

Homework 9

due Thursday, December 7

A. Read: Section 6.8, 7.1, 7.2.

B. Hand in:

1. pages 404-405, exercises 1, 2, one of 12 or 14.
2. Identify the coefficients a_0, a_1, a_2, a_3 in (1) by differentiating at $x_1 = x$ the Lagrange interpolation polynomial through $x_0 = x - h, x_1 = x, x_2 = x + h, x_3 = x + 2h$, so that the following approximation formula holds:

$$f'(x) = a_0 f(x - h) + a_1 f(x) + a_2 f(x + h) + a_3 f(x + 2h) + O(h^p) . \quad (1)$$

What is the value of p ?

3. Test the convergence rates for the right (first order correct), centered (second order correct) difference formulas for approximating the derivative, and for the formula you found at 2., by performing the following MATLAB experiment: Consider $f(x) = \sin(x)$ and $x = 1$. For $h_i = 2^{-i}, i = 1, \dots, 20$, and each of the difference formulas $l_1(x, h, f) = (f(x + h) - f(x))/h, l_2(x, h, f), l_3(x, h, f)$ compute $e_i = |\sin'(2) - l_\alpha(1, h_i, \sin)|$. Since it is expected that $e_i \approx Ch_i^p$ for some p , one can estimate p by computing $v_i = \ln(e_i)/\ln(h_i)$, as $h_i \rightarrow 0$. Verify the approximation order for each of the formulas by plotting the values of v_i on a single graph. You may use the provided codes `testorder.m`, `l1.m` (you would have to write `l2.m`, `l3.m`; usage: `vals=testorder(11); plot(vals)`). Please comment if you see anything unusual in the graphs.
4. pages 488-489, exercises 1, 6, 12.