Handout: IS/LM Model

IS Curve Derivation

Figure 4-4 in the textbook explains one derivation of the IS curve. This derivation uses the Induced Savings Function from Chapter 3. Here, I describe an alternative derivation of the IS curve using the 45°-line/Expenditure function model from Chapter 3. The results is the same but the graphs differ.

**IS Curve**: All combinations of interest rates and GDP for which the spending balance model is in equilibrium.

**Derivation**: The derivation begins with the “Keynes Cross” model developed in Chapter 3. Bear in mind that $I_p$ and $C_a$ from this model both depend on the interest rate in the economy, $r$. Graphically, the income determination model goes on the top set of axes and below it a set of axes with GDP graphed on the horizontal axis and the interest rate graphed on the vertical axis.

1. Begin at the equilibrium point $E_o$ in the spending balance model. $E_o$ identifies the equilibrium level of GDP ($Y_o^*$) and planned expenditure ($E_{po}$) in the model. In turn, the equilibrium level of planned expenditure depends on two factors, $I_p$ and $C_a$ that depend on the interest rate. To locate $A_{po}$, there must also be some corresponding interest rate, $r_o$, in the economy.

2. Map the equilibrium level of GDP, $Y_o^*$, and the corresponding interest rate, $r_o$, into the bottom panel ($r \times Y$). “Map” means locate the equilibrium point $E_o$ in the lower graph, by locating $Y_o^*$ and $r_o$ in this space. Note that, by definition, $E_o$ is on the IS Curve.
3. Change the interest rate. Suppose that interest rates rise from $r_0$ to $r_1$. An increase in interest rates reduces both $I_p$ and $C_a$, which reduces $A_p$ from $A_{p0}$ to $A_{p1}$. This shifts the $E_p$ line down and reduces equilibrium GDP from $Y^*_o$ to $Y^*_1$ and leads to a new equilibrium point, $E_1$.

4. Map the new equilibrium point in the spending balance model into the bottom panel ($r \times Y$). The $Y$ coordinate can be found by extending the vertical dashed line straight down. The $r$ coordinate must be located somewhere above the previous interest rate $r_0$ because the interest rate was assumed to increase. Note that the new equilibrium point $E_1$ is also, by definition, on the IS Curve.

More points on the IS Curve could be found by repeating this procedure and finding additional equilibrium points. This would result in a linear relationship between $r$ and $Y$. Moving along the IS Curve interest rates move in the opposite direction as GDP.
**Strong and Weak Policy**

**Monetary Policy**

- **Strong Monetary Policy**

- **Weak Monetary Policy**

**Fiscal Policy**

- **Strong Fiscal Policy**

- **Weak Fiscal Policy**
More on the Algebra of the IS/LM Model

The graphical presentation to the IS/LM model has a corresponding analytical representation. In many ways, the model in equations provides more insight than the graphical version of the model. The appendix to Chapter 5 contains a discussion of the algebraic solution to this model. Here, I present a slightly different version of this material.

Recall from Chapter 3, the effect of changes in the exogenous variables of the model on GDP depended on the multiplier

\[
\text{multiplier} = \frac{1}{\text{marginal leakage rate}}
\]  

(1)

And also recall that the equilibrium condition from the Chapter 3 model of income determination was

\[
Y = kA_p
\]

(2)

In a nutshell, the IS curve derivation in Chapter 4 simply makes Autonomous Planned Expenditure depend on the interest rate.

\[
A_p = A_p' - br
\]

(3)

Here, the parameter \( b \) simply reflects how sensitive \( A_p \) is to changes in the interest rate. Recall from the IS Curve derivation, when \( r \) changed, \( I_p \) and \( C_a \) changed, and the intercept of the \( E_p \) line shifted around; when \( r \uparrow \), the intercept fell and \( Y \downarrow \). This equation simply shows this algebraically. \( b \) is related to the effect of changes in the interest rate on \( I_p \) and \( C_a \).

This also changes the equilibrium condition from the model. Simply substitute the expanded equation for \( A_p \) into the equilibrium condition

\[
Y = k(A_p' - br)
\]

(4)

In fact, this new equilibrium condition is also an expression for the IS curve. To get this expression in slope-intercept form, we need to solve it for the variable graphed on the vertical axis, \( r \)

\[
Y = k(A_p' - br)
Y = kA_p' - kbr
kbr = kA_p' - Y
r = \frac{b}{k}A_p' - \frac{1}{kb}Y
r = \frac{1}{b}A_p' - \frac{1}{kb}Y
\]

In this equation, \( -\frac{1}{kb} \) is the slope of the IS Curve. It is negative, so the IS curve slopes down. The first term is the intercept of the IS curve. The exogenous factors in \( A_p \) shift this intercept - they shift the IS curve left and right.

