Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 3 – solutions to selected problems

- page 54, exercise 36
  solution:
  Keep examples simple: on the machine DRAGA-1 we use the decimal system, but we keep only one digit after the comma, that is, our numbers look like $\pm 0.q \cdot 10^m$ with $q \in \{0, 1, \ldots, 9\}$, and m is an integer.

  $$\mathrm{fl}(\mathrm{fl}(.3 \times .4) \times .5) = \mathrm{fl}(\mathrm{fl}(0.012) \times .5) = \mathrm{fl}(0.1 \cdot 10^{-1} \times .5) = 0.5 \cdot 10^{-2} \ .$$

  However,

  $$\mathrm{fl}(.3 \times \mathrm{fl}(.4 \times .5)) = \mathrm{fl}(.3 \times 0.2 \cdot 10^{-1}) = 0.6 \cdot 10^{-2} \ .$$

- page 60, exercise 9 (a, b, d, i)
  solution:

  $$\sqrt{x^2+1} - x = \frac{1}{\sqrt{x^2+1}+x} \ .$$

  $$\log x - \log y = \log \frac{x}{y} \ .$$

  $$\sinh(x) - \tanh(x) = \tanh(x)(\cosh(x) - 1) = 2\tanh(x)\sinh^2(x/2) \ .$$

- Consider the matrix:

  $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 6 \\ -1 & -4 & -1 \end{bmatrix} \ .$$

  Use Gaussian elimination to produce an LU factorization of A, and write L as a product of elementary matrices. Verify your answers using Matlab's `lu` function (type `help lu` to learn about it).

solution:
Gaussian elimination gives:

$$E_{32}(-2) \cdot E_{31}(1) \cdot E_{21}(-2)A = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{U} .$$

Hence

$$A = E_{21}(2) \cdot E_{31}(-1) \cdot E_{32}(2) \cdot U = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}}_{L} \cdot U .$$

- page 158, exercise 7
  solution:
  Assume
  $$L = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}, U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} ,$$
  and $LU = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Then
  $$\begin{aligned} l_{11}u_{11} &= 0 \\ l_{11}u_{12} &= 1 \\ l_{21}u_{11} &= 1 \\ l_{21}u_{12} + l_{22}u_{22} &= 1 . \end{aligned}$$
  The second and third equations imply that both $l_{11} \neq 0$ and $u_{11} \neq 0$, thus contradicting the first equation.

- page 158, exercise 12
  solution:
  A system similar to the one above gives rise to the set of solutions
  $$L = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & a \\ 0 & b - a\lambda \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} .$$