Tuesday, 04/11/12:

Moscow State Aviation Technology University
Dept of Applied — division of informatics and of mathematics
C, C++, libraries
diff. eqn., analysis, num. methods, probability, optimization

Computer accounts are needed:
- UniAccount (→ eduroam, IT Servicecenter)
- department account
- account in Pool-Raum 2421 = computer lab

Lectures: Tu 15:15 – 16:45, We. 13:15 – 14:45
Lab: We. 15:15 – 16:45
Open lab: Th. 11:15 – 12:45
} in computerlab 2421

HPCF-2010-2 Sec. 2:

$$-\Delta u = -u_{xx} - u_{yy} = f \quad \text{at} \quad (x_i, y_j)$$

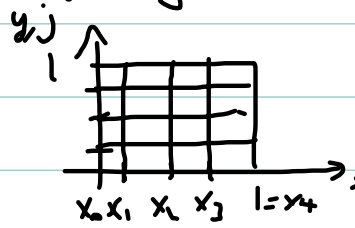
$$f(x_i, y_j) \approx \frac{-u(x_i, y_{j-1}) - u(x_{i-1}, y_i) + 4u(x_i, y_j) - u(x_{i+1}, y_j) - u(x_i, y_{j+1})}{h^2}$$

FD = finite difference method = take this approximate equation as the determining equation for the approximation $u_{ij} \approx u(x_i, y_j)$ $\Leftrightarrow$

$$\underbrace{f(x_i, y_j)}_{=: f_{ij}} = \frac{-u_{ij-1} - u_{i-1j} + 4u_{ij} - u_{i+1j} - u_{ij+1}}{h^2}$$

$=: f_{ij}$ = shorthand notation

Example of mesh with $N = 3$:




$x_i = ih$

$i = 0, 1, 2, 3, 4$

$N + 1 = 4$

$y_j = jh$,

$j = 0, \ldots, 4$

We have Dirichlet BC (boundary conditions)

$u = 0$ on $\partial\Omega$ $\Rightarrow$ The values of $u_{ij}$ with $i = 0$ or $N+1$ or $j = 0$ or $N+1$ are known and thus will be dropped from the system of equations.

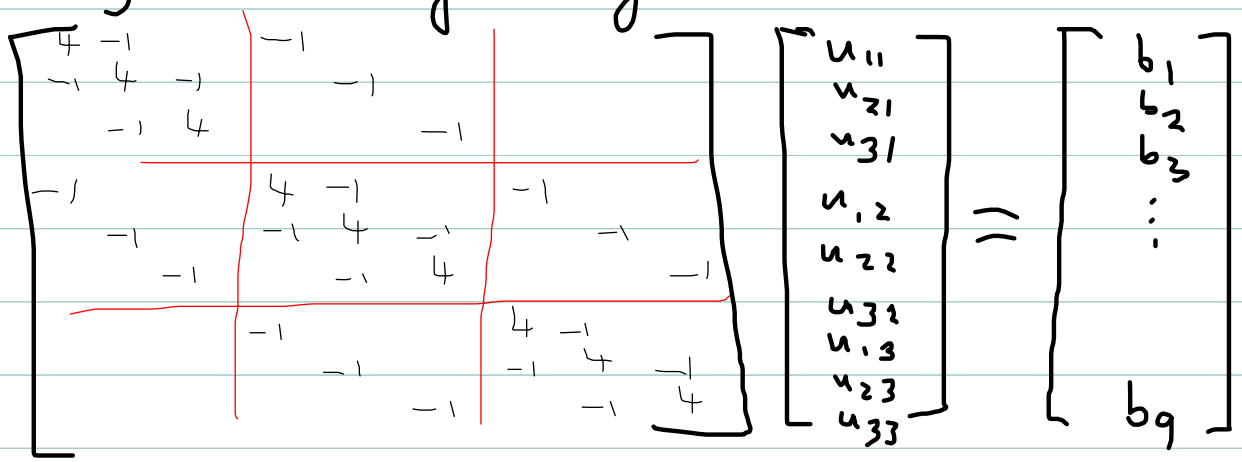
$N = 3 \Rightarrow$ interior points for $i = 1, 2, 3$, $j = 1, 2, 3 \Rightarrow$ 9 unknowns $u_{ij}$ and 9 equations.

$$
\begin{array}{cccccccccc}
4u_{11} & -u_{21} & & -u_{12} & & & & & & = f_{11} h^2 \\
-u_{11} & +4u_{21} & -u_{31} & & -u_{22} & & & & & = f_{21} h^2 \\
 & -u_{21} & +4u_{31} & & & -u_{32} & & & & = f_{31} h^2 \\
-u_{11} & & & +4u_{12} & -u_{22} & & -u_{13} & & & = f_{12} h^2 \\
 & -u_{21} & & -u_{12} & +4u_{22} & -u_{32} & & -u_{23} & & = f_{22} h^2 \\
 & & -u_{31} & & -u_{22} & +4u_{32} & & & -u_{33} & = f_{32} h^2 \\
 & & & -u_{12} & & & 4u_{13} & -u_{23} & & = f_{13} h^2 \\
 & & & & -u_{22} & & -u_{13} & +4u_{23} & -u_{33} & = f_{23} h^2 \\
 & & & & & -u_{32} & & -u_{23} & +4u_{33} & = f_{33} h^2
\end{array}
$$

The above is a system of linear equations. We need a particular ordering of the unknowns ! (Others are possible.) We can write it as $A\vec{u} = \vec{b}$ if we introduce $A$ and $\vec{b}$ appropriately.

Typical choice is to multiply by $h^2$ $\Rightarrow$

$A$ is just matrix of integers $0, -1, 4$ then.




Here we need $\vec{b}_k = h^2 f_{ij}$ with $k = i + N(j-1)$

and $\vec{u}_k = u_{ij}$

Now have $A\vec{u} = \vec{b}$ with $A$ in tech. rep.