

# Additional material for the lecture Numerical Methods for Hyperbolic and Parabolic Conservation Laws

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## Example 3.4

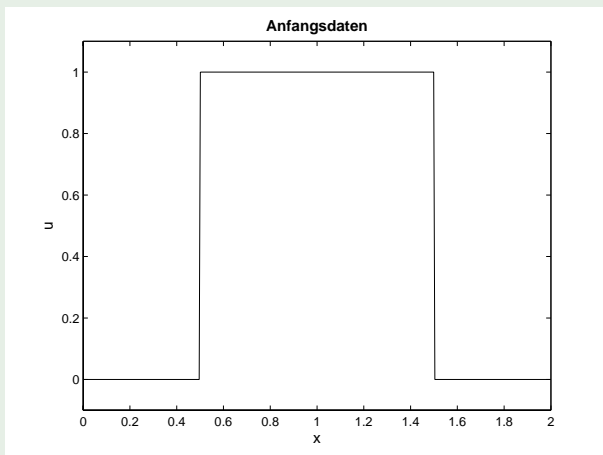
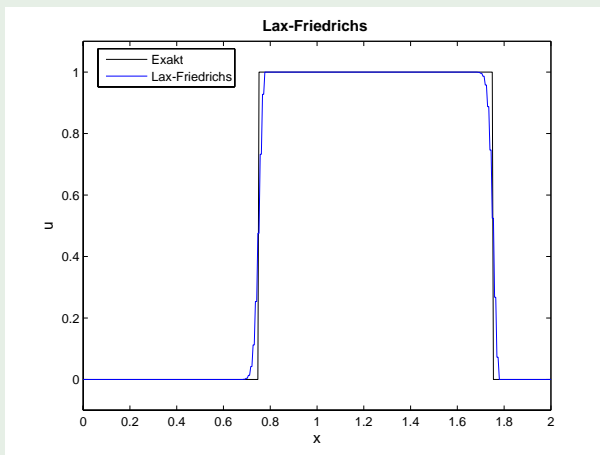


Figure: Initial distribution

## Example 3.4



**Figure:** Approximate solution of the Lax-Friedrichs-scheme at  $t = 0.5$ .

## Example 3.4

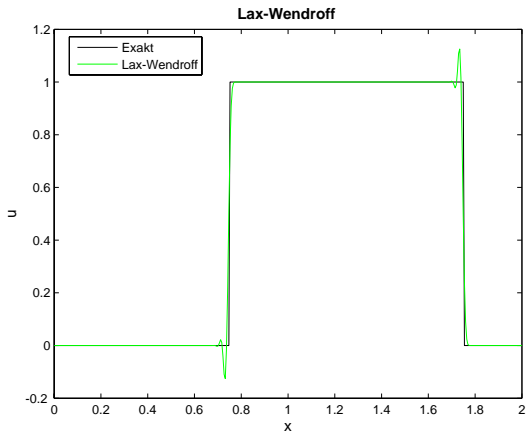


Figure: Approximate solution of the Lax-Wendroff-scheme at  $t = 0.5$ .

## Example 3.4

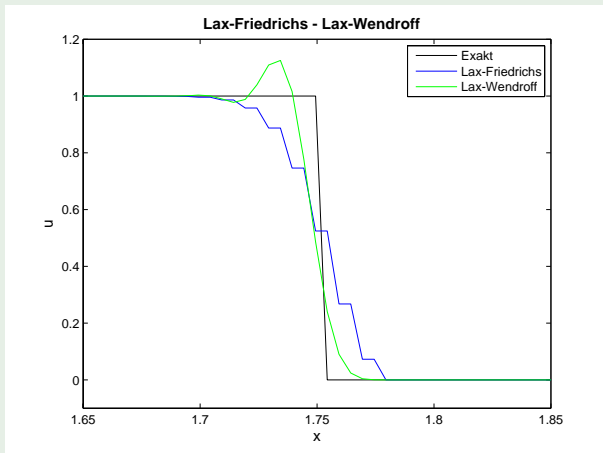


Figure: Comparison between Lax-Friedrichs and Lax-Wendroff

## Example 3.4

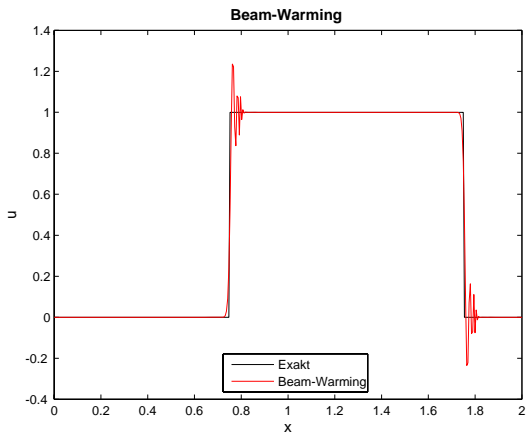


Figure: Approximate solution of the Beam-Warming-scheme at  $t = 0.5$ .

