Assessment of Simple and Alternative Bayesian Ranking Methods Utilizing Parallel Computing

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Abstract

The U.S. Census Bureau (USCB) assists the federal government in distributing approximately $400 billion of aid by providing a complete ranking of the states according to certain criteria, such as average poverty level. It is imperative that this ranking be as accurate as possible in order to ensure the fairness of the allocation of funds. Currently, the USCB ranks states based on point estimates of their true poverty level. Dr. Klein and Dr. Wright of the USCB have compared the performance of this method against more sophisticated procedures in simulation trials, but have found that they do not consistently outperform the existing method. We investigate this phenomenon by revisiting some of these procedures, and we expand on this work to produce new ranking algorithms. We utilize parallel programming to expedite Dr. Klein's procedures. In addition, we specify two new prior distributions on the population means — using previous years' census data as well as regression. We discuss the results of our methods in conjunction with Klein and Wright's corresponding simulation results. In our final report, we compare the performance of our techniques to that of the USCB's current method and show the resulting state ranks for each procedure.

1 Introduction

The U.S. Census Bureau collects data through the American Community Survey (ACS) to quantify characteristics such as percentage of the population that is below poverty level and average annual income in the states and the District of Columbia. The Bureau performs this task to assist the federal government in distributing over $400 billion of funds across the states in order to give more aid to states with a relatively high population of impoverished people when compared to the general population of the U.S. (http://www.census.gov/acs). The data the Bureau receives through the ACS does not yield the true values of the quantities of interest due to sampling error. If the true percentage below poverty level and average annual income of the states were known, then the Bureau would only need to rank these data and report the ordering to the federal government. Since this knowledge is impossible

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to obtain, robust ranking methods must be created to sift through uncertainty in the sample data and still produce accurate ranking results. Currently, the Bureau obtains 51 estimates and produces a ranking of the states directly from them.

The U.S. Census Bureau has previously explored this problem through Dr. Martin K. Klein and Dr. Tommy Wright's paper in which they discuss the performance of the Bureau's current ranking procedure [4]. It is the goal of this paper to invent new ranking procedures and compare results with Klein and Wright's methods. Parallel programming is used to expedite the calculations of all procedures. In our assessment, we analyze how Klein and Wright's non-informative methods compare to corresponding methods informed with the previous year's data. We also implement a procedure that computes the most likely ranking as well as a method utilizing regression. We conclude that when the informed methods use the last year's data, they produce rankings that too strongly reflect data from the previous year. When the error terms are relaxed, the methods perform better than any of the non-informative methods. In addition, our regression informed prior, with relaxed error terms, performs better than all other methods.

2 The Simple Model

The Census Bureau's current ranking method will be referred to as the "simple" method (SI). This model for ranking the population parameters, as obtained from [4], consists of estimating the ranks of certain unknown population parameters by ranking corresponding sample estimates. To obtain these estimates, we model the distribution of the sample estimates for each state, x_i, by

$$x_i|\boldsymbol{\theta} \overset{\text{ind}}{\sim} N(\theta_i, \sigma_i^2),\ i = 1, \dots, k, \tag{2.1}$$

where $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)$ is the vector of unknown population parameters, $\boldsymbol{x} = (x_1, \dots, x_k)$ is a vector of estimates for $\boldsymbol{\theta}$, and $\boldsymbol{\sigma^2} = (\sigma_1^2, \dots, \sigma_k^2)$ is the vector of variances of $\boldsymbol{x}$. Since the Census Bureau performs its calculations on such very large samples, this assumption of normality is justified.

The simple method determines the rank of θ_i by ordering the sample estimates and associating each x_i with a corresponding rank, $\hat{r}_i$. Such a method does not take into consideration any additional information beyond $\boldsymbol{x}$, and our question is whether there exists a more appropriate ranking algorithm.

To assess the accuracy of this method, we let $\boldsymbol{\theta}$ take on a vector of k known values. For each x_i, we draw n simulated values from $N(\theta_i, \sigma_i^2)$, where $1 \leq i \leq k$. We store the resulting estimates in an $n \times k$ matrix where the n row vectors represent independent trials. We rank the row estimates from 1 to k with 1 being the smallest value and k being the largest. For comparison purposes, we follow Klein and Wright's approach and are primarily concerned with computing the probabilities that our estimated ranks, $\hat{r}_i$, are actually the true rank, r_i, and the probabilities that the distance between $\hat{r}_i$ and r_i is less than some constant c_0. That is, we want to calculate $P(\hat{r}_i = r_i)$ and $P(|\hat{r}_i - r_i| < c_0)$ for some fixed c_0. When we discuss methods other than the SI method, we follow this framework for consistency.

3 Empirical Bayesian Approach

For all methods other than than the SI method, we apply a Bayesian approach and utilize Bayes' Theorem [1] to estimate $\boldsymbol{\theta}$, given values of $\boldsymbol{x}$:

$$P(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{P(\boldsymbol{x}|\boldsymbol{\theta})P(\boldsymbol{\theta})}{P(\boldsymbol{x})}, \tag{3.1}$$

where

$$\begin{aligned} P(\boldsymbol{\theta}|\boldsymbol{x}) &= \text{posterior distribution of } \boldsymbol{\theta} \text{ given } \boldsymbol{x}, \\ P(\boldsymbol{x}|\boldsymbol{\theta}) &= \text{likelihood of } \boldsymbol{x} \text{ given } \boldsymbol{\theta}, \\ P(\boldsymbol{\theta}) &= \text{prior density of } \boldsymbol{\theta}, \text{ and} \\ P(\boldsymbol{x}) &= \text{marginal density of } \boldsymbol{x}. \end{aligned}$$

More specifically, most of the ranking procedures in this paper utilize empirical Bayesian models. In these methods, Klein and Wright [4] assume that the θ_i's in (2.1) are identically distributed according to the following:

$$\theta_1, \ldots, \theta_k | \mu, \tau \overset{\text{iid}}{\sim} N(\mu, \tau^2). \tag{3.2}$$

This is also known as the prior distribution. In order to estimate a θ_i, estimates of hyperparameters μ and τ^2 must be found. This can be done empirically by solving the system of nonlinear equations (3.3) derived from maximizing the marginal density of $\boldsymbol{x}$, as shown in [4]. These equations are solved numerically for μ and τ^2 by the R library 'nleqslv' [3].

$$g_1(\mu, \tau^2) \equiv \mu - \frac{\sum_{i=1}^{k} \frac{x_i}{\sigma_i^2 + \tau^2}}{\sum_{i=1}^{k} \frac{1}{\sigma_1^2 + \tau^2}} = 0, \quad g_2(\mu, \tau^2) \equiv \tau^2 - \frac{\sum_{i=1}^{k} \frac{(x_i - \mu)^2 - \sigma_i^2}{(\sigma_i^2 + \tau^2)^2}}{\sum_{i=1}^{k} \frac{1}{(\sigma_i^2 + \tau^2)^2}} = 0 \tag{3.3}$$

Once the values of μ and τ^2 are found, the posterior density of θ is given by:

$$\theta_i \;\; \boldsymbol{x}, \mu, \tau \overset{\text{ind}}{\sim} N[\delta_i(x_i, \mu, \tau), \omega_i^2(\tau)], \tag{3.4}$$

where δ and ω^2 are functions for the posterior mean and variance respectively given by:

$$\delta_i(x_i, \mu, \tau) = \frac{x_i \tau^2 + \mu \sigma_i^2}{\tau^2 + \sigma_i^2}, \quad \omega_i^2(\tau) = \frac{1}{1/\sigma_i^2 + 1/\tau^2}. \tag{3.5}$$

4 Ranking Algorithms and Implementation

4.1 Ranking Procedure 1 (P1)

The first Bayesian ranking procedure uses the posterior probabilities to compute a ranking on $\boldsymbol{\theta}$. Let $\mathcal{P}$ be the set of k populations to be ranked and let $s = 1$. For each θ_i in

$\mathcal{P}$, where $i \in \{1, \ldots, k\}$, compute the probability that θ_i is rank s. That is, compute $P(\theta_1 \leq \theta_s, \ldots, \theta_{s-1} \leq \theta_s, \theta_{s+1} \leq \theta_s, \ldots \theta_k \leq \theta_s | \boldsymbol{x})$. Let θ_j denote the population with the maximum posterior probability and associate rank s with θ_j. Reduce the population set by letting $\mathcal{P} = \mathcal{P} - \{\theta_j\}$ and increment s. Continue until all θ_i's are ranked.

To implement this method, we draw an $n \times k$ matrix X of x_i's where each row represents an independent trial. Let $s = 1$ as in above. For each iteration of the method, a new $n \times k - s$ matrix Θ of estimated θ_i's is drawn. To estimate $P(\theta_1 \leq \theta_s, \ldots, \theta_{s-1} \leq \theta_s, \theta_{s+1} \leq \theta_s, \ldots \theta_k \leq \theta_s | \boldsymbol{x})$, we place a 1 in the position of the smallest estimated θ_i, and 0's everywhere else. We compute the column sums and divide by n to obtain the required probabilities. Let column j denote the column with the maximum probability. Associate θ_j with rank s. Delete column j from Θ. Increment s and repeat until Θ has only one column. Associate the last θ_i with rank k.

4.2 Ranking Procedure 2 (P2)

The procedure for the second Bayesian ranking method is identical to P1, except that the rankings are computed in reverse order. That is, let $s = k$ on the initial iteration and decrement s until all population have been ranked. The last θ_i is associated with rank 1.

4.3 Posterior Means Ranking Procedure (PM)

The posterior means Bayesian method ranks the unknown population means by sorting their corresponding Bayesian estimate: the posterior mean. To implement this method, compute posterior means by evaluating δ_i in (3.5) for $i \in \{1, \ldots, k\}$. The implementation is identical to that of the simple method.

The above methods are all implemented by Klein and Wright. Detailed descriptions of them may be found in [4]. Both P1 and P2 lose information in their ranking process by deleting previously ranked populations from consideration. PM reflects the simple method. The maximum probability procedure which follows, however, computes the most likely ranking arrangement by considering the frequency of the ranks themselves. The method still utilizes Bayesian statistics, but with a different approach.

4.4 Maximum Probability Procedure (MXPR)

The "Maximum Probability Method", or MXPR method, is our first addition to the methods proposed by [4]. Where P1 and P2 look for the estimated θ that has the highest probability of being a minimum or maximum, this method looks for the most likely overall ranking of the estimated θ's. Once a matrix of θ_i have been drawn (each row being an independent set of $\boldsymbol{\theta}$), a simple R function `rank` is applied to each row. The MXPR method then begins to assess the matrix row by row, checking for identical ranking matches, increasing a counter every time a match is found. For a matrix that contains n rows of estimates, it is only necessary to proceed until one ranking has a counter of size $n/2 + 1$. Once this ranking is found, there is no other unique ranking in the matrix that can have a higher number of occurrences. This ranking is then returned as the most probable ranking of the estimates.

