Applications of Econometric Models I

Su, Chapter 8, Section IV

Goals

1. Make IS/LM Model Dynamic
2. Understand dynamic properties of macroeconomic models
3. Use Excel as a simulation tool
4. Model taken from “Forecasting and Analysis with an Econometric Model,” Daniel B. Suits, *American Economic Review*, March 1962, pp. 104-132
5. Four equation econometric model, parameters come from econometric estimates

Structural Model

- Four equation model: Income Identity, Consumption Function, Investment Function, Tax Function

$$Y \equiv C + I + G \tag{8.2a}$$

$$C = \alpha_1 + \beta_1(Y - T) \tag{8.2b}$$

$$I = \alpha_2 + \beta_2 Y_{-1} + \gamma_2 R \tag{8.2c}$$

$$T = \alpha_3 + \beta_3 Y \tag{8.2d}$$

- **Exogenous Variables**: G, R
- **Endogenous Variables**: Y, C, I, T
- **Parameters**: α_1, α_2, α_3, β_1, β_2, β_3, γ_2
- No money market, R exogenous

Parameterized Structural Model

- Obtained by statistical techniques - data were obtained and these parameters were estimated by regression methods

$$Y \equiv C + I + G \tag{8.3a}$$

$$C = 16 + 0.7(Y - T) \tag{8.3b}$$

$$I = 6 + 0.1Y_{-1} - 0.3R \tag{8.3c}$$

$$T = 0.0 + 0.2Y \tag{8.3d}$$

$$\begin{array}{llll} (C) & \alpha_1 = 16 & \beta_1 = 0.7 & \\ (I) & \alpha_2 = 6 & \beta_2 = 0.1 & \gamma_2 = -0.3 \\ (T) & \alpha_3 = 0.0 & \beta_3 = 0.2 & \end{array}$$

Reduced Form Equations

- The "solution" to a structural model is called "reduced form equations"

$$Y = 50 + 0.2273Y_{-1} - 0.6818R + 2.2727G \quad (8.4a)$$
$$C = 44 + 0.1273Y_{-1} - 0.3818R + 1.2727G \quad (8.4b)$$
$$I = 6 + 0.01Y_{-1} - 0.03R \quad (8.4c)$$
$$T = 10 + 0.0455Y_{-1} - 0.1364R + 0.4545G \quad (8.4d)$$

- The numbers are reduced form parameters
- Note that an explicit reduced form equation for Y has been solved for
- First-order linear difference equations
- Endogenous on Right Hand Side, Exogenous on Left Hand Side

Reduced Form Equations

- Note that the general form of a reduced form model is

$$\begin{aligned} Y &= a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G \\ C &= a_{20} + a_{21}Y_{-1} + a_{22}R + a_{23}G \\ I &= a_{30} + a_{31}Y_{-1} + a_{32}R + a_{33}G \\ T &= a_{40} + a_{41}Y_{-1} + a_{42}R + a_{43}G \end{aligned}$$

- One equation for each endogenous variable
- A constant parameter for each reduced form equation
- One reduced form parameter for each endogenous variable

Finding the Reduced Form Equation for Y

$$
\begin{aligned}
Y &\equiv C + I + G \\
Y &= \alpha_1 + \beta_1(Y - \alpha_3 - \beta_3 Y) + \alpha_2 + \beta_2 Y_{-1} + \gamma_2 R + G \\
Y &= \alpha_1 + \alpha_2 + \beta_1 Y - \beta_1\alpha_3 + \beta_1\beta_3 Y + \beta_2 Y_{-1} + \gamma_2 R + G \\
Y &= \alpha_1 + \alpha_2 - \beta_1\alpha_3 + \beta_1 Y + \beta_1\beta_3 Y + \beta_2 Y_{-1} + \gamma_2 R + G \\
Y &= \alpha_1 + \alpha_2 - \beta_1\alpha_3 + (\beta_1 + \beta_1\beta_3)Y + \beta_2 Y_{-1} + \gamma_2 R + G
\end{aligned}
$$

$$
\begin{aligned}
Y - (\beta_1 + \beta_1\beta_3)Y &= \alpha_1 + \alpha_2 - \beta_1\alpha_3 + \beta_2 Y_{-1} + \gamma_2 R + G \\
(1 - \beta_1 + \beta_1\beta_3)Y &= \alpha_1 + \alpha_2 - \beta_1\alpha_3 + \beta_2 Y_{-1} + \gamma_2 R + G \\
Y &= \frac{\alpha_1 + \alpha_2 - \beta_1\alpha_3 + \beta_2 Y_{-1} + \gamma_2 R + G}{1 - \beta_1 + \beta_1\beta_3}
\end{aligned}
$$

$$
\begin{aligned}
Y &= \frac{\alpha_1 + \alpha_2 - \beta_1\alpha_3}{1 - \beta_1 + \beta_1\beta_3} + \frac{\beta_2}{1 - \beta_1 + \beta_1\beta_3} Y_{-1} + \frac{\gamma_2}{1 - \beta_1 + \beta_1\beta_3} R + \frac{1}{1 - \beta_1 + \beta_1\beta_3} G \\
Y &= a_{10} + a_{11} Y_{-1} + a_{12} R + a_{13} G
\end{aligned}
$$

