Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 7

due Thursday, November 2

A. Read: Section 6.1.

B. Hand in:

1. pages 323-327, exercises 9, 11, 14, 21, 25, 26.

C. Matlab:

2. **A large linear system.** Use the provided Matlab code (`get2Dbdvp.m`) from the course web page to retrieve the sparse matrix A corresponding to the two-dimensional boundary value problem

$$\begin{cases} -\partial_{xx}^2 u - \partial_{yy}^2 u = f \\ u|_{\partial\Omega} = 0 \end{cases},$$

where Ω is the unit square $[0,1] \times [0,1]$. The command `[A, b]=get2Dbdvp(n)` will construct a *sparse* matrix corresponding to a discretization with n intervals in each direction, and a right-hand side vector b corresponding to a point heat source in the center of the square. A has the constant value 4 on the diagonal, and up to four entries of -1 in each row, corresponding to the four neighboring points in the discretization. Note that A is a $(n-1)^2 \times (n-1)^2$ matrix. For a very small value of n (use $3, 4$), view the matrix by using `full(A)`. Perform the following experiment: for $n = 200, 400, 800, \ldots$ solve the system $Ax = b$ by using both the Matlab direct solver (`x=A\b`) and the Matlab provided conjugate gradient algorithm (the function is called `pcg`) using a relative tolerance of 10^{-2} and a maximum number of iterations of 2000. I recommend using the provided `test_cg_2dbvp.m` function. Report in a table for each value of n the following: the size of the matrix A, the number of iterations needed by `pcg` to converge for the given tolerance, the running time (wall-clock using `tic, toc`) for `pcg`, and the relative error

$$\frac{\|x_1 - x\|}{\|x\|} \, ,$$

where x_1 is the solution given by `pcg`. Keep doubling n until the direct solver does not work anymore (do not run other applications on that computer while performing the experiment). Interpret the results in light of the condition number. **Note: octave may not support these operations, you need to use Matlab.**