Undergraduate Research Experiences in Computational Mathematics

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Fall 2005 CBMS report

- Fall enrollment growth: $+21\%$ at 4-year colleges & universities since 1990 (total: 1.6 million students)
- Math enrollment growth: -2.3%
- All math/stat bachelors degrees: -2%
- Number of bachelor's degrees awarded by doctoral departments was up by 41% over 2000 but those by master's & 4-year schools declined by 19–27%
- Fraction of 4-year colleges offering Advanced Engineering Mathematics once every 2 years: 7% Advanced Calculus: 57% Algebra: 52%

“Big picture” questions

- Should math departments have the primary responsibility for teaching undergraduate courses in mathematics?
- Should mathematically talented undergraduates major in mathematics?
- How can mathematics departments contribute meaningfully to fast-growing disciplines like genomics, climate, and materials science?
- How can mathematics departments contribute to students’ professional development besides homework problem sets?

My interests and philosophy

- Professional goals: Create modern, high-impact programs for mathematics undergraduates that can serve as national models
- Objective: Educate mathematically literate scientists (not necessarily more mathematicians *per se*)
- Develop skills in scientific collaboration, public speaking, and technical writing
- Undergraduate education should be an opportunity to learn a little about a lot of things—premature specialization is undesirable
- Interdisciplinary collaboration is essential

A sample undergraduate course catalog

Calculus I, II, III	Differential Equations I, II
Advanced Calc I, II	Linear Algebra
Probability	Mathematical Statistics
Number Theory	Modern Algebra
Complex Variables	Vector Analysis
Numerical Analysis	Engineering Math
Logic	Computer Programming

The post-Sputnik problem

- It's the ASU program in 1958–59!

The post-Sputnik problem

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- Undergraduates see few applications of mathematics after 1950
  - Metropolis algorithm
  - Fast Fourier transform
  - Simplex algorithm
  - Public-key cryptography
  - Kalman-Bucy filter
  - Viterbi algorithm
  - Shotgun genome sequencing

The standard undergraduate curriculum fails most students

- The #1 FAQ: What can I do with a math degree?
- Little if any exposure to:
  - Real data
  - Mathematical modeling of "real world" problems
  - Uncertainty in data and models
  - Computation beyond a numerical analysis course
  - Teamwork and scientific collaboration
  - Technical writing beyond short proofs in homework
- If it takes too long for students to see interesting applications, then they lose interest
- Course catalogs are opaque

Some relevant numbers

- $\sim 10^6$ students enrolled in calculus courses at any given time
- $\sim 10^3$ Ph.D. degrees awarded per year in the U.S.
- $\sim 10^2$ tenure-track job openings per year in the U.S.
- Mathematics departments need to prepare students at undergraduate and graduate levels for jobs in industry and government
- payscale.com study: the top 15 highest-paying bachelor's degrees, both after graduation and 15 years later, all involve applied mathematics and statistics
- 2009 AMS data: median starting salary for Ph.D.'s in academia: \$60 K in industry: \$100 K

Flexibility in undergraduate programs is essential

- Programs must give students an idea of what's possible
- The majority of our math majors transfer from other programs—so programmatic requirements must accommodate them
- Undergraduate programs must provide options besides graduate school
- Courses in public speaking, leadership, business, finance, entrepreneurship, etc. should be allowable options
- Mentoring (by faculty) is essential

Undergraduate programs at ASU

- Current enrollment: 70,000 students on four campuses
- About 52,000 undergraduates
- Total mathematics enrollments: 15,000 students in fall, 13,500 in spring
- $\sim 40\%$ precalculus & below; $\sim 30\%$ business math; $\sim 30\%$ in engineering calculus & above
- Approximately 4,000 engineering majors
- Approximately 550 math majors
- Minimum high school competency: 4 years of mathematics through precalculus (but nobody checks the rigor of the courses)

