ENCH 630: Transport Phenomena

Problem Set 5

1. Do problem 10.B.15 from BSL. **Skip parts d, e, and f.** For part a, do not use a shell balance and instead work directly from the thermal energy balance. For part b, you do not need to use dimensionless variables and instead you can use dimensional variables.

2. Do problem 10.D.1 from BSL.

   Hint: The differential equation describing this problem has the modified Bessel functions I and K as solutions. Properties of these functions (such as the derivatives) can be found in standard mathematical handbooks as shown in the ENCH 630 course website.

3. Derive an expression for the temperature distribution $T(x)$ in a viscous fluid in steady, fully developed, laminar flow between large flat parallel plates, as shown in Fig. 2B.3 of BSL. Both plates are maintained at a constant temperature T_0. Take into account explicitly the heat generated by viscous dissipation. Neglect the temperature dependence of the viscosity and thermal conductivity. Your final answer should be written in terms of the maximum velocity in the fluid ($v_{\max}$), the half width between the plates (B), and the fluid viscosity (μ), and the fluid thermal conductivity (k). Note that for steady, fully developed, laminar flow between parallel plates the velocity profile is given by:

$$v_z = (\Delta P/L)\, B^2/(2\,\mu)\, [1 - (x/B)^2]$$

where the x and z directions are shown in Fig. 2B.3 of BSL.

4. A standard device used to measure the mass flow rate in a fluid employs a tiny platinum film deposited on a flat glass substrate which is located in some type of device (see Fig. 1 in the figures document). The film is heated electrically by passing a fixed current through it, and the resistance is measured. Since the resistance increases with temperature, the resistance measurement provides a temperature measurement, which can be used to infer the shear rate at the wall and therefore also the mass flow rate in the device.

   a. Determine the temperature profile at the surface of the platinum film, i.e., $T(x,0)$, in terms of the temperature of the fluid far from the film (T_0), the heat flux from the film (q), which is assumed to be uniform along the film length (L) and positive in value, and the fluid thermal conductivity (k), density (ρ) and heat capacity (C_p). You may assume that heat transfer in the x direction is due entirely to convection in the fluid, i.e., neglect the conduction of heat in the x direction in both the platinum film and in the fluid. Also assume that the fluid velocity in the y direction is zero, and that the fluid velocity in the x direction in the vicinity of the film is given by the relation:

$$u = \beta \cdot y$$

where β is constant so that u has no x dependence.

Hint: Assume that the temperature in the fluid above the heated film is given by a relation of the following self-similar form involving the unknown function *f*:

$$T - T_0 = (q/k)(3\alpha/\beta)^{1/3} x^{1/3} f(y\ [\beta/(3x\alpha)]^{1/3})$$

In the above equation $\alpha = k/(\rho C_p)$

b. One way to employ the platinum film described above is to use an electronic feedback circuit to keep the electrical resistance in the film constant, and to then measure the electrical current in the film. If the film resistance is constant, then it can be assumed that the temperature profile in the film is fixed since the resistance in the film depends on its temperature. Use your result from part a to determine how the measured electrical current varies with β under these conditions, and how to plot electrical current versus β so that, if a few calibration experiments are performed, then β can be determined for any measured electrical current by linear interpolation.

Hint: $q\ A = i^2\ R$ where A is the film surface area, i is the current and R is the film resistance.

Note: You will get full credit for this problem if in Part *a* you develop an ordinary differential equation for the function *f* that needs to be solved numerically. The numerical solution of this equation for *f* and all of part *b* of this problem will be completed in class.

5. Consider the steady state, laminar, nonisothermal flow of liquid water in a cylindrical tube which is 1 cm in diameter and 0.25 m in length. At the entrance of the tube, the fluid temperature is 298 K, and the fluid is in uniform plug flow in the axial direction. The no-slip condition applies at the tube walls, which are at 325 K.

5.a. Perform a set of simulations using COMSOL Multiphysics to determine the velocity of water at the inlet of the tube that corresponds to case where the temperature at the centerline of the tube at the tube outlet is 90% of the temperature value at the centerline that would be attained in an infinitely long tube (*i.e.*, determine the velocity of water at the tube inlet for the case where the centerline temperature at the tube outlet is 298 + 0.95 * (325 – 298) = 322.3 K).

5.b. For the inlet velocity that you determine in Part 5.a., make a set of graphs of the temperature profiles across the tube diameter at a variety of axial positions inside the tube.

Note: If you were able to complete the COMSOL problem in Problem Set 3, you can simply add a heat transfer node to the COMSOL computer program you developed for that problem set to create a new COMSOL computer program that will be applicable to this problem. If you were unable to complete the COMSOL problem in Problem Set 3, you can alternatively use a working version of a COMSOL computer program for that problem set that will be sent to you.