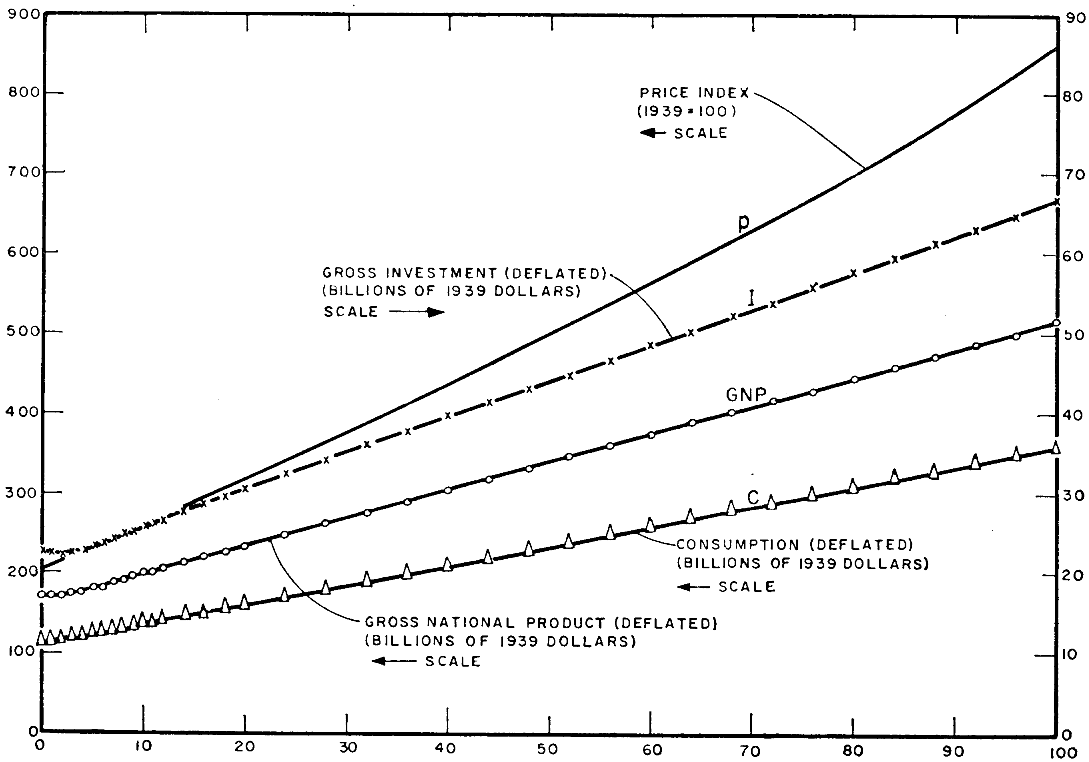
- What is the main point of this exercise?
- How do dynamic models relate to the economy?
- What can we learn from dynamic models?
- Do these cycles look like the historical business cycle?

Methods with Random Shocks

- As computing technology advanced in the 1950s, economists began building large macro-econometric models
- The *Klein-Goldberger* model was one prominent example, contained 25 equations including behavioral equations and accounting identities, used parameters values estimated from data using econometric techniques, was most complete business cycle model of the time
- The adequate reproduction of the historical business cycle behavior has been an important part of economic research in this area - seen as a method of validating models
- Adelman and Adelman (*Econometrica*, vol. 27, October 195, pp. 596-625) combined the Klein-Goldberger Model with perturbation analysis to determine how business cycles were generated in the past