## Example 3.4

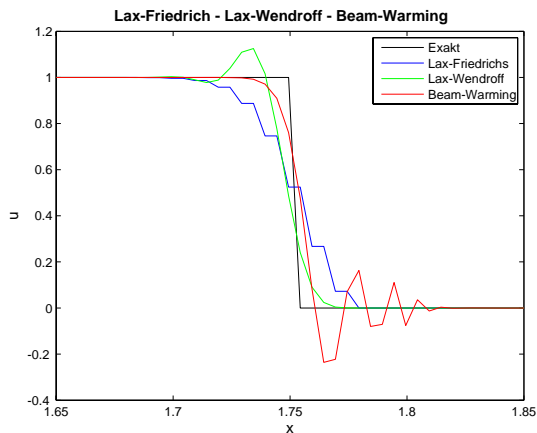
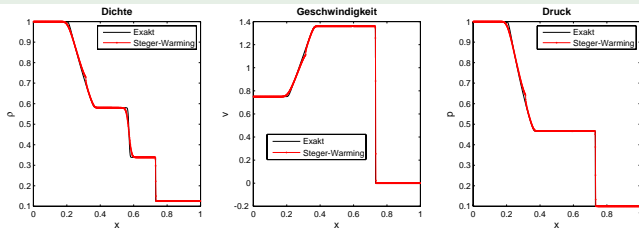


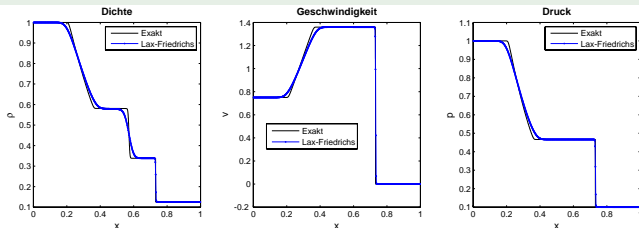
Figure: Vergleich Lax-Friedrichs, Lax-Wendroff, Beam-Warming