The mathematics programs at ASU

- 56 tenure-track faculty, ~ 50 lecturers/instructors, ~ 45 TAs
- Three undergraduate degrees: BA Math, BS Math (with statistics option), BS in Computational Mathematical Sciences
- Combined BS/MA degree option
- Four new Ph.D. programs:
  - Applied mathematics
  - Core mathematics
  - Statistics
  - Mathematics education

The Computational Mathematical Sciences B.S. degree

- Started 10 years ago; accounts for about 1/4 of all math majors
- Goal: Create an interdisciplinary program that would appeal to good science and math students
- Must accommodate transfer students easily
- Encourages independent study and summer internships
- Takes a broad view of "scientific computing"

Other considerations

- Natural home should be mathematics
- Must leverage existing courses as much as possible
- All interested faculty should be able to participate
- Can accommodate students with a variety of scientific careers in mind

CMS core areas

- Mathematics: 3 semester calculus sequence, ordinary differential equations, linear algebra (18 credits total)
- Science: 2 one-year sequences in life/physical sciences (16 credits)
- Computing: 2 semesters of introductory programming and data structures plus Scientific Computing and Introduction to Numerical Methods (12 credits)
- Advanced courses: At least 3 in math/stat and 1 internship/research course (12 credits)
- Liberal studies: 5 courses in humanities/social sciences, 2 “science and society” courses (21 credits)

The Scientific Computing course

- Audience: advanced undergraduate and beginning graduate students in mathematics and physical sciences
- Emphasis: Software development skills for scientific and high-performance computing
- Topics: Scripting (shell, Python, makefiles); Fortran 95/2003, C99, C++; LAPACK.
- Prerequisites: At least 2 semesters of programming, differential equations and/or linear algebra.
- Parallelism: In development as a collaboration with Intel: brief introduction to OpenMP and/or MPI

The CSUMS program at NSF

- Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS)
- Funded through Divisions of Mathematical Sciences and Undergraduate Education at the National Science Foundation
- First call for proposals in June 2006
- A 5-year effort; starting year 4 now
- http://nsf.gov/pubs/2006/nsf06559/nsf06559.htm

Quotes from the program solicitation

- Goal: help prepare math/stat majors for careers and graduate study in fields that require integrated strengths in computation and the mathematical sciences
- Core: Long-term research experiences for cohorts of at least six undergraduates
- Requirements: Projects must be genuine research experiences rather than rehearsals of research methods

The CSUMS project at ASU

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- 47 students in overlapping cohorts since Jan. 2008
- New cohort of 10–13 students begins each year
- 17 women, 17 first-generation college students, ages 15–31
- Weekly "pizza seminar" during the academic year & 8-week summer research project
- Conference presentation, honors thesis, hopefully a paper
- Sliding window of faculty participants

CSUMS participation timeline

- Basic requirements: Math major with 3.5+ GPA, differential equations, linear algebra, 2 semesters of programming
- Year 1: Weekly pizza seminar
- Summer 1: Research projects
- Year 2: Weekly pizza seminar, conference, paper, graduation
- Option: Continue to BS/MA program