Klein-Goldberger Model Simulation

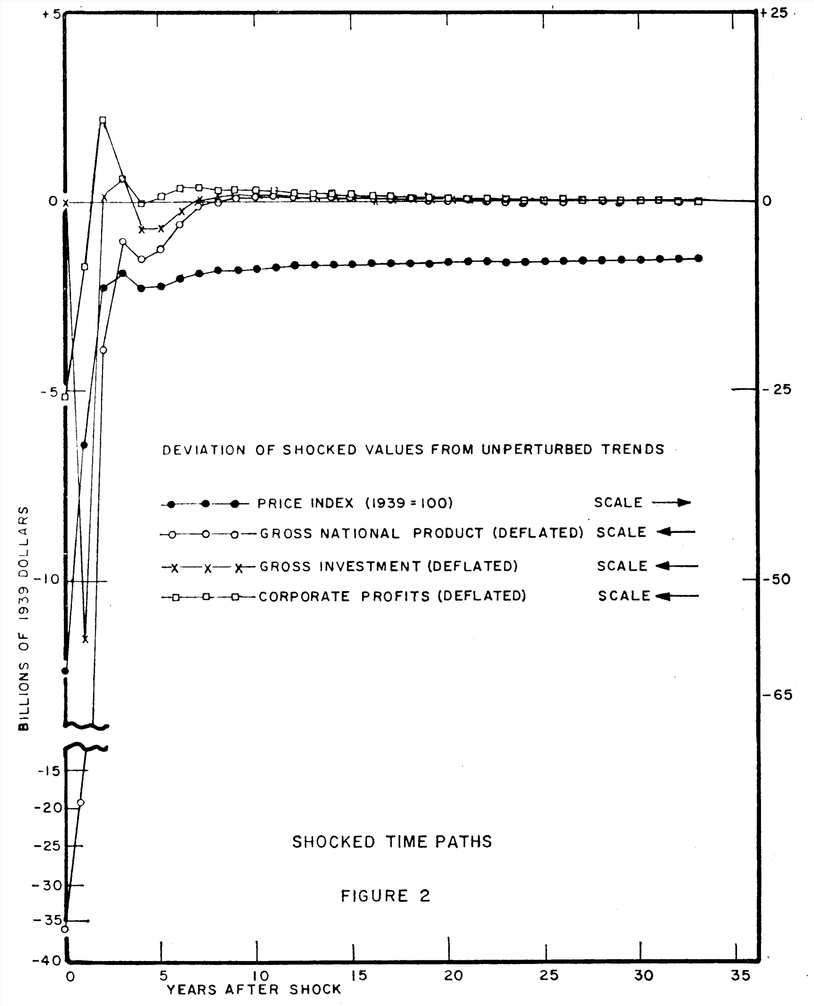
- Was a short-run predictive system, not designed to model growth or long-run time paths of variables
- Similar to IS/LM model, but with more equations, the model was dynamic but the cyclical properties of the model were unclear
- In order to investigate the dynamic properties, they looked at a 100 year time-path of the key endogenous variables - basically simulated the time paths like our lab exercises
- Only a brief "settling down period" before the model showed monotonic convergence, no internally generated cycles



KLEIN-GOLDBERGER TIME PATHS
FIGURE 1

Tests of Stability

- In order to explore the stability of the model, used a 100 period baseline simulation
- Model was "heavily shocked" in period 9: Introduced a large change in an exogenous variable in order to see if this led to divergent behavior - shock was a decrease in government spending of 75% that lasted 1 year before returning to the pre-shock level in period 10
- Shock caused an immediate depression in period 9
- Model immediately moved to recovery, after a few strong oscillations
- Took model 24 periods to converge back to pre-shock values
- Showed that the model was stable - a large change in an exogenous variable (a *shock*) did not lead to divergent behavior