5 Fully Informed Prior Distribution

For each of the Bayesian methods, we must first place a prior distribution on $\boldsymbol{\theta}$. This prior distribution is described by [4] as $\theta_1, \ldots, \theta_k|\mu, \tau, \overset{iid}{\sim} N(\mu, \tau^2)$.

As described in Section 3, a non-linear equation solver is used to solve for the most likely μ and τ. These values of μ and τ are then used to draw vectors of $\boldsymbol{\theta}$ which are subsequently used for the various ranking procedures. This is referred to as the "non-informative" prior distribution in this paper. Due to the fact that the same μ and τ are used across all populations, we assume that all populations are identically distributed. It is our suggestion that a more useful assumption can be made about the prior distribution on $\boldsymbol{\theta}$. To improve on this, we have developed a method of "Fully Informed Priors" (FIP), in which data from previous years is used to help determine the distribution of $\boldsymbol{\theta}$. The previous year's mean and variance for each state is used as the mean and variance of the prior distribution of theta for the respective state in the current year. As a result, the FIP method yields a different prior distribution for each state, whereas the priors used by Klein were identical and hence non-infomative). We believe that using this informed prior allows us to achieve a more probable list of estimated ranks.

Two different sets of data are used as prior information: 2007 estimates with μ and standard error τ, and the 3-year averages from 2005-2007. Our simulation data, see Section 11, shows that using the prior information greatly increases the probabilities of a correct ranking. However the estimated rankings are altered significantly. We believe this is due to the τ values used in the prior distribution being too small, or too specific, and therefore weighing in too heavily on the ranking decision for the current year's estimates. The prior information is essentially too informative.

6 General Improvements to the Informed Prior

6.1 Relaxation of the Hyper Parameter τ

Using the prior years' values as the hyper-parameters of the prior distribution leaves us with an estimated ranking that heavily reflects past data, and is therefore not an ideal way for us to inform the prior distribution. To compensate for this, we progressively relax the τ value by adding a multiplier. The three multipliers used for testing are: $\tau \times 1$ (normal strength of τ), $\tau \times 2$, and $\tau \times 4$. Of these values, we have decided that a multiplier of 2 yields the best results. By relaxing the value for τ, we can produce a ranking that is more indicative of the current year's data while still producing higher probabilities of a correct ranking. For example, figure 6.1 shows the P1FIP and P1RIP methods with a progressive relaxation of τ by adding the multiplier to the informed prior versions. This data is compared to the

U.S. Census Bureau's current ranking procedure, the Simple Method (SI). Note that with no multiplier on τ the rankings are harshly affected by the prior distribution.

As the multiplier is increased, the rankings become more representative of the current year's data. Even with an overly-relaxed multiplier of 4 on τ, the probability is higher than that of the non-informative prior version in almost every case.

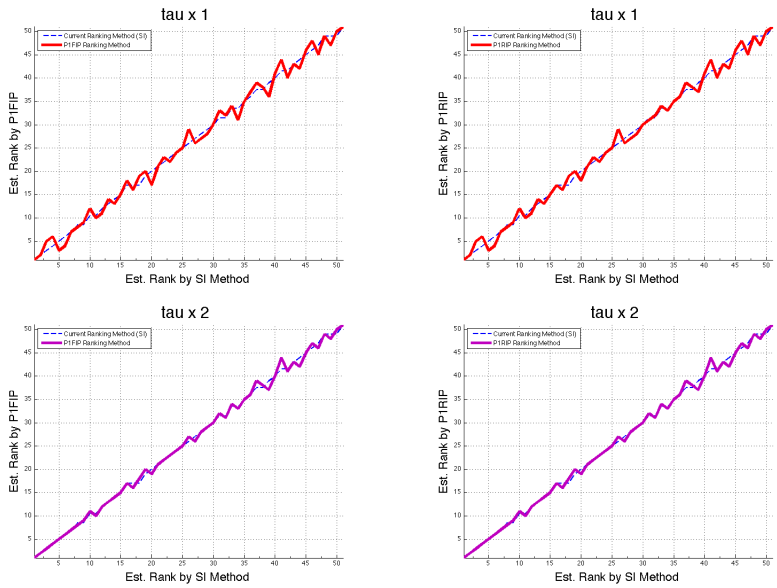


Table 6.1: P1FIP and P1RIP methods with multipliers on τ, as compared to the SI Method.

6.2 Regression: Version 1

Another approach used to find a middle ground between the non-informative and FIP is a regression procedure. This is a Bayesian approach to regression that is based on the following relationship:

$$\begin{aligned} \boldsymbol{\theta} &= \beta_0 + \beta_1 \boldsymbol{\theta_{prev}} + \boldsymbol{\varepsilon}, \\ \boldsymbol{\theta} &= \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \end{aligned} \tag{6.1}$$

where

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_k \end{bmatrix}, \quad \boldsymbol{X} = \begin{bmatrix} 1 & \theta_{prev_1} \\ 1 & \theta_{prev_2} \\ \vdots & \vdots \\ 1 & \theta_{prev_k} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}, \text{ and } \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}.$$

In (6.1), $\boldsymbol{\theta}$ and $\boldsymbol{\theta_{prev}}$ are, respectively, the current and previous data, $\boldsymbol{\beta}$ is the coefficient matrix describing the relationship between the two sets of data, and $\boldsymbol{\varepsilon}$ is the vector of error terms.

As described in [2], the posterior distribution of $\boldsymbol{\beta}$, given σ^2, can be found to be

$$\boldsymbol{\beta} | \sigma^2, \boldsymbol{\theta} \sim N(\hat{\beta}, V_\beta \sigma^2), \tag{6.2}$$

In order to find $\hat{\beta}$ and V_β, a linear relationship between $\boldsymbol{x_{prev}}$ and $\boldsymbol{x}$ is found via a linear regression [5] based on

$$\boldsymbol{x} = \hat{\beta}_0 + \hat{\beta}_1 \boldsymbol{x_{prev}}. \tag{6.3}$$

The value of σ^2 must then be found. The marginal posterior distribution of σ^2 has a scaled inverse-χ^2 distribution with degrees of freedom $k-2$ and scale s^2 where

$$s^2 = \frac{(\boldsymbol{\theta} - \boldsymbol{X}\hat{\boldsymbol{\beta}})'(\boldsymbol{\theta} - \boldsymbol{X}\hat{\boldsymbol{\beta}})}{k-2}. \tag{6.4}$$

Once all of these values have been found, it is simple to draw from (6.2) to assign values to β_0 and β_1. Next, $\boldsymbol{\varepsilon}$ is drawn from a $N(0, \sigma^2)$ distribution. This allows us to estimate $\boldsymbol{\theta}$ by substituting all of these values into (6.1).

Unfortunately, once implemented, this method proved to be unsuccessful in accurately estimating $\boldsymbol{\theta}$, yielding inconsistent rankings and less than sufficient probabilities of a correct ranking.

6.3 Regression: Version 2

The final approach to reducing the strength of the effect of the prior information on the estimated rankings is also a regression procedure. This procedure is a simplification of Regression Version 1 where we reduce the posterior distribution of β_0, β_1 to only the mean of the distribution in (6.2). We do this by running a linear regression using the prior information as the explanatory variable and the current information as the response variable as in

$$\boldsymbol{x} = \beta_0 + \beta_1 \boldsymbol{x}_{prev} \tag{6.5}$$

in order to obtain coefficients β_0 and β_1. Once these have been obtained, the mean $\boldsymbol{\mu}$ of the prior distribution is specified using the following:

$$\boldsymbol{\mu} = \beta_0 + \beta_1 \boldsymbol{x}_{prev}. \tag{6.6}$$

In contrast with Regression Version 1 which found a posterior distribution for σ, a common variance for all sample estimates, RIP uses the standard deviations (τ_i) of the previous data as the standard deviations for prior distribution. These standard deviations vary from state to state.

Using this approach for determining the mean and variance of the prior distribution, rankings were then generated using ranking procedures P1, P2, PM, and MXPR. This method was much more successful than the first version of regression implemented. Results are discussed in the following section.

6.4 Simulation Results

Once all methods have been implemented, simulations are run to measure the accuracy of the rankings produced by the P1FIP and P1RIP with different τ multipliers for each. Table 6.2 shows each simulation set of data used.

Data Set	$\boldsymbol{\theta}$	$\boldsymbol{\sigma}$
(i)	10.0, 10.2, 10.4, 10.6, 10.8	0.07, 0.07, 0.07, 0.07, 0.07
(ii)	10.0, 10.2, 10.4, 10.6, 10.8	0.05, 0.05, 0.2, 0.2, 0.2
(iii)	10.0, 10.2, 10.7, 11.2, 11.4	0.15, 0.15, 0.25, 0.15, 0.15
(iv)	10.0, 10.5, 10.7, 11.0, 11.2	0.1, 0.3, 0.3, 0.1, 0.5
(v)	9.8, 10.5, 10.7, 10.9, 11.6	0.5, 0.1, 0.1, 0.1, 0.5
Prev. Set	$\boldsymbol{\theta}_{prev}$	
(1)	10.1, 10.3, 10.4, 10.7, 10.9	
(2)	10.1, 10.2, 10.8, 10.4, 11.0	
(3)	10.7, 10.5, 10.3, 10.1, 9.9	

Table 6.2: Simulation Settings: $\boldsymbol{\theta}$ = unknown population means, $\boldsymbol{\sigma}$ = known standard deviations of $\boldsymbol{x}$, $\boldsymbol{\theta}_{prev}$ = unknown prior population means

	True Rank	SI		P1FIP				P1RIP			
				$\tau \times 1$		$\tau \times 2$		$\tau \times 1$		$\tau \times 2$	
	r_i	$\hat{r}_i$	$P(\hat{r}_i = r_i)$	$\hat{r}_i$	$P(\hat{r}_i = r_i)$	$\hat{r}_i$	$P(\hat{r}_i = r_i)$	$\hat{r}_i$	$P(\hat{r}_i = r_i)$	$\hat{r}_i$	$P(\hat{r}_i = r_i)$
PrevSet(1)	1	1	0.93	1	0.99	1	0.96	1	0.99	1	0.96
	2	2	0.56	2	0.75	2	0.63	2	0.78	2	0.65
	3	3	0.43	3	0.71	3	0.54	3	0.74	3	0.56
	4	4	0.56	4	0.77	4	0.64	4	0.80	4	0.67
	5	5	0.63	5	0.79	5	0.67	5	0.79	5	0.68
PrevSet(2)	1	1	0.93	1	0.98	1	0.96	1	0.96	1	0.95
	2	2	0.56	2	0.94	2	0.74	2	0.86	2	0.71
	3	3	0.43	4	0.33	3	0.51	4	0.31	3	0.47
	4	4	0.56	3	0.38	4	0.62	3	0.36	4	0.55
	5	5	0.63	5	0.89	5	0.72	5	0.80	5	0.69
PrevSet(3)	1	1	0.93	1	0.56	1	0.88	1	0.99	1	0.92
	2	2	0.56	3	0.23	2	0.51	2	0.84	2	0.58
	3	3	0.43	2	0.22	3	0.39	3	0.79	3	0.48
	4	4	0.56	5	0.42	4	0.50	4	0.79	4	0.63
	5	5	0.63	4	0.39	5	0.57	5	0.79	5	0.70

Table 6.3: Probability results, $P(\hat{r}_i = r_i)$, as compared to current SI ranking method, using Data Set (iv) and each of the 3 Prev. settings. $\tau = \sigma$.