# Finite Volume Scheme in one space dimension

## Example 3.9: Results Steger-Warming



## Example 3.9: Results Lax-Friedrichs





## Example 3.9: Shock

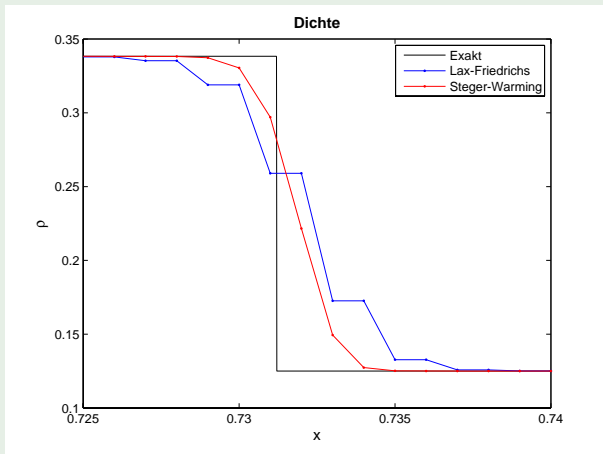


Figure: Comparison between Lax-Friedrichs and Steger-Warming

## Example 3.9: Contact discontinuity

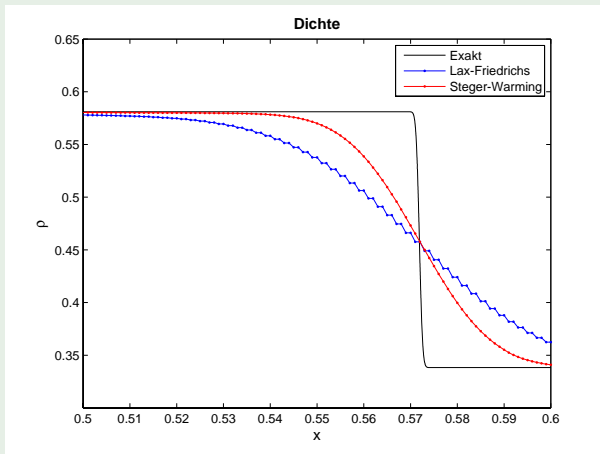


Figure: Comparison between Lax-Friedrichs and Steger-Warming

## Example 3.9: Expansion fan

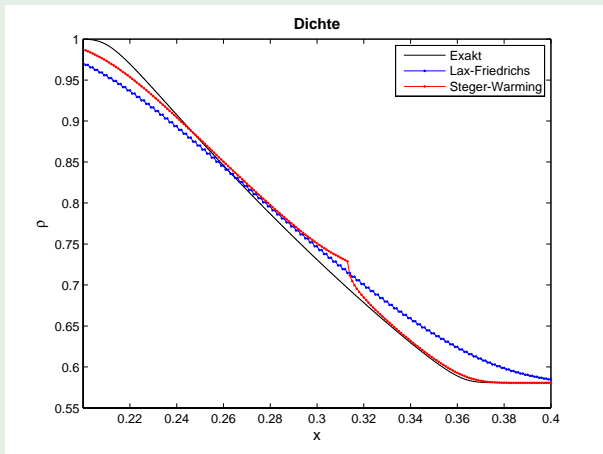


Figure: Comparison between Lax-Friedrichs and Steger-Warming

## Distribution of the mass flux w.r.t. the Mach number

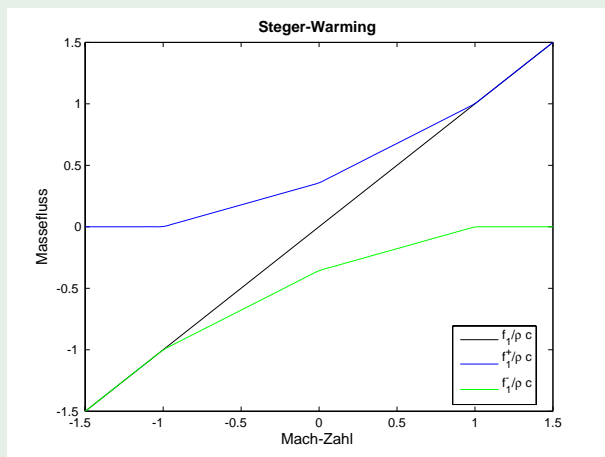


Figure: Steger-Warming

# Finite Volume Scheme in one space dimension

## Distribution of the momentum flux w.r.t. the Mach number

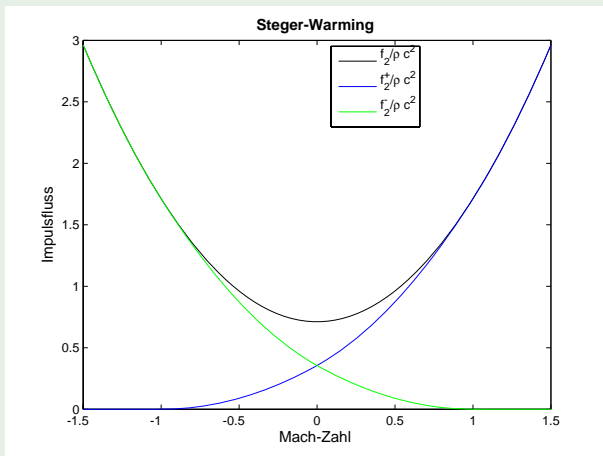


Figure: Steger-Warming

## Distribution of the energy flux w.r.t. the Mach number

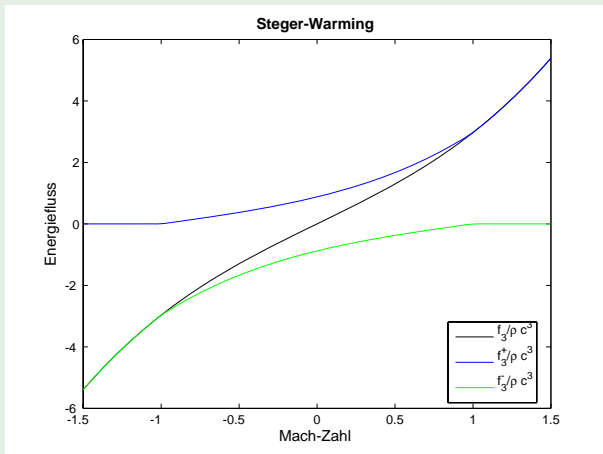


Figure: Steger-Warming

# Finite Volume Scheme in one space dimension

## Distribution of the mass flux w.r.t. the Mach number

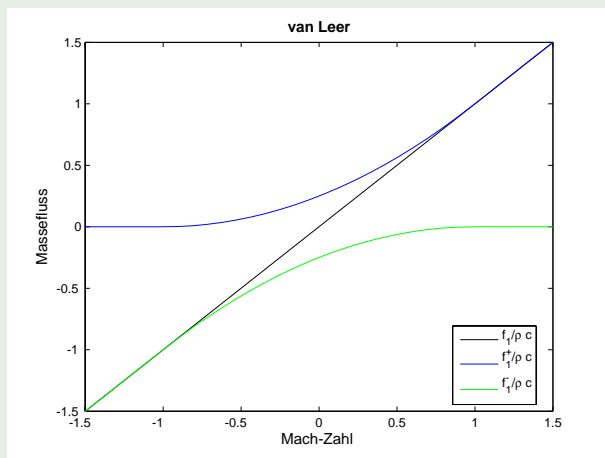


Figure: Van Leer

# Finite Volume Scheme in one space dimension

## Distribution of the momentum flux w.r.t. the Mach number

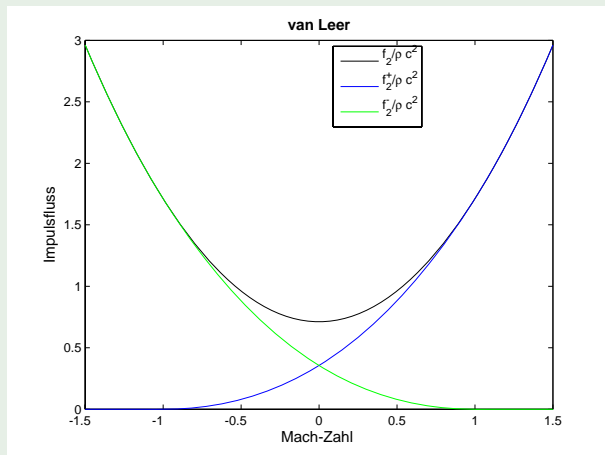


Figure: Van Leer



# Finite Volume Scheme in one space dimension

## Distribution of the energy flux w.r.t. the Mach number

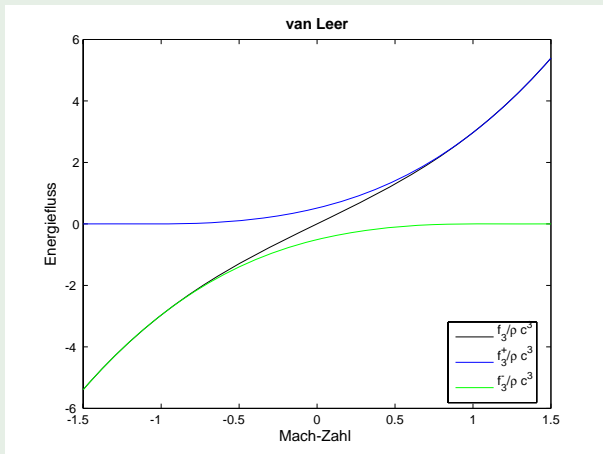
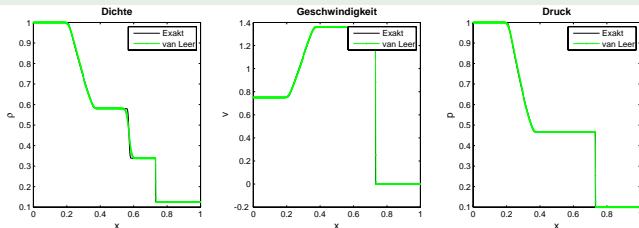