Verification of Numeric Solution

$$\begin{array}{llll} (C) & \alpha_1 = 16 & \beta_1 = 0.7 & \\ (I) & \alpha_2 = 6 & \beta_2 = 0.1 & \gamma_2 = -0.3 \\ (T) & \alpha_3 = 0.0 & \beta_3 = 0.2 & \end{array}$$

$$Y = \frac{\alpha_1 + \alpha_2 - \beta_1 \alpha_3}{1 - \beta_1 + \beta_1 \beta_3} + \frac{\beta_2}{1 - \beta_1 + \beta_1 \beta_3} Y_{-1} + \frac{\gamma_2}{1 - \beta_1 + \beta_1 \beta_3} R + \frac{1}{1 - \beta_1 + \beta_1 \beta_3} G$$

$$Y = \frac{16 + 6 - 0.7 \times 0.0}{1 - 0.7 + 0.7 \times 0.2} + \frac{0.1}{1 - 0.7 + 0.7 \times 0.2} Y_{-1} + \frac{-0.3}{1 - 0.7 + 0.7 \times 0.2} R + \frac{1}{1 - 0.7 + 0.7 \times 0.2} G$$

$$Y = \frac{16 + 6}{1 - 0.7 + 0.14} + \frac{0.1}{1 - 0.7 + 0.14} Y_{-1} + \frac{-0.3}{1 - 0.7 + 0.14} R + \frac{1}{1 - 0.7 + 0.14} G$$

$$Y = \frac{22}{0.44} + \frac{0.1}{0.44} Y_{-1} + \frac{-0.3}{0.44} R + \frac{1}{0.44} G$$

$$Y = 50 + 0.2273 Y_{-1} - 0.6818 R + 2.2727 G$$

Finding the Reduced Form Equation for T

$$Y = \frac{\alpha_1 + \alpha_2 - \beta_1\alpha_3}{1 - \beta_1 + \beta_1\beta_3} + \frac{\beta_2}{1 - \beta_1 + \beta_1\beta_3} Y_{-1} + \frac{\gamma_2}{1 - \beta_1 + \beta_1\beta_3} R + \frac{1}{1 - \beta_1 + \beta_1\beta_3} G$$

$$\begin{aligned}
T &= \alpha_3 + \beta_3 Y \\
T &= \alpha_3 + \beta_3 \left(\frac{\alpha_1 + \alpha_2 - \beta_1\alpha_3}{1 - \beta_1 + \beta_1\beta_3} + \frac{\beta_2}{1 - \beta_1 + \beta_1\beta_3} Y_{-1} + \frac{\gamma_2}{1 - \beta_1 + \beta_1\beta_3} R + \frac{1}{1 - \beta_1 + \beta_1\beta_3} G \right) \\
T &= \alpha_3 + \beta_3 \left(a_{10} + a_{11} Y_{-1} + a_{12} R + a_{13} G \right) \\
T &= \alpha_3 + \frac{\beta_3\alpha_1 + \beta_3\alpha_2 - \beta_1\alpha_3\beta_3}{1 - \beta_1 + \beta_1\beta_3} + \frac{\beta_3\beta_2}{1 - \beta_1 + \beta_1\beta_3} Y_{-1} + \frac{\beta_3\gamma_2}{1 - \beta_1 + \beta_1\beta_3} R + \frac{\beta_3}{1 - \beta_1 + \beta_1\beta_3} G \\
T &= \alpha_3 + \beta_3 a_{10} + \beta_3 a_{11} Y_{-1} + \beta_3 a_{12} R + \beta_3 a_{13} G \\
T &= a_{40} + a_{41} Y_{-1} + a_{42} R + a_{43} G
\end{aligned}$$

Finding the Reduced Form Equation for I

$$
\begin{aligned}
I &= \alpha_2 + \beta_2 Y_{-1} + \gamma_2 R \\
I &= a_{30} + a_{31} Y_{-1} + a_{32} R + a_{33} G
\end{aligned}
$$

- Note that $a_{33} = 0$

Finding the Reduced Form Equation for C

$$
\begin{aligned}
C &= \alpha_1 + \beta_1(Y - T) \\
Y - T &= a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G - (\alpha_3 + \beta_3 a_{10} + \beta_3 a_{11}Y_{-1} + \beta_3 a_{12}R + \beta_3 a_{13}G) \\
\alpha_3 &= 0 \\
Y - T &= a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G - (\beta_3 a_{10} + \beta_3 a_{11}Y_{-1} + \beta_3 a_{12}R + \beta_3 a_{13}G) \\
Y - T &= a_{10} - \beta_3 a_{10} + a_{11}Y_{-1} - \beta_3 a_{11}Y_{-1} + a_{12}R - \beta_3 a_{12}R + a_{13}G - \beta_3 a_{13}G \\
Y - T &= (1 - \beta_3)(a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G)
\end{aligned}
$$