The LM curve emerges from the money market model. Interest rates move to equilibrate money demand and money supply, so the equilibrium condition in the money market is

\[
\left( \frac{M^S}{P} \right) = \left( \frac{M^S}{P} \right)^d = hY - fr
\]

(5)

The LM Curve emerges from this equilibrium condition. Again, simply solve the equilibrium condition for \( r \), the variable graphed on the vertical axis.
\[ r = \frac{hY - \frac{M^S}{P}}{f} \]  

Now we have a system of two equations (the IS Curve and the LM Curve) in two unknowns. These two equations can be solved to get a \textit{reduced form equation} for GDP. To recap, we have

\textbf{IS Curve} \quad \textbf{LM Curve}

\begin{align*}
\text{Spending Market Equilibrium} & \quad \text{Money Market Equilibrium} \\
Y = k(A'_p - br) & \quad \left(\frac{M^S}{P}\right) = hY - fr
\end{align*}

There are many ways to algebraically solve this system. For example, the IS Curve could be set equal to the LM Curve, and the resulting equation solved for \( Y \). The approach in the appendix is to plug the expression for the LM Curve into the right hand side of the expression for the spending balance model equilibrium, substituting for \( r \) [Note the typo in equation (7) in the text]

\[ Y = k(A'_p - br) = k \left[ A'_p + \frac{bY}{f} + \frac{b}{f} \left(\frac{M^S}{P}\right)\right] \]  

(7)

Add \( kbbY/f \) to both sides and divide both sides by \( k \) to solve for \( Y \)

\[ Y \left(\frac{1}{k} + \frac{bh}{f}\right) = A'_p + \frac{b}{f} \left(\frac{M^S}{P}\right) \]

And divide both sides by the term in parentheses on the left hand side.

\[ Y = \frac{A'_p + \frac{b}{f} \left(\frac{M^S}{P}\right)}{\frac{1}{k} + \frac{bh}{f}} \]  

(8)

This equation is a \textit{reduced form equation} for GDP for the IS/LM Model. It shows the overall impact on \( Y \) of changes in the exogenous variables in the model when both the money market and the spending balance model are in equilibrium.

Chapter 5 is primarily concerned with the relative strength of monetary policy and fiscal policy. How do \( \Delta M^S \) and \( \Delta G \) affect \( Y \) in the model? Note that \( G \) is part of \( A'_p \). To make the analysis easier, the reduced form equation can be rewritten a bit

\[ Y = \frac{ \frac{A'_p + b}{f} \left(\frac{M^S}{P}\right) }{ \frac{1}{k} + \frac{bh}{f} } \]

\[ Y = \frac{1}{\frac{1}{k} + \frac{bh}{f}} A'_p + \frac{b}{k + \frac{bh}{f}} \left(\frac{M^S}{P}\right) \]

In this equation, the policy variables of interest are \( M^S \) and \( G \) which is part of \( A'_p \). The two messy fractions in front of these terms can be simplified into two parameters

\[ Y = k_1A'_p + k_2 \left(\frac{M^S}{P}\right) \]  

(9)

Where the parameters are simply

\[ k_1 = \frac{1}{\frac{1}{k} + \frac{bh}{f}} \]  

(10)
\[ k_2 = \frac{b/f}{k_1 + bh} = \left( \frac{b}{f} k_1 \right) \]  

(11)

And again, the simplified Reduced Form Equation For GDP from the IS/LM model is

\[ Y = k_1 A'_p + k_2 \left( \frac{M^S}{P} \right) \]  

(12)

Some Algebraic Insight

The graphical depictions of the conditions for strong and weak policy discussed in Chapter 5 and shown above also have algebraic counterparts. Insight into the conditions under which monetary and fiscal policy are relatively strong and weak can be seen from the reduced form solution for GDP in the IS-LM Model.

Reduced Form Solution: \[ Y = k_1 A'_p + k_2 \left( \frac{M^S}{P} \right) \]

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>Fiscal Policy</th>
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</thead>
<tbody>
<tr>
<td>[ \Delta Y = k_1 A'_p + k_2 \left( \frac{\Delta M^S}{P} \right) ]</td>
<td>[ \Delta Y = k_1 \Delta A'_p + k_2 \left( \frac{M^S}{P} \right) ]</td>
</tr>
</tbody>
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Effect of \( \Delta M \) on \( \Delta Y \) depends on:  
Size of the effect of \( \Delta G \) on \( \Delta Y \) depends on:

- \( k_2 = \left( \frac{b}{f} \right) k_1 \)
- \( b \uparrow \Rightarrow k_2 \uparrow \)
- \( f \uparrow \Rightarrow k_2 \downarrow \)

An Example: As the LM Curve gets steeper, fiscal policy gets weaker. Recall:

\[ r = \frac{h}{f} Y - \frac{1}{f} \frac{M^S}{P} \]

The strength of fiscal policy depends on \( k_1 \). When the LM curve gets steeper, \( \frac{h}{f} \) gets larger. Note that when \( \frac{h}{f} \) gets larger, the denominator of \( k_1 \) gets larger, and \( k_1 \) gets smaller. That makes fiscal policy (\( \Delta G \)) have a smaller effect on \( Y \).

This concept is illustrated with a sample exam question on the following page. Answer this question before looking at the solution on the next page.
Sample Exam Question: Suppose that money demand is not very sensitive to changes in the interest rate. In this case, monetary policy is strong. (True/False/Uncertain)
Solution: True. If money demand is not sensitive to changes in the interest rate, then the parameter $f$ is small. As $f$ gets smaller, $\frac{h}{f}$ (the slope of the LM curve) gets bigger - the LM curve gets steeper. The steeper the LM curve, the less effective is monetary policy.