Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 2

due Thursday, September 14

A. Read: Sections 1.1, 2.1
B. Hand in:

1. Prove that for all $x \in \mathcal{R}$, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, by showing that the remainder $R_n(x) = e^x - T_n(x)$ converges to 0 as $n \to \infty$, where $T_n(x)$ is the n^{th} Taylor polynomial of e^x around 0.

2. Which of the following are true (justify your answers)?

   (a) $xy = O(\sqrt{x^2 + y^2})$ as $(x, y) \to 0$.
   (b) $\sqrt{x^2 + y^2} = O(xy)$ as $(x, y) \to 0$.

3. Write a Matlab function that computes $\ln(x)$ with a given approximation from the formula

   $$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots.$$

   Verify your answers against the Matlab provided ln function for three numbers of your choice (pick numbers in the interval where the series converges). The header of the function is the following `function value=logtaylor(x, delta)`. Write your function such that the sum contains no more than 10^4 terms.

4. Compute the Taylor polynomials T_0, T_1, T_2, T_3 of the function $f(x, y) = \ln(x + y)$ around the point $(x, y) = (1, 0)$.

5. page 14, exercise 36.

6. pages 51-53, exercises 2, 10, 16.