SHOCKED TIME PATHS

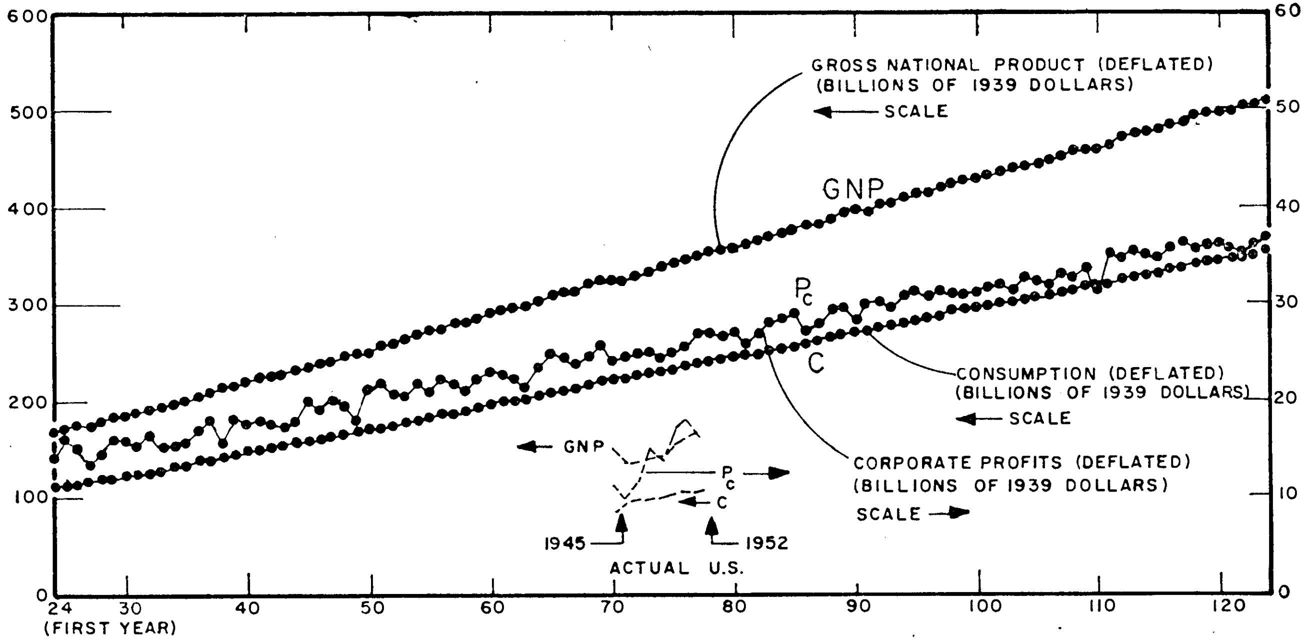
FIGURE 2

Experiments

- Assessed the ability of the model to replicate the US business cycle by introducing **random shocks** to the model - these were generated by a random number generation process
- In these experiments, each shock is determined by a random factor determining when it starts and by an exogenous value for the mean and variance of the random shock
- Note that Russian economist Eugene Slutsky had suggested in the 1930s that business cycles might be due to random shocks - this procedure formalized Slutsky's conjecture
- *Research Question*: does the introduction of uncorrelated random shocks generate business cycles in the Klein-Goldberger Model?

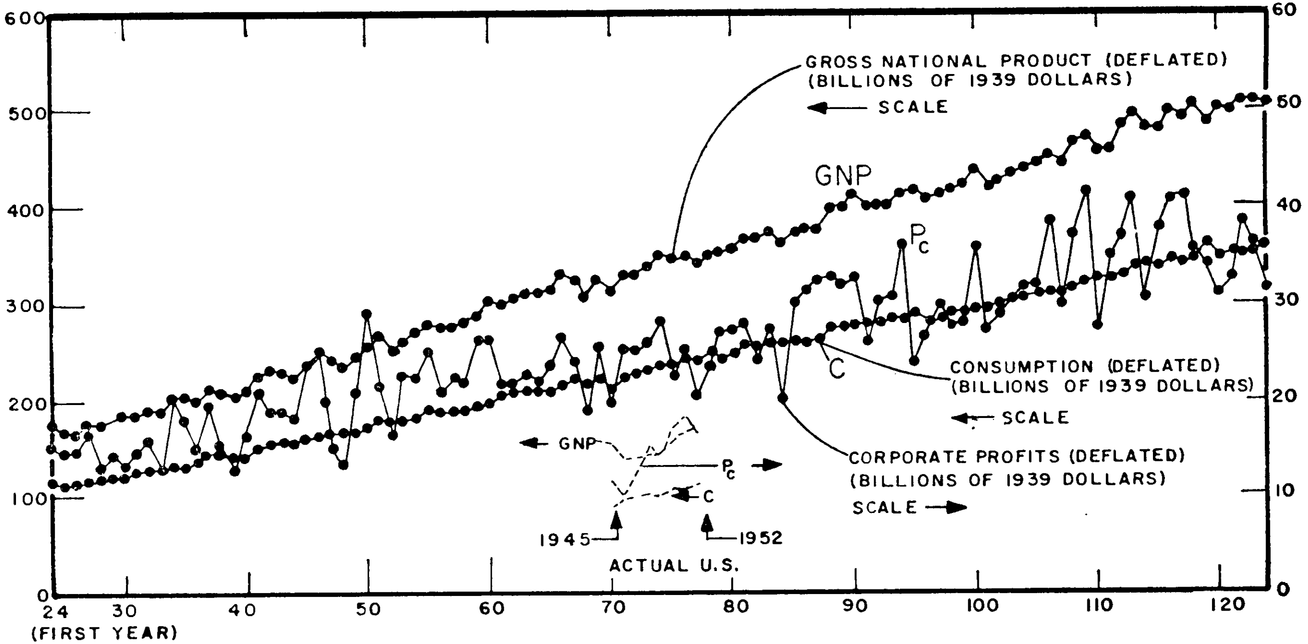
Types of Shocks

- **Type I Shocks**: Normally distributed with mean zero and constant variance, applied to exogenous variables in the model; variance was calibrated to match the historical variance of the exogenous variables; added to the trend growth of these variables
- **Type II Shocks**: Applied to all endogenous variables (behavioral relationships), magnitude of the shocks were determined by the disturbance variance in each reduced form equation ($\hat{\sigma}^2$), mean of shocks was zero
- Why put Type II shocks in model
- Inherent randomness in human behavior not captured in structural (behavioral) model



SELECTED TIME PATHS UNDER TYPE I IMPULSES

FIGURE 3



SELECTED TIME PATHS UNDER TYPE II IMPULSES

FIGURE 4

Results

- Type I shocks did not produce cycles that looked like the historical business cycle record of the US economy - too small and infrequent
- Type II shocks produced cycles that looked like the historical business cycle record of the US economy - periods of 3-4 years, relatively large amplitudes
- The behavioral relationships in the model are inexact representations of the underlying economic behavior
- Type II shocks introduce an element of unpredictability into the model and these shocks best resemble the actual business cycle
- *Implication*: As long as randomness exists in human behavior, business cycles will exist