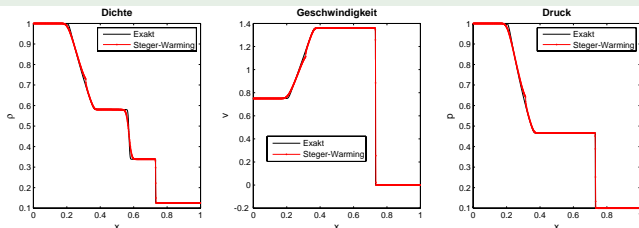
Figure: Van Leer

# Finite Volume Scheme in one space dimension

## Example 3.10: Results van Leer



## Example 3.10: Results Steger-Warming



## Example 3.10: Shock

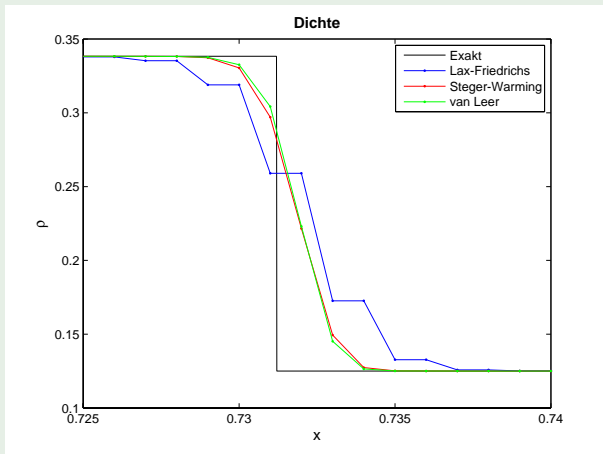


Figure: Comparison between Lax-Friedrichs, Steger-Warming and van Leer

## Example 3.10: Contact discontinuity

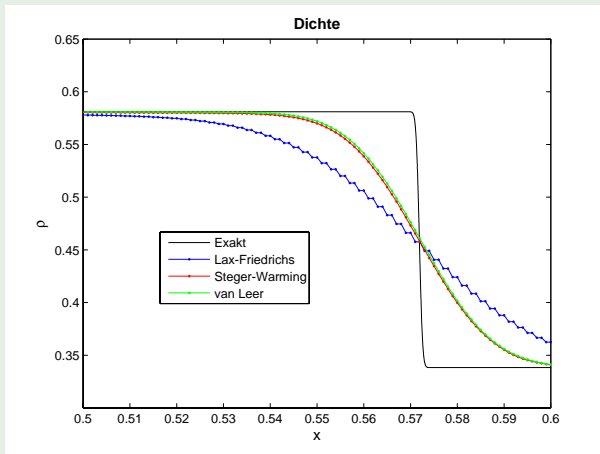


Figure: Comparison between Lax-Friedrichs, Steger-Warming and van Leer

## Example 3.10: Expansion fan

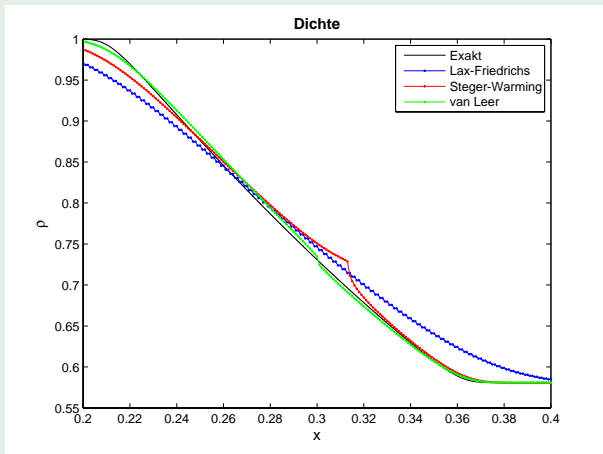
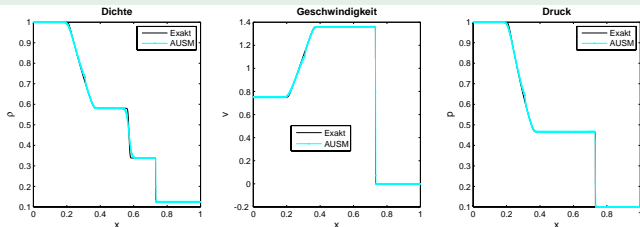


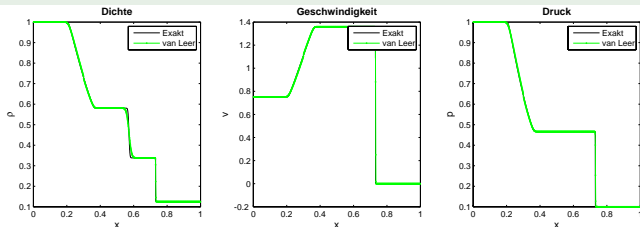
Figure: Comparison between Lax-Friedrichs, Steger-Warming and van Leer