Note that Prev. Set (1) is in the same ascending order as $\boldsymbol{\theta}$, Prev. Set (2) is also in the same ascending order with one set of results in the middle flipped, and Prev. Set (3) is in the opposite and descending order of $\boldsymbol{\theta}$ as the most extreme case. These $\boldsymbol{\theta}_{prev}$ settings are strategically picked to display whether or not the ranking method is weighing more heavily on the current estimates or prior information. For these simulations, prior standard deviations ($\boldsymbol{\tau}$) are taken to be the same as $\boldsymbol{\sigma}$. Table 6.3 shows the results of these simulations, specifically the estimated ranking and Monte Carlo estimates of the probabilities of correct ranks, compared with that of the non-informative, simple method.

These results indicate that for a τ multiplier of 1, both informed methods were susceptible to ranking errors due to relying heavily on the prior information. However, for $\tau \times 2$, both P1FIP and P1RIP were successful in finding the correct ranking. For both Prev. Set (1) and Prev. Set (2), the informed methods performed comparably, each better than the simple method. In contrast, P1RIP out-performed the other informed method while producing results comparable to those of SI, with better results for some rankings.

7 Description of the Bootstrap Estimates

The probability of a correct ranking cannot be computed in the same manner as in the simulation settings, because the true values of $\boldsymbol{\theta}$ for the states are unknown. In the simulations, we know the true rankings and it is easy to compute the probability that the rank is correct. This probability is simply $P(\hat{r}_i = r_i)$.

It is not the same, however, when we do not know the true value of $\boldsymbol{\theta}$, as in the cases of the state rankings. We perform a parametric bootstrap as outlined in [4]. Once an estimated ranking $\hat{r}_1, \ldots, \hat{r}_k$ is established through any of the various methods, we then draw m independent sets which are referred to as $x_1^*, \ldots, x_k^*$. The variable m, in this case, is the number of replications performed within the parametric bootstrap. Each independent set of $x_1^*, \ldots, x_k^*$ is then ranked using the exact same procedure used on the current years data, $\boldsymbol{x}$. These rankings of $x_1^*, \ldots, x_k^*$ are referred to as $\hat{r}_i^*, \ldots, \hat{r}_k^*$.

We use the same probability equation as used in [4] to obtain comparison results. There, the bootstrap estimate equation is

$$P(|r_i - \hat{r}_i| \leq c_0) = \frac{1}{m} \sum_{z=1}^{m} I(|\hat{r}_i - \hat{r}_{i,b}^*| \leq c_0), \tag{7.1}$$

where m is the number of replications in the bootstrap, and c_0 is the desired value for size of error to test for. These estimates are reflected in the data tables in Section 11.

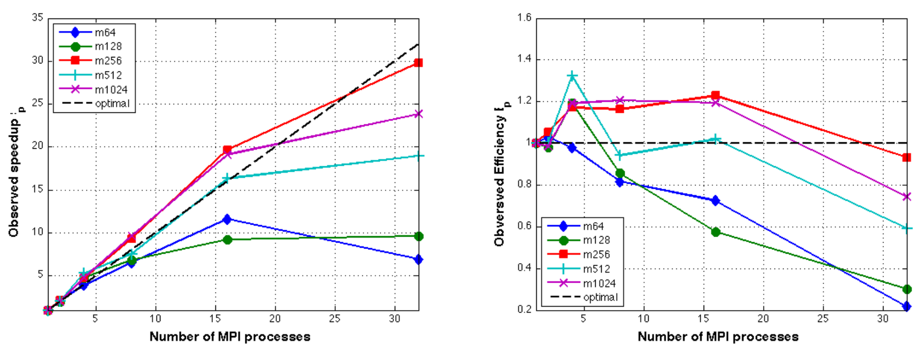
8 Parallelization of the Bootstrap Process Using "SNOW"

Using the R package, "SNOW" [6] and MPI (Message Passing Interface) communications, we are able to drastically reduce the amount of time required to compute the bootstrap estimates for the various ranking procedures. This is important because creates a more real-time analysis of data, allowing us to draw conclusions and notice trends much faster, making for more productive research. The bootstrap performs m replications, and, while using a serial program, this workload goes to a single processor. By splitting the workload among multiple processors, we reduce the number of replications performed on each processor. This new number, l_m, the local value of m for each processor, is defined by $l_m = \frac{m}{p}$, where p is the number of processors assigned to the task. It can be seen by Table 8.1 that as the number of replications increases from top to bottom, so does the amount of time required compute the bootstrap estimates. As the number of processors assigned to the job increases from left to right, the time required decreases, due to the workload being decreased for each processor.

This effect is achieved by using a SNOW cluster. A cluster of size p is created and the local number of replications, l_m, to be computed on each processor is found. This number is then sent to each processor, which performs the tasks using the `clusterCall` function, sending the exact same data to each processor. For the ranking methods that are required to draw random values from a distribution, it is important that each processor receive a different seed for its random number generator as to maintain an overall level of randomness. This is achieved by using a `setupClusterRNG` function prior to the ranking method being called on each processor.

m	$p = 1$	$p = 2$	$p = 4$	$p = 8$	$p = 16$	$p = 32$
64	9.77 min	4.75 min	2.50 min	1.50 min	50.57 sec	1.41 min
128	17.97 min	9.17 min	3.76 min	2.63 min	1.95 min	1.87 min
256	38.11 min	18.07 min	8.13 min	4.09 min	1.94 min	1.28 min
512	1.17 hrs	35.15 min	13.31 min	9.35 min	4.32 min	3.72 min
1024	2.35 hrs	1.19 hrs	29.54 min	14.59 min	7.37 min	5.91 min

Table 8.1: Time study for parallelized bootstrap for increasing bootstrap sizes, m, using the P1 ranking procedure with Klein and Wright prior and p processes.



(a) Speedup (b) Efficiency

Figure 8.1: Performance study of P1 ranking procedure with Klein and Wright prior as applied to 2008 states data set.

The speedup and efficiency plots for this time study can be seen in Figure 8.1. Note that the best speedup and overall efficiency is achieved at $m = 256$. This may be due to the amount of data fitting evenly into cache size chunks.

Using MPI communications causes some time lag due to the transfer of information between nodes. This is most notable in the instance of $m = 64$, where $p = 16$ actually performs faster than $p = 32$, which, while using twice the processors, should be twice as fast as $p = 16$. This is because computation takes much less time than communication.

When the task is split up among the 16 processors in this case, it computes much faster than it does if it sends data to twice the amount of processors, requiring twice the amount of communication. This only occurs in our study at a very low m size. Once m becomes reasonably large, it takes slightly less time to run on 32 processors than it does on 16.

9 Conclusions and Future Work

9.1 Conclusions

We conclude that informing the prior distribution gives a more realistic model than the Klein and Wright prior. This finding is most clear when using procedure 1 with a regressively informed prior distribution, P1RIP. This method estimates rankings more accurately with higher corresponding probabilities more often than other methods when the previous data has a linear trend. Procedure 1 with a fully informed prior (P1FIP) and no regression, however, performs better than P1RIP when no linear trends in the previous data exist. Both techniques suffer when certain ordering techniques are violated in our simulation studies: P1RIP, when linear trends do not exist in the previous data, and P1FIP, when the order of the previous mean estimates do not match the order of the current population means. A previous data standard deviation, τ, with a constant multiplier reduces the influence of the previous data and enables the methods to achieve correct rankings.

9.2 Future Work

We suggest that future researchers investigate informing the prior distribution of our Bayesian approach as a starting point in an investigation to improve the Census Bureau's current method. In particular, investigating regression priors seem very useful. Researchers could also add optimization techniques to find a suitable constant multiplier for τ when informing our prior distributions. Another method may be to combine several approaches by adding decision criteria to let programs choose a suitable ranking procedure. For example, if the regression coefficient is within a certain distance from zero, then the program could decide not to use regression and instead choose a procedure that can better handle non-linear data.

10 Acknowledgments

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11 Data and Tables

		SI		P1		P2		PM		MXPR
Setting(i)	$\hat{r}_1$	0.98	*0.98*	0.98	*0.98*	0.98	*0.98*	0.98	*0.98*	0.98
	$\hat{r}_2$	0.96	*0.96*	0.96	*0.96*	0.96	*0.96*	0.96	*0.96*	0.96
	$\hat{r}_3$	0.96	*0.96*	0.96	*0.96*	0.96	*0.96*	0.96	*0.96*	0.95
	$\hat{r}_4$	0.96	*0.97*	0.96	*0.97*	0.96	*0.97*	0.96	*0.97*	0.95
	$\hat{r}_5$	0.98	*0.98*	0.98	*0.98*	0.98	*0.98*	0.98	*0.98*	0.98
Setting(ii)	$\hat{r}_1$	0.97	*0.96*	0.99	*0.98*	0.99	*0.99*	0.99	*0.98*	0.99
	$\hat{r}_2$	0.81	*0.78*	0.86	*0.83*	0.86	*0.85*	0.85	*0.85*	0.84
	$\hat{r}_3$	0.57	*0.56*	0.61	*0.60*	0.61	*0.62*	0.61	*0.61*	0.59
	$\hat{r}_4$	0.55	*0.53*	0.52	*0.53*	0.52	*0.54*	0.54	*0.53*	0.54
	$\hat{r}_5$	0.73	*0.72*	0.70	*0.72*	0.70	*0.72*	0.72	*0.72*	0.71
Setting(iii)	$\hat{r}_1$	0.82	*0.82*	0.83	*0.82*	0.83	*0.82*	0.83	*0.82*	0.82
	$\hat{r}_2$	0.79	*0.79*	0.80	*0.79*	0.80	*0.80*	0.80	*0.80*	0.78
	$\hat{r}_3$	0.91	*0.91*	0.93	*0.94*	0.93	*0.94*	0.93	*0.93*	0.92
	$\hat{r}_4$	0.79	*0.78*	0.79	*0.79*	0.79	*0.79*	0.80	*0.79*	0.80
	$\hat{r}_5$	0.82	*0.82*	0.82	*0.82*	0.82	*0.82*	0.82	*0.82*	0.83
Setting(iv)	$\hat{r}_1$	0.92	*0.93*	0.98	*0.98*	0.98	*0.98*	0.98	*0.98*	0.98
	$\hat{r}_2$	0.57	*0.56*	0.62	*0.59*	0.62	*0.60*	0.61	*0.59*	0.61
	$\hat{r}_3$	0.45	*0.43*	0.42	*0.44*	0.42	*0.43*	0.45	*0.43*	0.44
	$\hat{r}_4$	0.57	*0.56*	0.30	*0.34*	0.30	*0.36*	0.30	*0.32*	0.33
	$\hat{r}_5$	0.63	*0.63*	0.30	*0.31*	0.30	*0.35*	0.31	*0.31*	0.31
Setting(v)	$\hat{r}_1$	0.91	*0.91*	0.30	*0.37*	0.30	*0.34*	0.29	*0.35*	0.28
	$\hat{r}_2$	0.83	*0.84*	0.31	*0.36*	0.30	*0.34*	0.29	*0.34*	0.28
	$\hat{r}_3$	0.78	*0.80*	0.60	*0.63*	0.60	*0.63*	0.62	*0.64*	0.61
	$\hat{r}_4$	0.83	*0.85*	0.30	*0.34*	0.30	*0.35*	0.30	*0.34*	0.31
	$\hat{r}_5$	0.91	*0.92*	0.30	*0.34*	0.30	*0.37*	0.30	*0.35*	0.29

Table 11.1: Probability results, $P(\hat{r}_i = r_i)$, by ranking method, using multiple test settings. All methods utilize KWP to specify the prior distribution. Italicized columns refer to data from Klein [4] for comparison.