$$
\begin{aligned}
C &= \alpha_1 + \beta_1((1 - \beta_3)(a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G)) \\
C &= \alpha_1 + (\beta_1 - \beta_1\beta_3)(a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G) \\
C &= \alpha_1 + (\beta_1 - \beta_1\beta_3)\left(\frac{\alpha_1 + \alpha_2 - \beta_1\alpha_3}{1 - \beta_1 + \beta_1\beta_3} + \frac{\beta_2}{1 - \beta_1 + \beta_1\beta_3}Y_{-1}\right. \\
&\quad \left. + \frac{\gamma_2}{1 - \beta_1 + \beta_1\beta_3}R + \frac{1}{1 - \beta_1 + \beta_1\beta_3}G\right) \\
C &= \alpha_1 + \left(\frac{\beta_1 - \beta_1\beta_3}{1 - \beta_1 + \beta_1\beta_3}\right)(\alpha_1 + \alpha_2 - \beta_1\alpha_3 + \beta_2 Y_{-1} + \gamma_2 R + G)
\end{aligned}
$$

A Practical Reduced Form Equation for C

$$
\begin{aligned}
C &= \alpha_1 + (\beta_1 - \beta_1\beta_3)(a_{10} + a_{11}Y_{-1} + a_{12}R + a_{13}G) \\
C &= \alpha_1 + (\beta_1 - \beta_1\beta_3)(a_{10}) + (\beta_1 - \beta_1\beta_3)a_{11}Y_{-1} + (\beta_1 - \beta_1\beta_3)a_{12}R + (\beta_1 - \beta_1\beta_3)a_{13}G
\end{aligned}
$$

- The second term in parentheses in the first equation is the reduced form equation for Y
- this can be used to find the reduced form equation for C

Recap - Reduced Form Equations

$$
\begin{array}{rcllll}
Y & = & \frac{\alpha_1+\alpha_2-\beta_1\alpha_3}{1-\beta_1+\beta_1\beta_3} & +\frac{\beta_2}{1-\beta_1+\beta_1\beta_3}Y_{-1} & +\frac{\gamma_2}{1-\beta_1+\beta_1\beta_3}R & +\frac{1}{1-\beta_1+\beta_1\beta_3}G
\end{array}
$$

$$
\begin{array}{rcllll}
Y & = & a_{10} & +a_{11}Y_{-1} & +a_{12}R & +a_{13}G \\
T & = & \alpha_3 + \beta_3 a_{10} & +\beta_3 a_{11}Y_{-1} & +\beta_3 a_{12}R & +\beta_3 a_{13}G \\
I & = & \alpha_2 & +\beta_2 Y_{-1} & +\gamma_2 R & \\
C & = & \alpha_1 + (\beta_1 - \beta_1\beta_3)(a_{10}) & +(\beta_1 - \beta_1\beta_3)a_{11}Y_{-1} & +(\beta_1 - \beta_1\beta_3)a_{12}R & +(\beta_1 - \beta_1\beta_3)a_{13}G
\end{array}
$$

$$
\begin{array}{lllll}
(Y) & a_{10} = \frac{\alpha_1+\alpha_2-\beta_1\alpha_3}{1-\beta_1+\beta_1\beta_3} & a_{11} = \frac{\beta_2}{1-\beta_1-\beta_1\beta_3} & a_{12} = \frac{\gamma_2}{1-\beta_1+\beta_1\beta_3} & a_{13} = \frac{1}{1-\beta_1+\beta_1\beta_3} \\
(C) & a_{20} = \alpha_1 + (\beta_1 - \beta_1\beta_3)(a_{10}) & a_{21} = (\beta_1 - \beta_1\beta_3)a_{11} & a_{22} = (\beta_1 - \beta_1\beta_3)a_{12} & a_{23} = (\beta_1 - \beta_1\beta_3)a_{13} \\
(I) & a_{30} = \alpha_2 & a_{31} = \beta_2 & a_{32} = \gamma_2 & a_{33} = 0 \\
(T) & a_{40} = \alpha_3 + \beta_3 a_{10} & a_{41} = \beta_3 a_{11} & a_{42} = \beta_3 a_{12} & a_{43} = \beta_3 a_{13}
\end{array}
$$

Multipliers

- The model is dynamic which complicates the analysis of the effects of changes in the exogenous variables, must distinguish between short-term impact of changes in the exogenous variables and long-term impact
- **Short-term multiplier**: Reflects the one-period effect of a change in an exogenous variable on some endogenous variable
- The *Short-term multiplier* is simply the reduced form parameter on each exogenous variable
- **Long-term multiplier**: Cumulative impact of a change in an exogenous variable on some endogenous variable; if m_S is the short-term multiplier on an exogenous variable, then

$$\textit{long term multiplier} = m_L = 1 + m_S + m_S^2 + m_S^3 + \ldots = \frac{1}{1 - m_S}$$