# Finite Volume Scheme in one space dimension

## Example 3.12: Results AUSM



## Example 3.12: Results van Leer



## Example 3.12: Shock

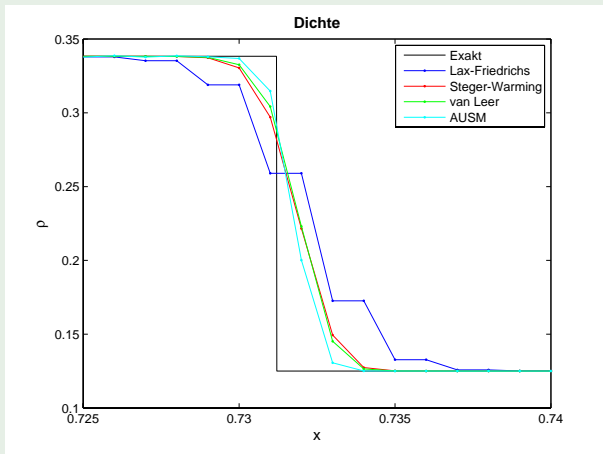


Figure: Comparison between Lax-Friedrichs, Steger-Warming, van Leer and AUSM

## Example 3.12: Contact discontinuity

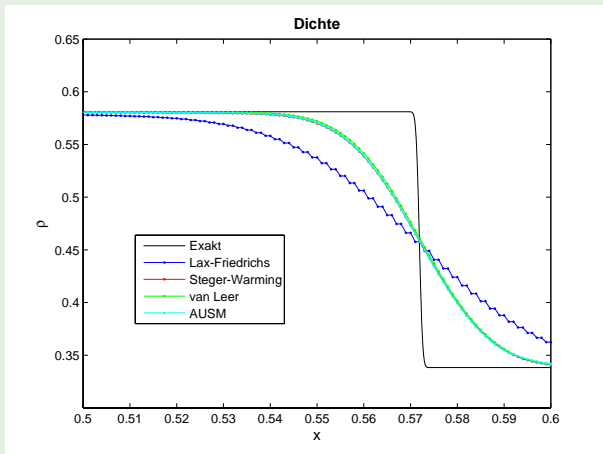


Figure: Comparison between Lax-Friedrichs, Steger-Warming, van Leer and AUSM



## Example 3.12: Expansion fan

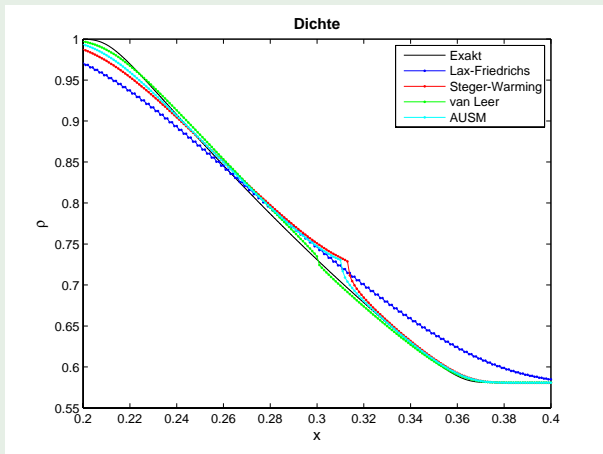


Figure: Comparison between Lax-Friedrichs, Steger-Warming, van Leer and AUSM

## Example 3.13: Stationary Contact discontinuity

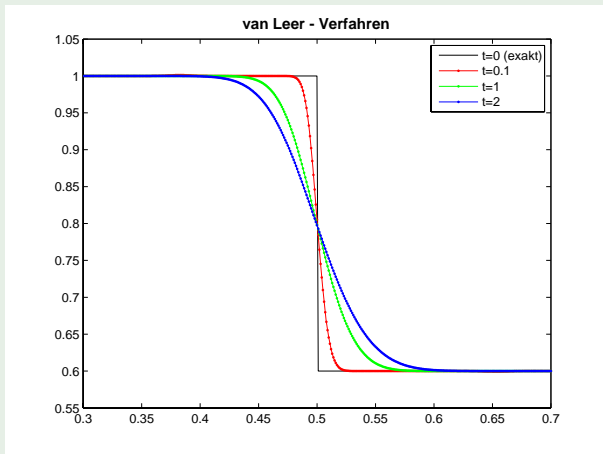


Figure: Comparison between exact solution and the results of van Leer

## Example 3.13: Stationary Contact discontinuity

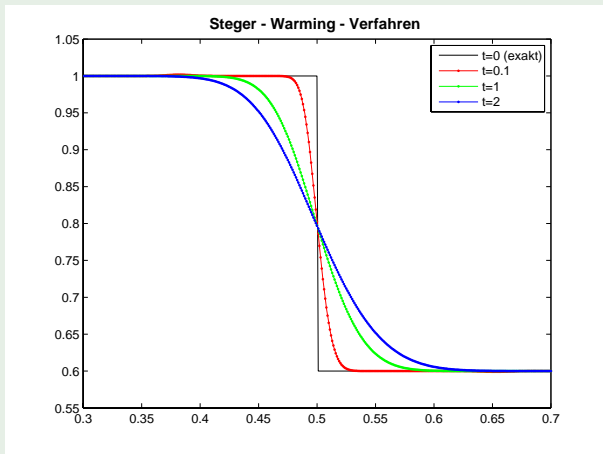


Figure: Comparison between exact solution and the results of Steger-Warming

## Example 3.13: Stationary Contact discontinuity

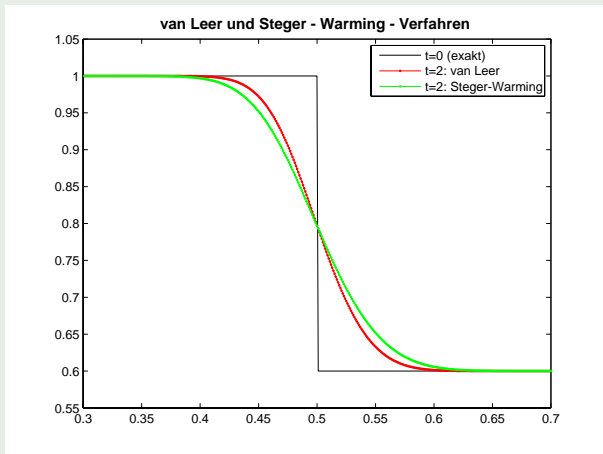


Figure: Comparison between the exact solution and the results of van Leer as well as Steger-Warming