Comparing Ranks

State Name	SI	P1			P2			PM			MXPR			RIP
		KWP	FIP	RIP	KWP	FIP	RIP	KWP	FIP	RIP	KWP	FIP	RIP	ver. 1
New Hampshire	1.0	1	1	1	1	1	1	1.0	1.0	1.0	1	1	1	1
Maryland	2.0	2	2	2	2	2	2	2.0	2.0	2.0	2	2	2	5
Alaska	3.0	3	3	3	3	3	3	3.0	3.0	3.0	3	3	3	7
New Jersey	4.0	4	4	4	4	4	4	4.0	4.0	4.0	5	4	4	2
Hawaii	5.0	5	5	5	5	5	5	5.0	5.0	5.0	7	5	5	4
Connecticut	6.0	6	6	6	6	6	6	6.0	6.0	6.0	6	6	6	3
Wyoming	7.0	7	7	7	7	7	7	7.0	7.0	7.0	4	7	7	8
Minnesota	8.5	8	8	8	8	8	8	8.0	8.0	8.0	8	8	8	6
Utah	8.5	9	9	9	9	9	9	9.0	9.0	9.0	9	9	9	18
Delaware	10.5	11	11	11	11	11	11	11.0	11.0	11.0	12	11	11	11
Massachusetts	10.5	10	10	10	10	10	10	10.0	10.0	10.0	10	10	10	9
Virginia	12.0	12	12	12	12	12	12	12.0	12.0	12.0	11	12	12	14
Wisconsin	13.0	13	13	13	13	13	13	13.0	13.0	13.0	13	14	14	13
Vermont	14.0	14	14	14	14	14	14	14.0	14.0	14.0	14	13	13	10
Nebraska	15.0	15	15	15	15	15	15	15.0	15.0	15.0	15	15	15	17
Kansas	17.0	18	17	17	17	17	17	17.0	17.0	17.0	18	16	17	23
Nevada	17.0	16	16	16	18	16	16	18.0	16.0	16.0	19	17	16	12
Washington	17.0	17	18	18	16	18	18	16.0	18.0	18.0	16	18	18	15
Colorado	19.0	19	20	20	19	20	20	19.0	20.0	20.0	21	20	20	25
Iowa	20.0	20	19	19	20	19	19	20.0	19.0	19.0	20	19	19	16
Rhode Island	21.0	21	21	21	21	21	21	21.0	21.0	21.0	22	21	21	26
North Dakota	22.0	22	22	22	22	22	23	22.0	22.0	22.0	17	22	24	32
Pennsylvania	23.0	23	23	23	23	23	22	23.0	23.0	23.0	23	23	22	21
Illinois	24.0	24	24	24	24	24	24	24.0	24.0	24.0	24	24	23	20
Maine	25.0	25	25	25	25	25	25	25.0	25.0	25.0	25	25	26	28
South Dakota	26.0	26	27	27	26	27	27	26.0	27.0	27.0	26	27	30	27
Idaho	27.0	27	26	26	27	26	26	27.0	26.0	26.0	27	26	25	22
Indiana	28.0	28	28	28	28	28	28	28.0	28.0	28.0	28	28	27	19
Florida	29.0	29	29	29	29	29	29	29.0	29.0	29.0	29	29	28	24
California	30.0	30	30	30	30	30	30	30.0	30.0	30.0	30	30	29	31
Missouri	31.5	31	32	32	31	32	32	31.5	32.0	32.0	32	32	31	33
Ohio	31.5	32	31	31	32	31	31	31.5	31.0	31.0	31	31	32	29
New York	33.5	33	34	34	34	34	34	34.0	34.0	34.0	34	34	34	38
Oregon	33.5	34	33	33	33	33	33	33.0	33.0	33.0	33	33	33	30
Michigan	35.0	35	35	35	35	35	35	35.0	35.0	35.0	35	35	35	35
North Carolina	36.0	36	36	36	36	36	36	36.0	36.0	36.0	36	36	36	40
Arizona	37.5	38	39	39	37	38	38	37.0	38.0	38.0	37	38	39	39
Georgia	37.5	39	38	38	38	37	37	38.0	37.0	37.0	38	37	37	34
Montana	39.0	37	37	37	39	39	39	39.0	39.0	39.0	39	39	38	36
Tennessee	40.0	40	40	40	40	40	40	40.0	40.0	40.0	40	40	40	41
Alabama	41.5	42	44	44	42	44	44	41.5	44.0	44.0	42	44	42	46
South Carolina	41.5	41	41	41	41	41	41	41.5	41.0	41.0	41	41	41	37
Texas	43.0	43	43	43	43	42	42	43.0	42.5	42.5	43	42	44	47
Oklahoma	44.0	44	42	42	44	43	43	44.0	42.5	42.5	44	43	43	42
West Virginia	45.0	46	45	45	46	45	45	46.0	45.0	45.0	47	46	45	43
New Mexico	46.0	47	47	47	47	47	47	47.0	47.0	47.0	45	47	47	48
Dist. of Columbia	47.0	45	46	46	45	46	46	45.0	46.0	46.0	49	45	46	44
Arkansas	49.0	48	49	49	50	49	49	48.0	49.0	49.0	50	48	49	49
Kentucky	49.0	50	48	48	49	48	48	50.0	48.0	48.0	48	49	48	45
Louisiana	49.0	49	50	50	48	50	50	49.0	50.0	50.0	46	50	50	50
Mississippi	51.0	51	51	51	51	51	51	51.0	51.0	51.0	51	51	51	51

Table 11.2: Table of estimated ranks for 2008 data using 2007 estimates as previous data with $\tau \times 2$.

Comparing Bootstrap Estimates of $P(|\hat{r}_i - r_i| \leq 1)$

State Name	SI	P1			P2			PM			MXPR			RIP
		KWP	FIP	RIP	KWP	FIP	RIP	KWP	FIP	RIP	KWP	FIP	RIP	ver. 1
New Hampshire	0.96	0.96	1.00	1.00	0.96	1.00	1.00	0.96	0.99	1.00	0.94	0.99	0.99	0.83
Maryland	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	1.00	0.99	0.98	0.98	0.44
Alaska	0.75	0.77	0.72	0.73	0.74	0.71	0.72	0.75	0.71	0.79	0.70	0.69	0.70	0.44
New Jersey	0.96	0.97	0.92	0.93	0.96	0.93	0.94	0.96	0.92	0.89	0.72	0.91	0.92	0.23
Hawaii	0.63	0.62	0.73	0.75	0.60	0.73	0.75	0.62	0.72	0.73	0.38	0.70	0.72	0.50
Connecticut	0.77	0.79	0.92	0.91	0.80	0.92	0.91	0.78	0.91	0.95	0.76	0.90	0.89	0.62
Wyoming	0.39	0.42	0.58	0.54	0.38	0.57	0.55	0.37	0.58	0.67	0.22	0.53	0.50	0.29
Minnesota	0.59	0.79	0.91	0.91	0.78	0.92	0.91	0.78	0.94	0.98	0.77	0.90	0.89	0.25
Utah	0.43	0.57	0.77	0.77	0.57	0.77	0.77	0.56	0.75	0.66	0.54	0.75	0.73	0.03
Delaware	0.32	0.43	0.51	0.51	0.40	0.52	0.52	0.45	0.54	0.64	0.35	0.47	0.46	0.26
Massachusetts	0.62	0.80	0.82	0.81	0.78	0.83	0.83	0.76	0.81	0.87	0.77	0.81	0.80	0.35
Virginia	0.80	0.77	0.85	0.85	0.77	0.84	0.84	0.77	0.83	0.81	0.70	0.82	0.82	0.19
Wisconsin	0.87	0.86	0.93	0.93	0.85	0.94	0.93	0.86	0.94	0.99	0.83	0.83	0.83	0.26
Vermont	0.46	0.47	0.58	0.58	0.46	0.58	0.58	0.47	0.57	0.75	0.42	0.51	0.50	0.33
Nebraska	0.77	0.75	0.83	0.82	0.74	0.84	0.83	0.77	0.85	0.84	0.68	0.79	0.78	0.26
Kansas	0.49	0.47	0.60	0.60	0.50	0.62	0.62	0.49	0.58	0.67	0.45	0.47	0.57	0.13
Nevada	0.45	0.39	0.55	0.56	0.39	0.55	0.55	0.41	0.55	0.84	0.34	0.49	0.54	0.24
Washington	0.62	0.59	0.72	0.72	0.45	0.72	0.73	0.47	0.72	0.72	0.43	0.71	0.71	0.18
Colorado	0.49	0.48	0.74	0.74	0.52	0.74	0.74	0.50	0.70	0.72	0.34	0.67	0.68	0.22
Iowa	0.54	0.54	0.58	0.58	0.55	0.59	0.59	0.54	0.58	0.89	0.51	0.52	0.53	0.28
Rhode Island	0.40	0.40	0.47	0.46	0.44	0.50	0.48	0.42	0.48	0.65	0.32	0.44	0.42	0.21
North Dakota	0.34	0.38	0.42	0.40	0.34	0.39	0.37	0.35	0.40	0.62	0.11	0.35	0.30	0.09
Pennsylvania	0.77	0.78	0.82	0.82	0.78	0.83	0.71	0.77	0.82	0.94	0.74	0.79	0.68	0.23
Illinois	0.78	0.76	0.80	0.81	0.79	0.81	0.81	0.78	0.80	0.96	0.77	0.80	0.68	0.19
Maine	0.56	0.59	0.58	0.59	0.56	0.58	0.59	0.57	0.59	0.82	0.55	0.58	0.53	0.18
South Dakota	0.52	0.52	0.57	0.58	0.55	0.57	0.57	0.53	0.54	0.50	0.49	0.48	0.16	0.14
Idaho	0.49	0.50	0.62	0.64	0.50	0.63	0.64	0.50	0.63	0.60	0.46	0.59	0.51	0.20
Indiana	0.67	0.68	0.86	0.84	0.71	0.84	0.83	0.67	0.84	0.94	0.64	0.82	0.72	0.09
Florida	0.77	0.74	0.90	0.89	0.78	0.89	0.88	0.77	0.88	0.77	0.75	0.86	0.85	0.22
California	0.71	0.71	0.90	0.89	0.71	0.90	0.89	0.71	0.88	0.83	0.68	0.88	0.75	0.19
Missouri	0.45	0.61	0.84	0.81	0.60	0.83	0.80	0.44	0.82	0.96	0.57	0.82	0.71	0.28
Ohio	0.44	0.58	0.76	0.75	0.58	0.76	0.75	0.44	0.76	0.85	0.58	0.77	0.82	0.21
New York	0.76	0.91	0.96	0.95	0.74	0.96	0.95	0.77	0.96	1.00	0.74	0.95	0.94	0.22
Oregon	0.59	0.61	0.75	0.76	0.72	0.75	0.76	0.71	0.73	0.95	0.67	0.72	0.72	0.29
Michigan	0.82	0.77	0.84	0.84	0.78	0.85	0.85	0.80	0.86	0.99	0.76	0.84	0.84	0.35
North Carolina	0.64	0.65	0.72	0.72	0.67	0.73	0.73	0.64	0.70	0.68	0.62	0.69	0.67	0.28
Arizona	0.56	0.75	0.58	0.57	0.68	0.81	0.80	0.71	0.80	0.78	0.69	0.79	0.55	0.35
Georgia	0.57	0.56	0.84	0.84	0.79	0.75	0.76	0.79	0.73	0.91	0.78	0.75	0.75	0.21
Montana	0.49	0.35	0.38	0.38	0.47	0.53	0.53	0.47	0.50	0.74	0.44	0.51	0.58	0.38
Tennessee	0.74	0.72	0.80	0.80	0.75	0.81	0.80	0.73	0.80	0.93	0.72	0.79	0.79	0.51
Alabama	0.39	0.59	0.63	0.61	0.61	0.64	0.61	0.41	0.64	0.94	0.56	0.64	0.54	0.52
South Carolina	0.39	0.61	0.88	0.87	0.59	0.88	0.87	0.41	0.88	0.98	0.60	0.88	0.87	0.22
Texas	0.82	0.85	0.93	0.93	0.84	0.75	0.74	0.82	0.71	0.77	0.82	0.74	0.63	0.16
Oklahoma	0.62	0.65	0.58	0.57	0.64	0.77	0.77	0.61	0.47	0.67	0.63	0.77	0.76	0.56
West Virginia	0.53	0.66	0.70	0.70	0.68	0.71	0.71	0.68	0.71	0.76	0.55	0.83	0.64	0.30
New Mexico	0.60	0.59	0.64	0.63	0.59	0.63	0.62	0.56	0.61	0.74	0.40	0.60	0.59	0.66
Dist. of Columbia	0.31	0.49	0.63	0.61	0.43	0.56	0.55	0.44	0.59	0.68	0.40	0.46	0.50	0.50
Arkansas	0.63	0.52	0.70	0.69	0.40	0.69	0.68	0.58	0.72	0.78	0.40	0.62	0.66	0.67
Kentucky	0.62	0.47	0.73	0.72	0.69	0.71	0.70	0.45	0.71	0.63	0.67	0.48	0.68	0.39
Louisiana	0.61	0.65	0.76	0.75	0.61	0.75	0.72	0.64	0.74	0.94	0.38	0.70	0.68	0.80
Mississippi	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11.3: Table of bootstrap estimates for 2008 data using 2007 estimates as previous data with $\tau \times 2$.

