Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 8

due Tuesday, November 21

A. Read: Sections 6.3, 6.4.

B. Hand in:

1. Let $f(x) = \sin x$. Use the cardinal functions A_i, B_i on page 344 of the textbook to find a polynomial P of appropriate degree that interpolates the values and first derivatives of f at $t_0 = 0, t_1 = \pi$. Compute the truncation error $P(\pi/2) - f(\pi/2)$ (use a plotting utility to verify that your P truly interpolates f as requested).

2. pages 361-364, exercises 5, 7, 19, 24.

C. Matlab:

3. **Convergence of polynomial interpolation.** Let

$$f(x) = 1/(1 + x^2) \ ,$$

and P_n be the Lagrange interpolation polynomial associated with the points obtained by dividing an interval $[-L, L]$ in n equal intervals. Use the provided Matlab code (`myinterp.m, runge.m`) from the course web page to identify a relatively large interval $[-L, L]$ on which $P_n(x)$ converges to $f(x)$ by completing the following convergence study for $L = 1, 1.2, 1.4, 1.6, \ldots,$ (I recommend using the provided script `test_interp_conv.m`):

   (i) Divide the interval $[-L, L]$ in n equal intervals for $n = 10, 20, 40$, and compute for each of them $\|P_n - f\|_\infty$[1]. If this quantity seems to be decreasing, then this signals convergence of $P_n(x)$ to $f(x)$.

[1] $\|f\|_\infty = \max_{x \in [-L,L]} |f(x)|$

(ii) Repeat (i) with an increased L until $\|P_n - f\|_\infty$ seems to increase, that is, $P_n(x)$ does not converge to $f(x)$.

Report the computed norms in a table. For L where divergence seems to happen plot $f(x), P_{10}(x), P_{20}(x)$ on the same figure.

4. **Order of approximation of linear splines.** Verify that

$$\left\| f(x) - S_1^h(x) \right\| = O(h^2) ,$$

where $S_1^h(x)$ is the linear spline interpolating f at a set of equidistant points with a spacing of h, and f is the function in exercise 3., by completing the following steps (I recommend using the provided script `test_interp_conv_sp.m`):

(i) Divide the interval $[-2, 2]$ ($L = 2$) in n equal intervals for $n = 10, 20, 40, 80, 160, 320$, and compute for each of them $v_i = \left\| S_1^{h_i} - f \right\|_\infty$ ($i = 1, 2, 3, 4, 5, 6,\ h_i = \frac{4}{10 \cdot 2^{i-1}}$). Examples:
`v1=test_interp_conv_sp(2,10)`
`v2=test_interp_conv_sp(2,20)`.

(ii) Since v_i represents the approximation error for h_i, for an approximation order of h^2 it is expected that $v_i/v_{i+1} \approx 4$ (use the command `format long`).

Report in a table the numbers v_i and the obtained quotients v_i/v_{i+1}. Plot on the same figure $S_1^{h_1}, S_1^{h_2}, f$.

5. **Order of approximation of cubic splines.** Find the approximation order p of cubic splines

$$\left\| f(x) - S_3^h(x) \right\| = O(h^p)$$

by repeating exercise 4. with cubic splines instead of linear splines. You will need to modify line 14 of the script
`test_interp_conv_sp.m` to include the 'spline' option:
`interp1(x,y,xi, 'spline')`. Report in a table the numbers v_i and the obtained quotients, and use the results to estimate p. Plot on the same figure $S_3^{h_1}, S_3^{h_2}, f$.

D. Practice:

- pages 361-364, exercises 12, 13, 14, 21, 22, 23.