## Example 3.13: Stationary Contact discontinuity

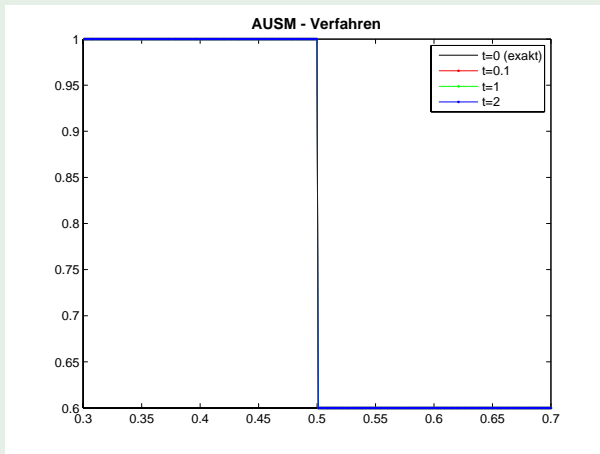


Figure: Comparison between exact solution and the results of AUSM

## Example ?..?: Linear Advection Equation

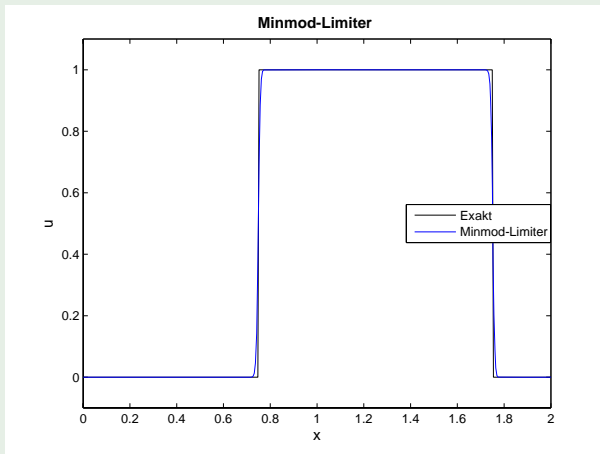


Figure: Minmod-Limiter

## Example ?.??: Linear Advection Equation

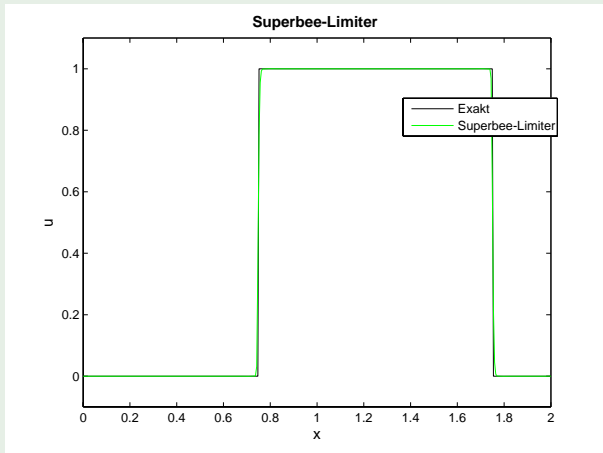


Figure: Superbee-Limiter

## Example .??: Linear Advection Equation

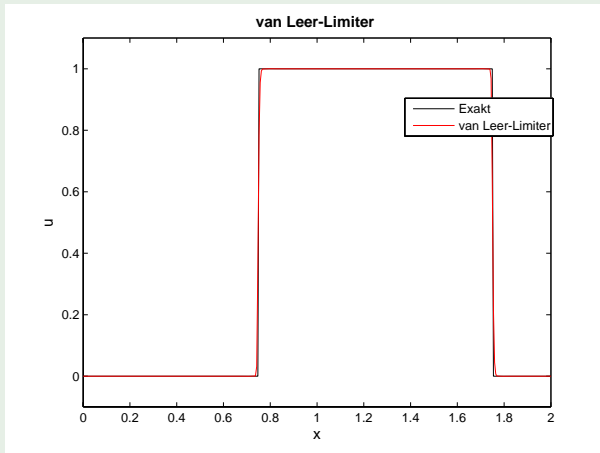


Figure: Van Leer Limiter



## Example ?.??: Linear Advection Equation

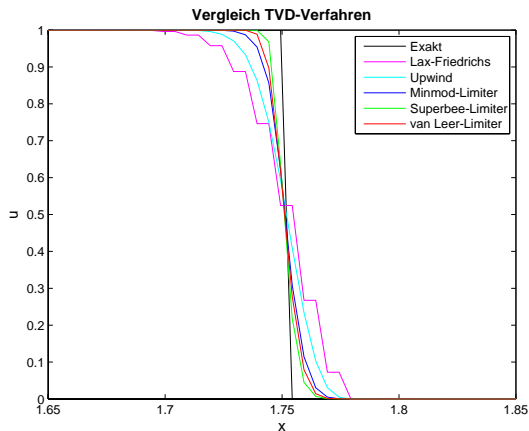
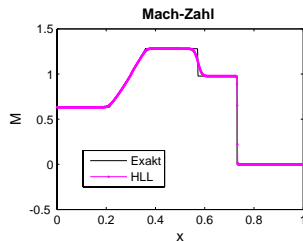
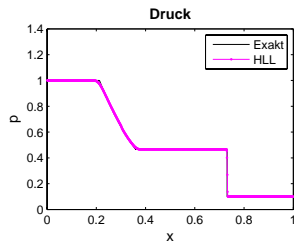
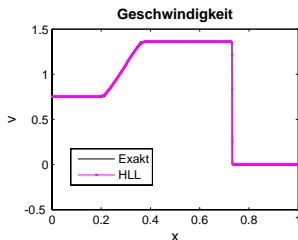
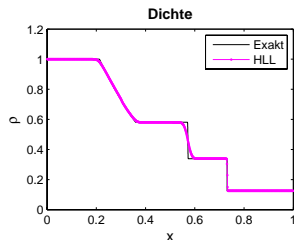


