Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 6

due Thursday, October 26

A. Read: Sections 4.4 (condition number), 4.6 (pages 207-218).

B. Hand in:

1. pages 193-197, exercises **14**, **15**, **39**, **40**.

2. pages 229-231, exercises **5**, **8**, **10** (for 8 we proved one direction in class, now prove that $\rho(A) < 1$ is **necessary** for $A^k \to 0$).

3. Find the eigenvalues and compute the condition number for the $(n-1) \times (n-1)$ matrix (n intervals) with 2 on the diagonal and -1 above and below the diagonal defined in the notes from Lecture 12 by completing the steps below.

   a. Given that the $n-1$ eigenvectors of A are $U^{(k)}, k = 1, \ldots, n-1$ defined by $U_i^{(k)} = \sin \frac{2\pi k i}{n} = \sin(2\pi k x)|_{x=i/n}$, with $i = 1, \ldots, n-1$, find the associated eigenvalues λ_k, that is, the values for which
   $$AU^{(k)} = \lambda_k U^{(k)} \ .$$
   (You will need the trig formulas for $\sin a - \sin b$ and $\cos a - \cos b$.)

   b. Find the largest and the smallest of the λ_k's.

   c. Verify that $\sigma(A) \in (0, 4]$ and that $\mathrm{cond}(A) = Ch^{-2}$, for $h = \frac{2\pi}{n}$, where C is a constant that is independent of h.

C. Matlab (hand in printouts):

4. Use the provided Matlab code from the course web page to retrieve a diagonally dominant 10×10 matrix A (use `get_gen_mat.m`), retrieve the diagonal part D and the lower triangular part L of A (use

extractL.m), and compute the spectral radii for $\rho(I - D^{-1}A), \rho(I - L^{-1}A)$ (use spect_rad.m). Run a Jacobi and Gauss-Seidel iteration (it_jacobi.m, it_gauss_seidel.m) to solve the system $Ax = b$ with a tolerance of 10^{-6}, and report the number of iterations needed for the system to converge. Plot the residual histories on a single picture using a logarithmic scale on the y-axis.

5. Use the provided Matlab code from the course web page to retrieve the sparse matrix A from Lecture 12 (get2ptbdvp.m) and a right-hand side vector b with $n = 40, 80$ intevals; report $\rho(I-D^{-1}A), \rho(I-L^{-1}A)$. (use the command format long in order to view accurate numbers). Verify correctness for the eigenvalues you found in Exercise 3.