CSUMS presentations at SIAM CSE 11

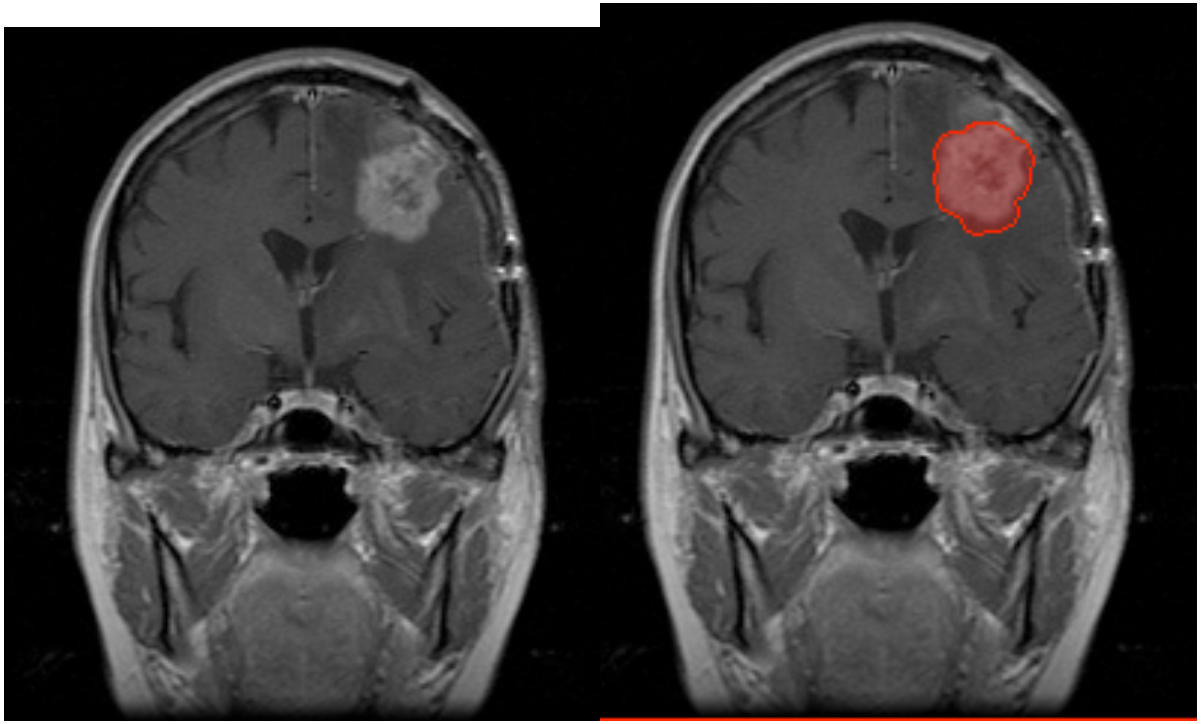
- John Ingraham, diffusion tensors for modeling glioma invasion
- Juan Durazo, surface flows of phytoplankton in the ocean
- Hélène Rhodes, MR image processing for tumor and edema
- Keith Voytek, modeling creation of hydroxyl radicals in radiotherapy
- Eric Adams, simulating radiation in a brain tumor model

CSUMS presentations at SIAM CSE 11, continued

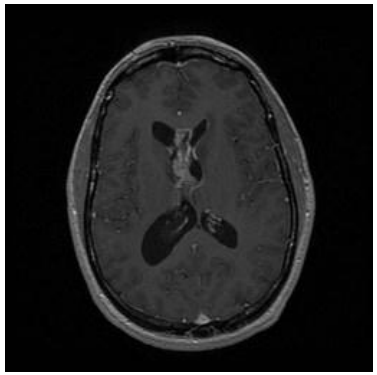
- Jordan Martel, radial basis functions for 2-d fluid flow on the sphere
- Maple So, glioma modeling on individual patient brain images
- Rebecca Borchering, multi-species rabies models with data from Arizona and Texas
- Miles Manning and April Chiu, model of a *Drosophila* larva heart

Glioma “painting” with MIPAV

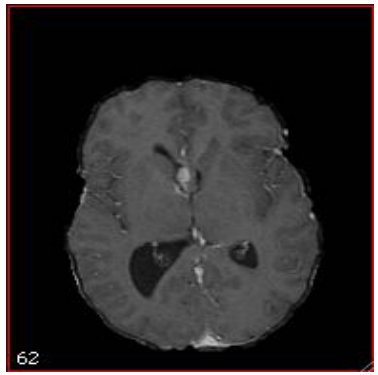
- Goal: estimate volume of tumor and edematous regions