Figure: Comparison between different limiters

# Finite Volume Scheme in one space dimension

## Example 3.15: Results HLL



## Example 3.15: Shock

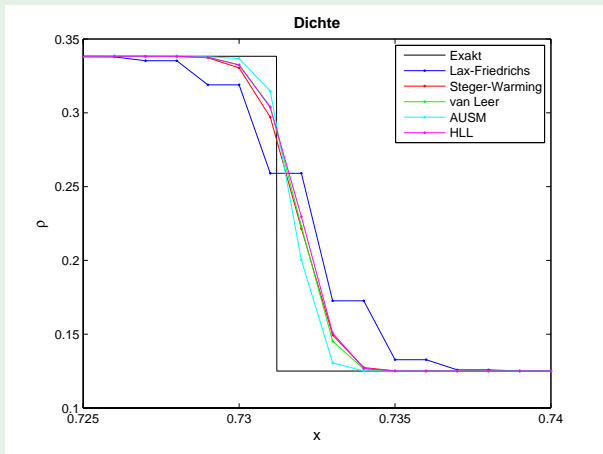


Figure: Comparison between different schemes

## Example 3.15: Contact discontinuity

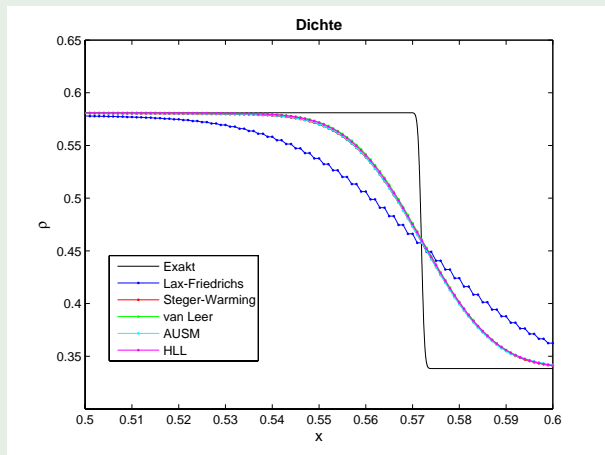


Figure: Comparison between different schemes

## Example 3.15: Expansion fan

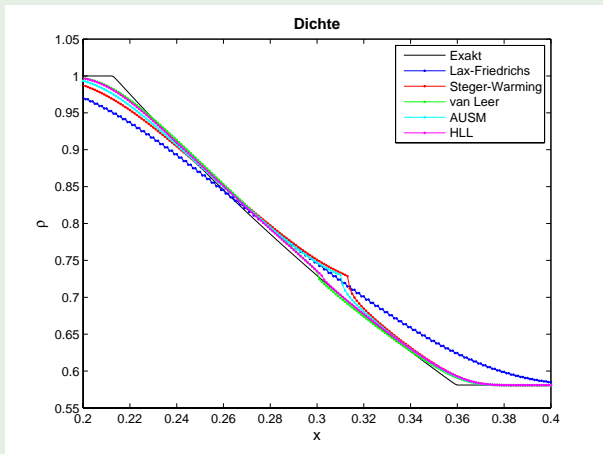
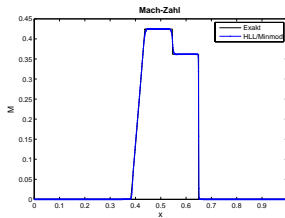
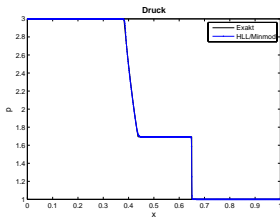
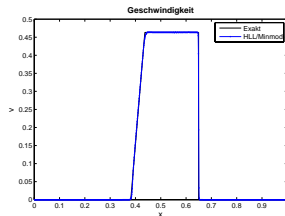
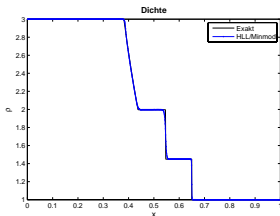