Basic Information

State Name	2008 Data		2007 Data		2005-2007 Data	
	x_i	σ_i	μ_i	τ_i	μ_i	τ_i
New Hampshire	7.6	0.36	7.1	0.36	7.7	0.24
Maryland	8.1	0.18	7.9	0.24	8.2	0.12
Alaska	8.4	0.49	8.0	0.30	10.4	0.30
New Jersey	8.7	0.18	8.3	0.24	8.7	0.12
Hawaii	9.1	0.43	8.6	0.18	9.1	0.24
Connecticut	9.3	0.24	8.7	0.73	8.2	0.12
Wyoming	9.4	0.55	8.9	0.49	8.9	0.36
Minnesota	9.6	0.18	9.5	0.18	9.6	0.12
Utah	9.6	0.30	9.7	0.30	10.3	0.12
Delaware	10.0	0.49	9.9	0.18	10.6	0.30
Massachusetts	10.0	0.18	9.9	0.18	10.1	0.12
Virginia	10.2	0.18	10.1	0.55	9.9	0.12
Wisconsin	10.4	0.18	10.5	0.55	10.8	0.12
Vermont	10.6	0.55	10.7	0.43	10.7	0.30
Nebraska	10.8	0.30	10.8	0.18	11.3	0.18
Kansas	11.3	0.30	11.0	0.30	11.9	0.18
Nevada	11.3	0.36	11.2	0.30	10.8	0.18
Washington	11.3	0.18	11.2	0.30	11.8	0.12
Colorado	11.4	0.30	11.4	0.18	11.8	0.18
Iowa	11.5	0.30	11.6	0.18	11.0	0.12
Rhode Island	11.7	0.49	11.9	0.18	11.7	0.24
North Dakota	12.0	0.55	12.0	0.24	11.9	0.30
Pennsylvania	12.1	0.12	12.0	0.36	11.9	0.06
Illinois	12.2	0.12	12.0	0.55	12.1	0.12
Maine	12.3	0.36	12.1	0.12	12.8	0.24
South Dakota	12.5	0.55	12.1	0.36	13.3	0.30
Idaho	12.6	0.55	12.1	0.55	13.0	0.30
Indiana	13.1	0.24	12.3	0.18	12.5	0.12
Florida	13.2	0.12	12.4	0.12	12.6	0.12
California	13.3	0.12	12.9	0.30	13.0	0.06
Missouri	13.4	0.18	13.0	0.24	13.4	0.12
Ohio	13.4	0.18	13.1	0.18	13.2	0.06
New York	13.6	0.12	13.1	0.49	14.0	0.06
Oregon	13.6	0.30	13.7	0.12	13.5	0.18
Michigan	14.4	0.18	14.0	0.18	13.7	0.12
North Carolina	14.6	0.24	14.1	0.49	14.8	0.12
Arizona	14.7	0.24	14.2	0.30	14.2	0.18
Georgia	14.7	0.18	14.3	0.18	14.5	0.12
Montana	14.8	0.55	14.3	0.18	14.0	0.30
Tennessee	15.5	0.24	15.0	0.30	15.9	0.18
Alabama	15.7	0.30	15.9	0.30	16.8	0.18
South Carolina	15.7	0.30	15.9	0.30	15.6	0.18
Texas	15.8	0.12	16.3	0.12	16.9	0.12
Oklahoma	15.9	0.30	16.4	0.85	16.6	0.24
West Virginia	17.0	0.43	16.9	0.30	17.7	0.24
New Mexico	17.1	0.43	16.9	0.36	18.4	0.30
Dist. of Columbia	17.2	0.79	17.3	0.30	18.8	0.61
Arkansas	17.3	0.43	17.9	0.36	17.5	0.18
Kentucky	17.3	0.30	18.1	0.49	17.1	0.18
Louisiana	17.3	0.36	18.6	0.30	19.3	0.24
Mississippi	21.2	0.55	20.6	0.43	21.1	0.30

Table 11.4: Table of information used for Maximum Probability, P1, P2, PM and SI methods.

SI Method: Estimated Ranking
and Bootstrap Estimate of $P(|\hat{r}_i - r_i| \leq 1)$

State Name	Rank	Bootstrap Estimate
New Hampshire	1.0	0.96
Maryland	2.0	0.98
Alaska	3.0	0.75
New Jersey	4.0	0.96
Hawaii	5.0	0.63
Connecticut	6.0	0.77
Wyoming	7.0	0.39
Minnesota	8.5	0.59
Utah	8.5	0.43
Delaware	10.5	0.32
Massachusetts	10.5	0.62
Virginia	12.0	0.80
Wisconsin	13.0	0.87
Vermont	14.0	0.46
Nebraska	15.0	0.77
Kansas	17.0	0.49
Nevada	17.0	0.45
Washington	17.0	0.62
Colorado	19.0	0.49
Iowa	20.0	0.54
Rhode Island	21.0	0.40
North Dakota	22.0	0.34
Pennsylvania	23.0	0.77
Illinois	24.0	0.78
Maine	25.0	0.56
South Dakota	26.0	0.52
Idaho	27.0	0.49
Indiana	28.0	0.67
Florida	29.0	0.77
California	30.0	0.71
Missouri	31.5	0.45
Ohio	31.5	0.44
New York	33.5	0.76
Oregon	33.5	0.59
Michigan	35.0	0.82
North Carolina	36.0	0.64
Arizona	37.5	0.56
Georgia	37.5	0.57
Montana	39.0	0.49
Tennessee	40.0	0.74
Alabama	41.5	0.39
South Carolina	41.5	0.39
Texas	43.0	0.82
Oklahoma	44.0	0.62
West Virginia	45.0	0.53
New Mexico	46.0	0.60
Dist. of Columbia	47.0	0.31
Arkansas	49.0	0.63
Kentucky	49.0	0.62
Louisiana	49.0	0.61
Mississippi	51.0	1.00

Table 11.5: Table of estimated ranking and bootstrap estimate using the SI Method

P1 Method: Estimated Ranking

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	1	1	1	1	1	1	1	1	1
Maryland	2	2	2	2	2	2	2	2	2
Alaska	3	5	3	5	3	8	6	8	6
New Jersey	4	6	4	6	4	4	3	4	3
Hawaii	5	3	5	3	5	6	5	6	5
Connecticut	6	4	6	4	6	3	4	3	4
Wyoming	7	7	7	7	7	5	7	5	7
Minnesota	8	8	8	8	8	7	8	7	8
Utah	9	9	9	9	9	11	9	11	9
Delaware	11	12	11	12	11	12	12	12	12
Massachusetts	10	10	10	10	10	10	10	10	10
Virginia	12	11	12	11	12	9	11	9	11
Wisconsin	13	14	13	14	13	13	13	13	13
Vermont	14	13	14	13	14	14	14	14	14
Nebraska	15	15	15	15	15	17	15	17	15
Kansas	18	18	17	17	17	21	19	21	19
Nevada	16	16	16	16	16	15	16	15	16
Washington	17	19	18	19	18	18	18	18	18
Colorado	19	20	20	20	20	19	20	19	20
Iowa	20	17	19	18	19	16	17	16	17
Rhode Island	21	21	21	21	21	20	21	20	21
North Dakota	22	23	22	23	22	22	22	22	22
Pennsylvania	23	22	23	22	23	23	23	23	23
Illinois	24	24	24	24	24	24	24	24	24
Maine	25	25	25	25	25	26	25	26	25
South Dakota	26	29	27	29	27	30	28	30	28
Idaho	27	26	26	26	26	28	26	27	26
Indiana	28	27	28	27	28	25	27	25	27
Florida	29	28	29	28	29	27	29	28	29
California	30	30	30	30	30	29	30	29	30
Missouri	31	33	32	31	32	32	32	32	32
Ohio	32	32	31	32	31	31	31	31	31
New York	33	34	34	34	34	35	34	34	34
Oregon	34	31	33	33	33	33	33	33	33
Michigan	35	35	35	35	35	34	35	35	35
North Carolina	36	37	36	36	36	39	39	39	39
Arizona	38	39	39	39	39	37	37	37	37
Georgia	39	38	38	38	38	38	38	38	38
Montana	37	36	37	37	37	36	36	36	36
Tennessee	40	41	40	41	40	41	40	41	40
Alabama	42	44	44	44	44	44	44	44	43
South Carolina	41	40	41	40	41	40	41	40	41
Texas	43	43	43	43	43	43	42	43	42
Oklahoma	44	42	42	42	42	42	43	42	44
West Virginia	46	46	45	46	45	47	46	47	46
New Mexico	47	48	47	48	47	48	48	48	48
Dist. of Columbia	45	45	46	45	46	49	49	49	49
Arkansas	48	49	49	49	49	46	47	46	47
Kentucky	50	47	48	47	48	45	45	45	45
Louisiana	49	50	50	50	50	50	50	50	50
Mississippi	51	51	51	51	51	51	51	51	51

