Math 441, Introduction to Numerical Analysis
Fall 2006

Homework 1

due Thursday, September 7

Analize the backward Euler method for the ODE

$$\frac{d}{dt}x = -a\,x\ , \quad x(0) = x_0\ , \tag{1}$$

with $a > 0$, following (if desired) the model of the analysis presented in class for the forward Euler method. The method emerges from the approximation

$$\frac{d}{dt}x(t_n) \approx \frac{x(t_n) - x(t_{n-1})}{t_n - t_{n-1}} = \frac{x(t_n) - x(t_{n-1})}{h}\ , \tag{2}$$

with $h > 0, t_n = nh$, which gives rise to the difference equation

$$\frac{U_{n+1} - U_n}{h} = -aU_{n+1}\ . \tag{3}$$

1. Solve the difference equation (3).
2. Is the method stable for all choice of $h > 0$?
3. Show that $U_n \to x(t_n)$, where $x(t) = x_0 e^{-at}$ is the exact solution of (1) (for notation refer also to the summary of Lecture 1, also available on line). What is the order of convergence of the backward Euler method.