Figure: Comparison between different schemes

# Finite Volume Scheme in one space dimension

## Example 3.18: Results HLL with limiter minmod



## Example 3.18: Shock

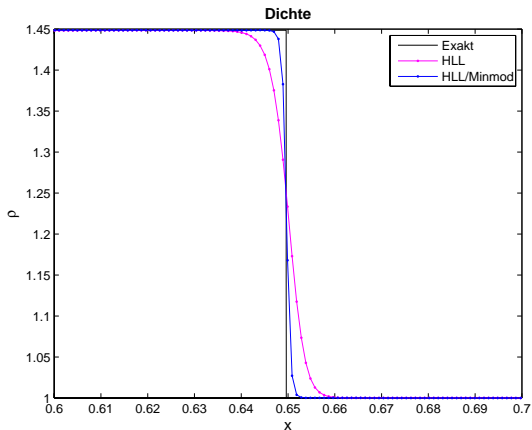


Figure: HLL with limiter minmod

## Example 3.18: Contact discontinuity

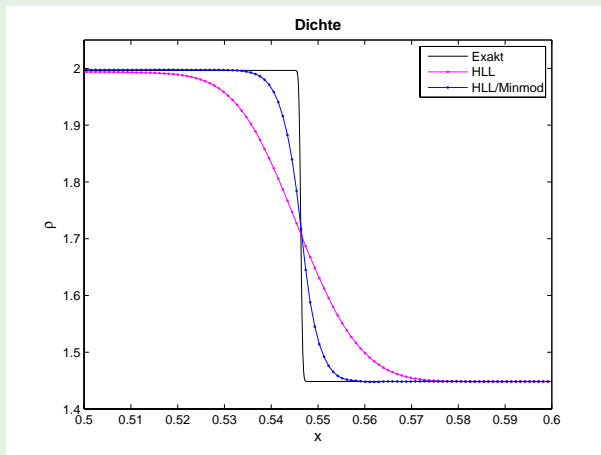


Figure: HLL with limiter minmod



## Example 3.18: Expansion fan

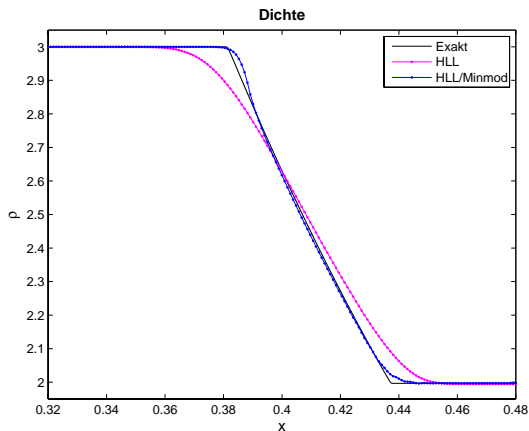
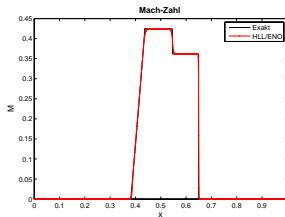
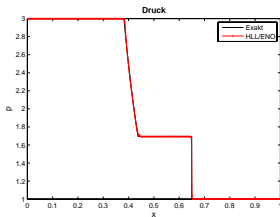
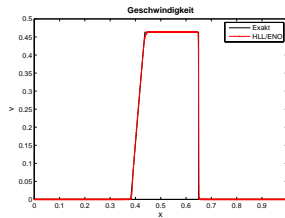
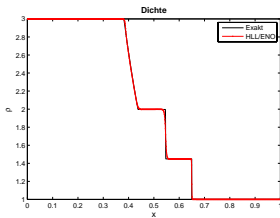


Figure: HLL with limiter minmod

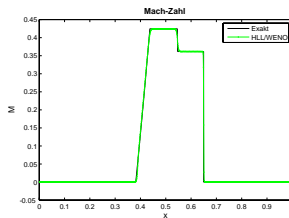
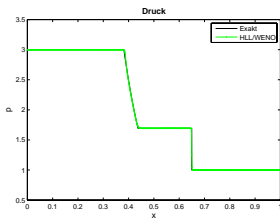
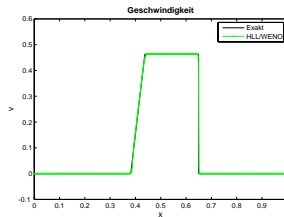
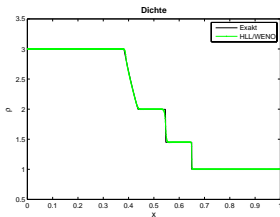
# Finite Volume Scheme in one space dimension

## Example 3.19: Results HLL with ENO-reconstruction



# Finite Volume Scheme in one space dimension

## Example 3.19: Results HLL with ENO-reconstruction



## Example 3.19: Shock

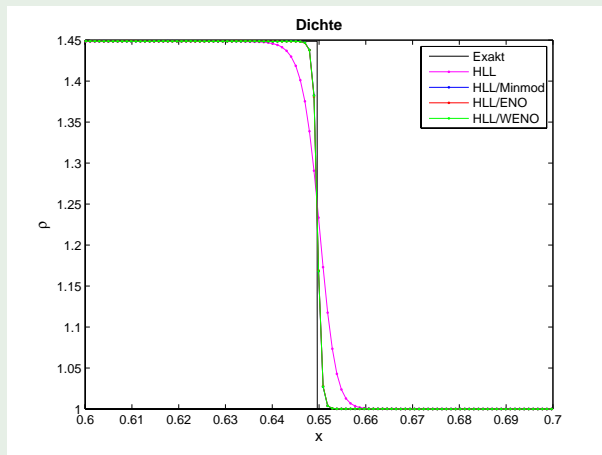


Figure: HLL with different reconstructions

## Example 3.19: Contact discontinuity

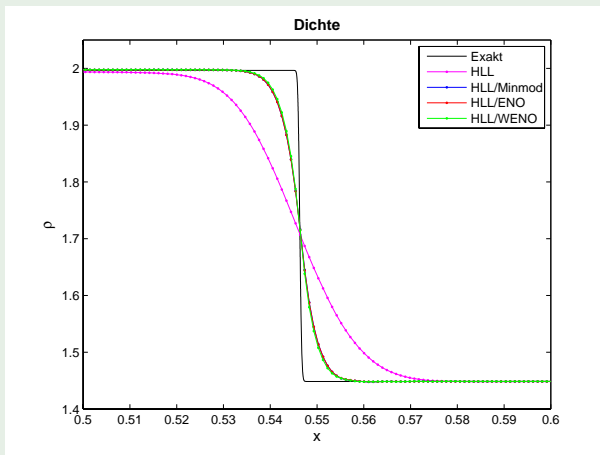


Figure: HLL with different reconstructions

## Example 3.19: Expansion fan

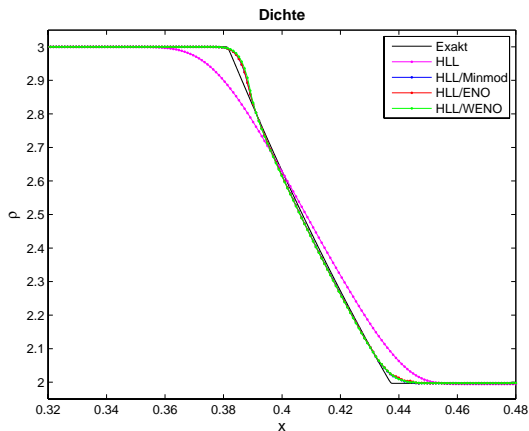


Figure: HLL with different reconstructions