Table 11.6: Table of estimated rankings using the P1 Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

P1 Method: Bootstrap Estimate of $P(|\hat{r}_i - r_i| \leq 1)$

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maryland	0.99	0.98	0.99	1.00	0.99	1.00	1.00	1.00	1.00
Alaska	0.77	0.73	0.72	0.65	0.73	0.94	0.79	0.95	0.77
New Jersey	0.97	0.80	0.92	0.75	0.93	1.00	0.87	1.00	0.88
Hawaii	0.62	0.93	0.73	0.83	0.75	1.00	0.74	1.00	0.72
Connecticut	0.79	0.76	0.92	0.71	0.91	1.00	0.97	1.00	0.95
Wyoming	0.42	0.86	0.58	0.83	0.54	1.00	0.67	1.00	0.67
Minnesota	0.79	0.99	0.91	0.98	0.91	1.00	0.98	1.00	0.98
Utah	0.57	0.95	0.77	0.95	0.77	1.00	0.66	1.00	0.65
Delaware	0.43	0.75	0.51	0.72	0.51	0.97	0.64	0.96	0.62
Massachusetts	0.80	0.91	0.82	0.90	0.81	0.97	0.87	0.97	0.86
Virginia	0.77	0.94	0.85	0.93	0.85	1.00	0.82	1.00	0.83
Wisconsin	0.86	0.97	0.93	0.97	0.93	1.00	0.99	1.00	0.98
Vermont	0.47	0.81	0.58	0.80	0.58	0.91	0.76	0.91	0.75
Nebraska	0.75	0.84	0.83	0.84	0.82	0.99	0.84	0.99	0.84
Kansas	0.47	0.75	0.60	0.74	0.60	0.73	0.71	0.71	0.71
Nevada	0.39	0.79	0.55	0.79	0.56	1.00	0.86	0.99	0.83
Washington	0.59	0.84	0.72	0.83	0.72	0.76	0.68	0.74	0.66
Colorado	0.48	0.84	0.74	0.78	0.74	0.76	0.73	0.75	0.72
Iowa	0.54	0.73	0.58	0.77	0.58	1.00	0.88	1.00	0.87
Rhode Island	0.40	0.68	0.47	0.66	0.46	0.71	0.67	0.69	0.65
North Dakota	0.38	0.42	0.42	0.42	0.40	0.87	0.62	0.87	0.63
Pennsylvania	0.78	0.86	0.82	0.86	0.82	0.99	0.96	0.99	0.96
Illinois	0.76	0.86	0.80	0.86	0.81	1.00	0.97	1.00	0.97
Maine	0.59	0.69	0.58	0.73	0.59	0.99	0.84	0.99	0.82
South Dakota	0.52	0.74	0.57	0.69	0.58	0.85	0.53	0.76	0.51
Idaho	0.50	0.74	0.62	0.79	0.64	0.86	0.63	0.82	0.61
Indiana	0.68	0.93	0.86	0.90	0.84	0.99	0.94	0.98	0.94
Florida	0.74	0.99	0.90	0.98	0.89	0.95	0.90	0.95	0.76
California	0.71	0.99	0.90	0.99	0.89	1.00	0.90	0.99	0.85
Missouri	0.61	0.68	0.84	0.71	0.81	1.00	0.96	1.00	0.96
Ohio	0.58	0.98	0.76	0.98	0.75	1.00	0.97	1.00	0.85
New York	0.91	1.00	0.96	1.00	0.95	1.00	1.00	0.99	1.00
Oregon	0.61	0.63	0.75	0.65	0.76	1.00	0.93	1.00	0.94
Michigan	0.77	0.96	0.84	0.96	0.84	0.98	0.99	1.00	0.99
North Carolina	0.65	0.92	0.72	0.73	0.72	1.00	0.79	1.00	0.67
Arizona	0.75	0.64	0.58	0.63	0.57	1.00	0.85	1.00	0.76
Georgia	0.56	0.91	0.84	0.90	0.84	1.00	0.89	1.00	0.89
Montana	0.35	0.60	0.38	0.52	0.38	0.98	0.73	0.98	0.73
Tennessee	0.72	1.00	0.80	0.99	0.80	1.00	0.95	1.00	0.94
Alabama	0.59	0.98	0.63	0.97	0.61	0.98	0.81	0.88	0.96
South Carolina	0.61	0.99	0.88	0.99	0.87	1.00	0.98	1.00	0.98
Texas	0.85	1.00	0.93	1.00	0.93	1.00	0.88	1.00	0.76
Oklahoma	0.65	0.96	0.58	0.95	0.57	0.91	0.91	0.83	0.68
West Virginia	0.66	1.00	0.70	0.99	0.70	1.00	0.74	0.99	0.75
New Mexico	0.59	0.92	0.64	0.91	0.63	1.00	0.77	1.00	0.78
Dist. of Columbia	0.49	0.81	0.63	0.79	0.61	0.97	0.59	0.98	0.63
Arkansas	0.52	0.93	0.70	0.91	0.69	1.00	0.79	1.00	0.78
Kentucky	0.47	0.97	0.73	0.95	0.72	1.00	0.68	1.00	0.62
Louisiana	0.65	1.00	0.76	0.99	0.75	1.00	0.94	1.00	0.94
Mississippi	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11.7: Table of bootstrap estimates using the P1 Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

P2 Method: Estimated Ranking

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	1	1	1	1	1	1	1	1	1
Maryland	2	2	2	2	2	2	2	2	2
Alaska	3	6	3	5	3	8	6	8	6
New Jersey	4	5	4	6	4	4	3	4	3
Hawaii	5	3	5	3	5	6	5	6	5
Connecticut	6	4	6	4	6	3	4	3	4
Wyoming	7	7	7	7	7	5	7	5	7
Minnesota	8	8	8	8	8	7	8	7	8
Utah	9	9	9	9	9	11	9	11	9
Delaware	11	12	11	12	11	12	12	12	12
Massachusetts	10	10	10	10	10	10	10	10	10
Virginia	12	11	12	11	12	9	11	9	11
Wisconsin	13	14	13	14	13	14	13	13	13
Vermont	14	13	14	13	14	13	14	14	14
Nebraska	15	15	15	15	15	17	15	17	15
Kansas	17	17	17	17	17	21	19	21	19
Nevada	18	16	16	16	16	15	16	15	16
Washington	16	19	18	19	18	18	18	18	18
Colorado	19	20	20	20	20	19	20	19	20
Iowa	20	18	19	18	19	16	17	16	17
Rhode Island	21	21	21	21	21	20	21	20	21
North Dakota	22	23	22	23	23	22	22	22	22
Pennsylvania	23	22	23	22	22	23	23	23	23
Illinois	24	24	24	24	24	24	24	24	24
Maine	25	25	25	25	25	26	25	26	25
South Dakota	26	29	27	29	27	30	28	30	28
Idaho	27	26	26	26	26	28	26	27	26
Indiana	28	27	28	27	28	25	27	25	27
Florida	29	28	29	28	29	27	29	28	29
California	30	30	30	30	30	29	30	29	30
Missouri	31	32	32	31	32	32	32	32	32
Ohio	32	31	31	32	31	31	31	31	31
New York	34	34	34	34	34	35	34	34	34
Oregon	33	33	33	33	33	33	33	33	33
Michigan	35	35	35	35	35	34	35	35	35
North Carolina	36	36	36	36	36	39	39	39	39
Arizona	37	39	38	39	38	37	37	37	37
Georgia	38	38	37	38	37	38	38	38	38
Montana	39	37	39	37	39	36	36	36	36
Tennessee	40	41	40	41	40	41	40	41	40
Alabama	42	44	44	44	44	44	44	44	43
South Carolina	41	40	41	40	41	40	41	40	41
Texas	43	43	42	43	42	43	42	43	42
Oklahoma	44	42	43	42	43	42	43	42	44
West Virginia	46	46	45	46	45	47	46	47	46
New Mexico	47	48	47	48	47	48	48	48	48
Dist. of Columbia	45	45	46	45	46	49	49	49	49
Arkansas	50	49	49	49	49	46	47	46	47
Kentucky	49	47	48	47	48	45	45	45	45
Louisiana	48	50	50	50	50	50	50	50	50
Mississippi	51	51	51	51	51	51	51	51	51

Table 11.8: Table of estimated rankings using the P2 Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