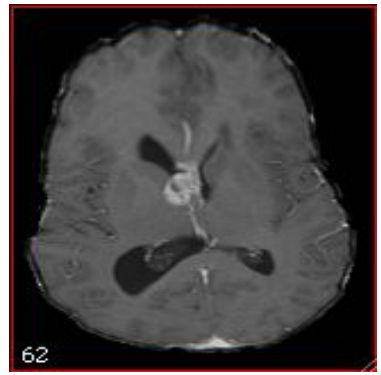


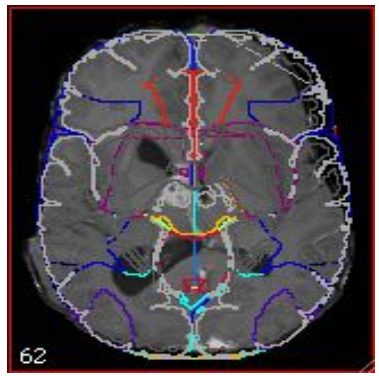
Standardize coordinates (Maple So)

Original 

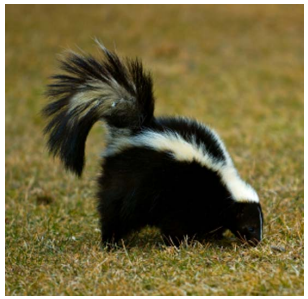
Skull Stripped 

ACPC Aligned 

Talairach 

Talairach Atlas VOIs 

Rabies epidemiology (Becky Borchering & Mara Steinhaus)



- Susceptible: $S_t = rS(1 - S/K) - \beta SI$
- Exposed: $E_t = \beta SI - (\sigma + m)E$
- Infected: $I_t = \sigma E - m_r I + \nabla \cdot (D \nabla I)$

Selected outcomes to date

- 14 talks and 6 posters at 4 SIAM conferences (CSE, Imaging, National meetings)
- 5 talks at undergrad conferences (SUNMARC, CSUMS), 3 talks at AAAS
- 2 (of 15 nationally) Honorable Mentions in Applied Mathematics in the NSF Graduate Fellowship Competition
- 13 honors theses defended or in progress

Selected outcomes to date, continued

- 2 research journal publications
- 12 alumni in Ph.D. programs in applied math, 5 on graduate fellowships
- 4 alumni in medical school (including MD/PhD programs)
- 1 in industry
- Follow-on internships at Max Planck Institute, Harvard Systems Biology program, Budapest Semesters in Mathematics, Hispanic College Fund
- 4 resignations

Things that have worked for us

- Mathematical preparation of vector calculus, ODE, and linear algebra suffices to do something interesting
- The 8-week summer format allows students to focus on their projects without other distractions—yet still leaves time for a break
- The first week includes a "boot camp" on basics of LaTeX and MATLAB
- The pizza seminar allows time for preparatory lectures leading up to the summer—as well as a time for guest lectures, student talk practice, etc.

Overall philosophy

- An undergraduate research project is not a miniature Ph.D. thesis
- Pedagogical goal is for students to decide whether they are interested in research in mathematical sciences
- Students need a couple weeks to learn about the topic and help to define a project that can be substantially completed in another 6 weeks
- New mathematics can be learned as needed along the way
- The computer is a tool for discovery, not the principal focus

Recruitment challenges

- Word of mouth is very helpful
- Otherwise promising students are sometimes turned off by the perception of a "nerd factor"
- Having a critical mass of female students and faculty mentors is important for recruiting women
- Well motivated students are essential—and not always obvious from transcripts and letters of recommendation
- Not all students will succeed

Faculty challenges

- A program of this size is a lot of work
- Proportionally smaller cohorts are appropriate depending on the student and faculty pool
- A “sliding window” of faculty mentors helps prevent burn-out
- Research-active faculty are essential
- The summer format may not be practical for some faculty

Community challenges

- Devising a durable "ecosystem" that supports undergraduate research
- Maintaining funding and momentum in a period of budget crises
- Expanding opportunities for students (and faculty!) at undergraduate institutions for summer research and professional development

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