P2 Method: Bootstrap Estimate of $P(|\hat{r}_i - r_i| \leq 1)$

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Maryland	0.99	0.98	0.99	1.00	0.99	1.00	1.00	1.00	1.00
Alaska	0.74	0.61	0.71	0.66	0.72	0.94	0.79	0.95	0.77
New Jersey	0.96	0.95	0.93	0.73	0.94	1.00	0.88	1.00	0.89
Hawaii	0.60	0.93	0.73	0.84	0.75	1.00	0.74	1.00	0.72
Connecticut	0.80	0.77	0.92	0.73	0.91	1.00	0.98	1.00	0.96
Wyoming	0.38	0.87	0.57	0.85	0.55	1.00	0.67	1.00	0.68
Minnesota	0.78	0.99	0.92	0.99	0.91	1.00	0.97	1.00	0.97
Utah	0.57	0.95	0.77	0.95	0.77	1.00	0.67	1.00	0.66
Delaware	0.40	0.76	0.52	0.73	0.52	0.96	0.67	0.96	0.65
Massachusetts	0.78	0.91	0.83	0.91	0.83	0.97	0.87	0.97	0.87
Virginia	0.77	0.95	0.84	0.93	0.84	1.00	0.82	1.00	0.83
Wisconsin	0.85	0.97	0.94	0.97	0.93	0.96	0.99	1.00	0.98
Vermont	0.46	0.82	0.58	0.81	0.58	0.93	0.75	0.91	0.74
Nebraska	0.74	0.84	0.84	0.84	0.83	0.99	0.84	0.98	0.84
Kansas	0.50	0.75	0.62	0.74	0.62	0.72	0.72	0.71	0.71
Nevada	0.39	0.78	0.55	0.78	0.55	1.00	0.85	0.99	0.83
Washington	0.45	0.84	0.72	0.83	0.73	0.77	0.69	0.75	0.68
Colorado	0.52	0.85	0.74	0.80	0.74	0.77	0.73	0.75	0.72
Iowa	0.55	0.76	0.59	0.77	0.59	1.00	0.88	1.00	0.87
Rhode Island	0.44	0.67	0.50	0.64	0.48	0.71	0.68	0.69	0.67
North Dakota	0.34	0.40	0.39	0.41	0.37	0.89	0.61	0.88	0.63
Pennsylvania	0.78	0.87	0.83	0.87	0.71	0.99	0.95	0.99	0.95
Illinois	0.79	0.86	0.81	0.85	0.81	1.00	0.97	1.00	0.97
Maine	0.56	0.68	0.58	0.71	0.59	0.99	0.84	0.99	0.82
South Dakota	0.55	0.74	0.57	0.71	0.57	0.85	0.53	0.77	0.50
Idaho	0.50	0.73	0.63	0.79	0.64	0.86	0.62	0.81	0.60
Indiana	0.71	0.94	0.84	0.91	0.83	0.99	0.93	0.99	0.93
Florida	0.78	0.98	0.89	0.98	0.88	0.95	0.89	0.95	0.77
California	0.71	0.99	0.90	0.99	0.89	1.00	0.89	0.99	0.84
Missouri	0.60	0.96	0.83	0.71	0.80	1.00	0.95	1.00	0.97
Ohio	0.58	0.70	0.76	0.98	0.75	1.00	0.97	0.99	0.84
New York	0.74	1.00	0.96	1.00	0.95	1.00	1.00	0.99	1.00
Oregon	0.72	0.63	0.75	0.65	0.76	1.00	0.93	1.00	0.94
Michigan	0.78	0.96	0.85	0.96	0.85	0.98	0.99	1.00	0.99
North Carolina	0.67	0.75	0.73	0.74	0.73	1.00	0.79	1.00	0.65
Arizona	0.68	0.63	0.81	0.62	0.80	1.00	0.85	1.00	0.77
Georgia	0.79	0.90	0.75	0.89	0.76	1.00	0.89	1.00	0.89
Montana	0.47	0.52	0.53	0.51	0.53	0.98	0.70	0.98	0.70
Tennessee	0.75	1.00	0.81	0.99	0.80	1.00	0.95	1.00	0.94
Alabama	0.61	0.98	0.64	0.98	0.61	0.98	0.81	0.88	0.95
South Carolina	0.59	0.99	0.88	0.99	0.87	1.00	0.98	1.00	0.98
Texas	0.84	1.00	0.75	1.00	0.74	1.00	0.88	1.00	0.77
Oklahoma	0.64	0.97	0.77	0.96	0.77	0.91	0.92	0.83	0.68
West Virginia	0.68	1.00	0.71	0.99	0.71	1.00	0.76	0.99	0.76
New Mexico	0.59	0.92	0.63	0.91	0.62	1.00	0.76	1.00	0.79
Dist. of Columbia	0.43	0.80	0.56	0.78	0.55	0.97	0.66	0.98	0.69
Arkansas	0.40	0.93	0.69	0.91	0.68	1.00	0.79	1.00	0.77
Kentucky	0.69	0.97	0.71	0.96	0.70	1.00	0.71	1.00	0.63
Louisiana	0.61	1.00	0.75	0.99	0.72	1.00	0.94	1.00	0.93
Mississippi	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11.9: Table of bootstrap estimates using the P2 Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

PM Method: Estimated Ranking

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Maryland	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
Alaska	3.0	6.0	3.0	8.0	6.0	8.0	6.0	5.0	3.0
New Jersey	4.0	5.0	4.0	4.0	3.0	4.0	3.0	6.0	4.0
Hawaii	5.0	3.0	5.0	6.0	5.0	6.0	5.0	3.0	5.0
Connecticut	6.0	4.0	6.0	3.0	4.0	3.0	4.0	4.0	6.0
Wyoming	7.0	7.0	7.0	5.0	7.0	5.0	7.0	7.0	7.0
Minnesota	8.0	8.0	8.0	7.0	8.0	7.0	8.0	8.0	8.0
Utah	9.0	9.0	9.0	11.0	9.0	11.0	9.0	9.0	9.0
Delaware	11.0	12.0	11.0	12.0	12.0	12.0	12.0	12.0	11.0
Massachusetts	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
Virginia	12.0	11.0	12.0	9.0	11.0	9.0	11.0	11.0	12.0
Wisconsin	13.0	14.0	13.0	13.0	13.0	13.0	13.0	14.0	13.0
Vermont	14.0	13.0	14.0	14.0	14.0	14.0	14.0	13.0	14.0
Nebraska	15.0	15.0	15.0	17.0	15.0	17.0	15.0	15.0	15.0
Kansas	17.0	18.0	17.0	21.0	19.0	21.0	19.0	17.0	17.0
Nevada	18.0	16.0	16.0	15.0	16.0	15.0	16.0	16.0	16.0
Washington	16.0	19.0	18.0	18.0	18.0	18.0	18.0	19.0	18.0
Colorado	19.0	20.0	20.0	19.0	20.0	19.0	20.0	20.0	20.0
Iowa	20.0	17.0	19.0	16.0	17.0	16.0	17.0	18.0	19.0
Rhode Island	21.0	21.0	21.0	20.0	21.0	20.0	21.0	21.0	21.0
North Dakota	22.0	23.0	22.0	22.0	22.0	22.0	22.0	23.0	22.0
Pennsylvania	23.0	22.0	23.0	23.0	23.0	23.0	23.0	22.0	23.0
Illinois	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0
Maine	25.0	25.0	25.0	26.0	25.0	26.0	25.0	25.0	25.0
South Dakota	26.0	29.0	27.0	30.0	28.0	30.0	28.0	29.0	27.0
Idaho	27.0	26.0	26.0	27.0	26.0	28.0	26.0	26.0	26.0
Indiana	28.0	27.0	28.0	25.0	27.0	25.0	27.0	27.0	28.0
Florida	29.0	28.0	29.0	28.0	29.0	27.0	29.0	28.0	29.0
California	30.0	30.0	30.0	29.0	30.0	29.0	30.0	30.0	30.0
Missouri	31.5	33.0	32.0	32.0	32.0	32.0	32.0	31.0	32.0
Ohio	31.5	32.0	31.0	31.0	31.0	31.0	31.0	32.0	31.0
New York	34.0	34.0	34.0	34.0	34.0	35.0	34.0	34.0	34.0
Oregon	33.0	31.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0
Michigan	35.0	35.0	35.0	35.0	35.0	34.0	35.0	35.0	35.0
North Carolina	36.0	36.0	36.0	39.0	39.0	39.0	39.0	36.0	36.0
Arizona	37.0	39.0	38.0	37.0	37.0	37.0	37.0	39.0	38.0
Georgia	38.0	38.0	37.0	38.0	38.0	38.0	38.0	38.0	37.0
Montana	39.0	37.0	39.0	36.0	36.0	36.0	36.0	37.0	39.0
Tennessee	40.0	41.0	40.0	41.0	40.0	41.0	40.0	41.0	40.0
Alabama	41.5	44.0	44.0	44.0	43.0	44.0	44.0	44.0	44.0
South Carolina	41.5	40.0	41.0	40.0	41.0	40.0	41.0	40.0	41.0
Texas	43.0	43.0	42.5	43.0	42.0	43.0	42.0	43.0	42.0
Oklahoma	44.0	42.0	42.5	42.0	44.0	42.0	43.0	42.0	43.0
West Virginia	46.0	46.0	45.0	47.0	46.0	47.0	46.0	46.0	45.0
New Mexico	47.0	48.0	47.0	48.0	48.0	48.0	48.0	48.0	47.0
Dist. of Columbia	45.0	45.0	46.0	49.0	49.0	49.0	49.0	45.0	46.0
Arkansas	48.0	49.0	49.0	46.0	47.0	46.0	47.0	49.0	49.0
Kentucky	50.0	47.0	48.0	45.0	45.0	45.0	45.0	47.0	48.0
Louisiana	49.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
Mississippi	51.0	51.0	51.0	51.0	51.0	51.0	51.0	51.0	51.0

Table 11.10: Table of estimated rankings using the PM Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

PM Method: Bootstrap Estimate of $P(|\hat{r}_i - r_i| \leq 1)$

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	0.96	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.99
Maryland	0.98	0.98	0.98	1.00	1.00	1.00	1.00	0.99	0.99
Alaska	0.75	0.63	0.71	0.96	0.79	0.95	0.83	0.67	0.73
New Jersey	0.96	0.95	0.92	1.00	0.89	1.00	0.90	0.74	0.93
Hawaii	0.62	0.95	0.72	1.00	0.73	1.00	0.76	0.85	0.74
Connecticut	0.78	0.77	0.91	1.00	0.95	1.00	0.97	0.71	0.91
Wyoming	0.37	0.85	0.58	1.00	0.67	1.00	0.68	0.82	0.55
Minnesota	0.78	1.00	0.94	1.00	0.98	1.00	0.98	0.99	0.93
Utah	0.56	0.93	0.75	1.00	0.66	1.00	0.65	0.93	0.74
Delaware	0.45	0.78	0.54	0.95	0.64	0.96	0.66	0.75	0.53
Massachusetts	0.76	0.92	0.81	0.98	0.87	0.98	0.88	0.90	0.80
Virginia	0.77	0.94	0.83	1.00	0.81	1.00	0.80	0.93	0.83
Wisconsin	0.86	0.98	0.94	1.00	0.99	1.00	0.99	0.98	0.94
Vermont	0.47	0.83	0.57	0.91	0.75	0.91	0.76	0.81	0.57
Nebraska	0.77	0.85	0.85	0.99	0.84	1.00	0.84	0.85	0.85
Kansas	0.49	0.76	0.58	0.72	0.67	0.74	0.69	0.71	0.57
Nevada	0.41	0.81	0.55	1.00	0.84	1.00	0.86	0.81	0.55
Washington	0.47	0.83	0.72	0.77	0.72	0.79	0.74	0.81	0.72
Colorado	0.50	0.84	0.70	0.77	0.72	0.78	0.73	0.78	0.70
Iowa	0.54	0.72	0.58	1.00	0.89	1.00	0.90	0.78	0.59
Rhode Island	0.42	0.65	0.48	0.71	0.65	0.72	0.67	0.63	0.48
North Dakota	0.35	0.40	0.40	0.87	0.62	0.87	0.60	0.40	0.39
Pennsylvania	0.77	0.86	0.82	0.99	0.94	0.99	0.94	0.85	0.81
Illinois	0.78	0.84	0.80	1.00	0.96	1.00	0.95	0.83	0.80
Maine	0.57	0.67	0.59	0.99	0.82	0.99	0.85	0.69	0.59
South Dakota	0.53	0.72	0.54	0.75	0.50	0.84	0.52	0.68	0.54
Idaho	0.50	0.74	0.63	0.81	0.60	0.87	0.61	0.79	0.65
Indiana	0.67	0.92	0.84	0.99	0.94	1.00	0.93	0.89	0.83
Florida	0.77	0.98	0.88	0.96	0.77	0.97	0.89	0.98	0.87
California	0.71	0.99	0.88	0.99	0.83	1.00	0.88	0.98	0.86
Missouri	0.44	0.66	0.82	1.00	0.96	1.00	0.94	0.71	0.79
Ohio	0.44	0.97	0.76	1.00	0.85	1.00	0.96	0.97	0.76
New York	0.77	1.00	0.96	1.00	1.00	1.00	1.00	1.00	0.95
Oregon	0.71	0.64	0.73	1.00	0.95	1.00	0.94	0.63	0.74
Michigan	0.80	0.97	0.86	1.00	0.99	0.98	0.99	0.96	0.86
North Carolina	0.64	0.72	0.70	1.00	0.68	1.00	0.79	0.70	0.69
Arizona	0.71	0.65	0.80	1.00	0.78	1.00	0.86	0.63	0.80
Georgia	0.79	0.92	0.73	1.00	0.91	1.00	0.90	0.91	0.73
Montana	0.47	0.54	0.50	0.99	0.74	0.99	0.74	0.53	0.51
Tennessee	0.73	0.99	0.80	1.00	0.93	1.00	0.94	0.99	0.79
Alabama	0.41	0.99	0.64	0.86	0.94	0.98	0.79	0.98	0.63
South Carolina	0.41	0.99	0.88	1.00	0.98	1.00	0.97	0.99	0.87
Texas	0.82	1.00	0.71	1.00	0.77	1.00	0.87	0.99	0.76
Oklahoma	0.61	0.96	0.47	0.83	0.67	0.89	0.91	0.94	0.75
West Virginia	0.68	0.99	0.71	0.99	0.76	1.00	0.76	0.99	0.71
New Mexico	0.56	0.92	0.61	1.00	0.74	1.00	0.71	0.90	0.61
Dist. of Columbia	0.44	0.82	0.59	0.99	0.68	0.98	0.63	0.81	0.58
Arkansas	0.58	0.96	0.72	1.00	0.78	1.00	0.81	0.94	0.71
Kentucky	0.45	0.96	0.71	1.00	0.63	1.00	0.71	0.95	0.71
Louisiana	0.64	1.00	0.74	1.00	0.94	1.00	0.95	1.00	0.72
Mississippi	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11.11: Table of bootstrap estimates using the PM Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

Maximum Probability Method: Estimated Ranks

State Name	KWP	Prior: 2007 Data				Prior: 2005-2007 Data			
		FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	1	1	1	1	1	1	1	1	1
Maryland	2	2	2	2	2	2	2	2	2
Alaska	3	6	3	4	3	8	7	8	6
New Jersey	5	5	4	6	4	4	3	4	3
Hawaii	7	3	5	3	5	6	5	6	5
Connecticut	6	4	6	5	6	3	4	3	4
Wyoming	4	7	7	7	7	5	6	5	7
Minnesota	8	8	8	8	8	7	8	7	8
Utah	9	9	9	9	9	11	9	11	11
Delaware	12	12	11	12	11	12	12	12	9
Massachusetts	10	10	10	10	10	10	10	10	10
Virginia	11	11	12	11	12	9	11	9	12
Wisconsin	13	14	14	14	14	14	13	13	13
Vermont	14	13	13	13	13	13	14	14	14
Nebraska	15	15	15	15	15	17	15	17	15
Kansas	18	17	16	17	17	21	19	21	19
Nevada	19	16	17	16	16	15	16	15	16
Washington	16	19	18	19	18	18	18	18	18
Colorado	21	21	20	21	20	19	20	19	20
Iowa	20	18	19	18	19	16	17	16	17
Rhode Island	22	20	21	20	21	20	21	20	21
North Dakota	17	23	22	24	24	23	23	22	22
Pennsylvania	23	22	23	22	22	22	22	23	23
Illinois	24	24	24	23	23	24	24	24	24
Maine	25	25	25	25	26	26	25	26	25
South Dakota	26	29	27	29	30	30	28	30	28
Idaho	27	26	26	26	25	27	27	27	27
Indiana	28	27	28	27	27	25	26	25	26
Florida	29	28	29	28	28	28	29	28	30
California	30	30	30	30	29	29	30	29	29
Missouri	32	33	32	32	31	31	32	32	32
Ohio	31	32	31	33	32	32	31	31	31
New York	34	34	34	34	34	35	34	34	34
Oregon	33	31	33	31	33	33	33	33	33
Michigan	35	35	35	35	35	34	35	35	35
North Carolina	36	37	36	36	36	39	39	39	38
Arizona	37	39	38	37	39	37	37	37	37
Georgia	38	38	37	38	37	38	38	38	39
Montana	39	36	39	39	38	36	36	36	36
Tennessee	40	41	40	41	40	41	41	41	40
Alabama	42	44	44	44	42	44	44	44	44
South Carolina	41	40	41	40	41	40	40	40	41
Texas	43	43	42	43	44	43	42	43	42
Oklahoma	44	42	43	42	43	42	43	42	43
West Virginia	47	46	46	46	45	47	48	47	45
New Mexico	45	48	47	48	47	48	47	48	48
Dist. of Columbia	49	45	45	45	46	49	49	49	49
Arkansas	50	49	48	49	49	46	46	46	47
Kentucky	48	47	49	47	48	45	45	45	46
Louisiana	46	50	50	50	50	50	50	50	50
Mississippi	51	51	51	51	51	51	51	51	51

Table 11.12: Table of estimated ranks using the Maximum Probability Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).

Maximum Probability Method: Bootstrap Estimate of $P(\|\hat{r}_i - r_i\| \leq 1)$									
		Prior: 2007 Data				Prior: 2005-2007 Data			
State Name	KWP	FIP		RIP		FIP		RIP	
		$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$	$\tau \times 1$	$\tau \times 2$
New Hampshire	0.94	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00
Maryland	0.99	0.97	0.98	0.99	0.98	1.00	1.00	1.00	1.00
Alaska	0.70	0.61	0.69	0.52	0.70	0.92	0.66	0.94	0.77
New Jersey	0.72	0.94	0.91	0.73	0.92	1.00	0.88	1.00	0.88
Hawaii	0.38	0.95	0.70	0.82	0.72	1.00	0.73	1.00	0.72
Connecticut	0.76	0.79	0.90	0.87	0.89	1.00	0.96	1.00	0.94
Wyoming	0.22	0.84	0.53	0.82	0.50	1.00	0.71	1.00	0.66
Minnesota	0.77	1.00	0.90	1.00	0.89	1.00	0.96	1.00	0.96
Utah	0.54	0.94	0.75	0.93	0.73	1.00	0.67	1.00	0.62
Delaware	0.35	0.74	0.47	0.71	0.46	0.96	0.62	0.95	0.28
Massachusetts	0.77	0.90	0.81	0.89	0.80	0.98	0.86	0.98	0.86
Virginia	0.70	0.96	0.82	0.95	0.82	1.00	0.80	1.00	0.59
Wisconsin	0.83	0.97	0.83	0.97	0.83	0.97	0.99	0.99	0.98
Vermont	0.42	0.80	0.51	0.79	0.50	0.93	0.71	0.89	0.70
Nebraska	0.68	0.84	0.79	0.84	0.78	0.99	0.81	0.99	0.81
Kansas	0.45	0.72	0.47	0.72	0.57	0.74	0.69	0.72	0.66
Nevada	0.34	0.80	0.49	0.79	0.54	0.99	0.84	0.99	0.83
Washington	0.43	0.83	0.71	0.83	0.71	0.79	0.70	0.76	0.68
Colorado	0.34	0.94	0.67	0.92	0.68	0.77	0.71	0.76	0.70
Iowa	0.51	0.77	0.52	0.77	0.53	1.00	0.86	1.00	0.84
Rhode Island	0.32	0.56	0.44	0.55	0.42	0.69	0.64	0.69	0.63
North Dakota	0.11	0.37	0.35	0.42	0.30	0.82	0.55	0.88	0.55
Pennsylvania	0.74	0.86	0.79	0.86	0.68	1.00	0.95	0.99	0.93
Illinois	0.77	0.87	0.80	0.76	0.68	1.00	0.94	1.00	0.95
Maine	0.55	0.69	0.58	0.72	0.53	0.99	0.84	0.99	0.81
South Dakota	0.49	0.73	0.48	0.68	0.16	0.85	0.48	0.77	0.46
Idaho	0.46	0.73	0.59	0.77	0.51	0.83	0.63	0.82	0.63
Indiana	0.64	0.92	0.82	0.88	0.72	0.99	0.76	0.99	0.76
Florida	0.75	0.98	0.86	0.98	0.85	0.98	0.88	0.97	0.72
California	0.68	0.99	0.88	0.98	0.75	1.00	0.88	1.00	0.86
Missouri	0.57	0.67	0.82	0.95	0.71	1.00	0.95	1.00	0.97
Ohio	0.58	0.97	0.77	0.67	0.82	1.00	0.94	1.00	0.82
New York	0.74	1.00	0.95	1.00	0.94	1.00	1.00	1.00	1.00
Oregon	0.67	0.65	0.72	0.60	0.72	1.00	0.92	1.00	0.92
Michigan	0.76	0.95	0.84	0.95	0.84	0.98	0.99	1.00	0.99
North Carolina	0.62	0.88	0.69	0.71	0.67	1.00	0.80	1.00	0.92
Arizona	0.69	0.61	0.79	0.64	0.55	1.00	0.86	1.00	0.80
Georgia	0.78	0.93	0.75	0.92	0.75	1.00	0.91	1.00	0.63
Montana	0.44	0.60	0.51	0.43	0.58	0.98	0.71	0.98	0.70
Tennessee	0.72	0.99	0.79	0.98	0.79	1.00	0.98	1.00	0.93
Alabama	0.56	0.99	0.64	0.98	0.54	0.98	0.80	0.88	0.66
South Carolina	0.60	1.00	0.88	0.99	0.87	1.00	0.90	1.00	0.98
Texas	0.82	0.99	0.74	0.99	0.63	1.00	0.88	1.00	0.78
Oklahoma	0.63	0.97	0.77	0.96	0.76	0.91	0.91	0.84	0.91
West Virginia	0.55	0.98	0.83	0.99	0.64	1.00	0.48	1.00	0.50
New Mexico	0.40	0.91	0.60	0.89	0.59	1.00	0.63	1.00	0.76
Dist. of Columbia	0.40	0.77	0.46	0.75	0.50	0.98	0.63	0.99	0.67
Arkansas	0.40	0.92	0.62	0.90	0.66	1.00	0.61	1.00	0.78
Kentucky	0.67	0.97	0.48	0.96	0.68	1.00	0.72	1.00	0.86
Louisiana	0.38	0.99	0.70	0.99	0.68	1.00	0.93	1.00	0.92
Mississippi	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11.13: Table of bootstrap estimates using the Maximum Probability Method with various priors: non-informative, 2007 estimates, 3 year (2005-2007) average estimates. The informed prior versions are also represented with two different multipliers of τ ($\tau \times 2$ being the most relaxed of the informed priors).