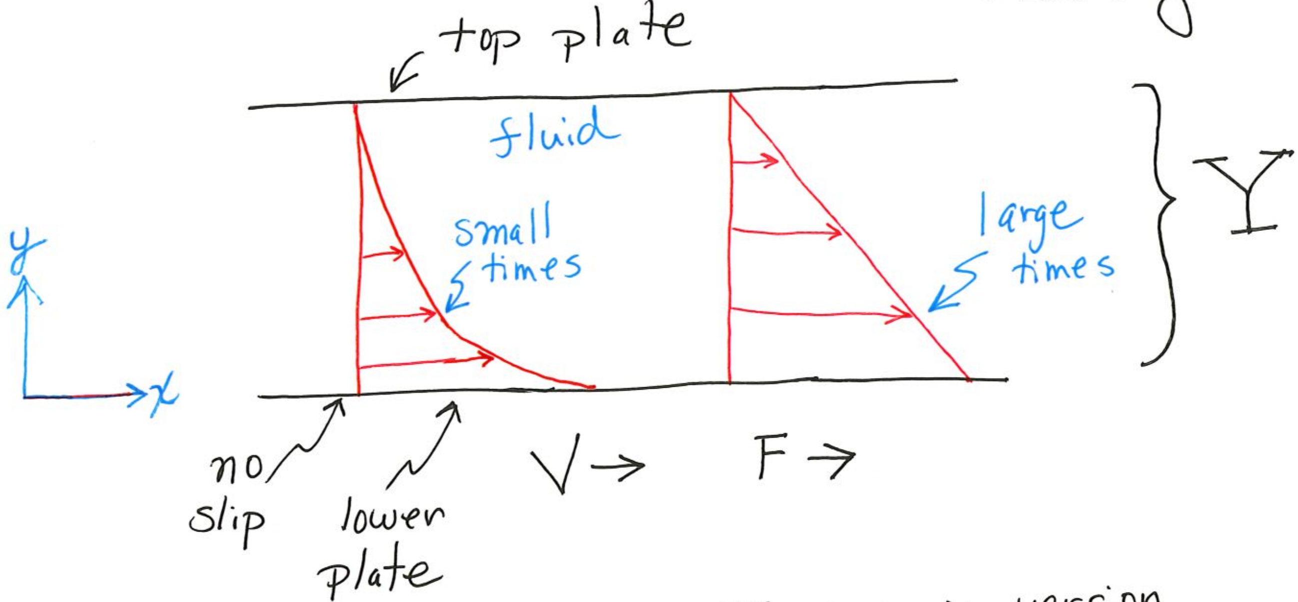
ENCH 630 — Lecture 2: Newton's Law of Viscosity



$$\frac{F}{A} = \mu \frac{V}{Y}$$

Macroscopic version of Newton's Law of viscosity

$\frac{V}{Y}$: velocity gradient

μ: viscosity

$\frac{F}{A}$: Shear force per unit area = shear "stress"

Microscopic version:

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

Momentum transferred in direction opposite to gradient (from high to low velocity)

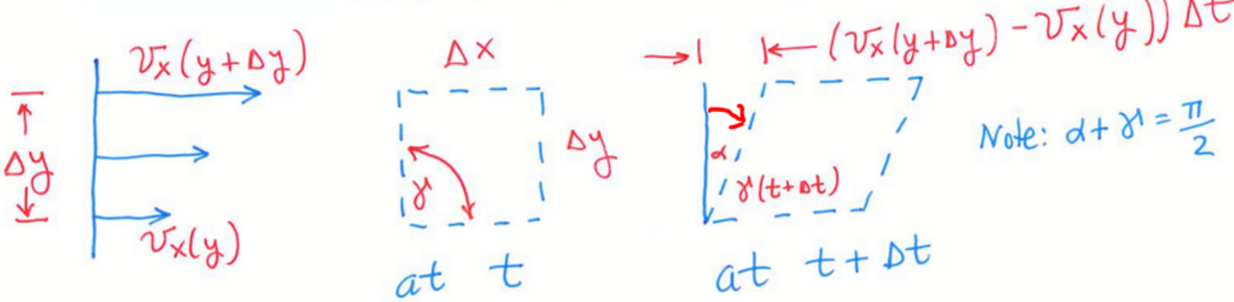
$\frac{dv_x}{dy}$: rate of strain

τ_{yx}: shear stress in $\underline{x}$ direction on a plane of constant $\underline{y}$

Units ?

For fluids, the material continuously deforms when stress is applied.

rate of strain $= -d\alpha/dt$



$$\frac{d\alpha}{dt} = \lim_{\Delta t \to 0} \frac{\alpha(t+\Delta t) - \alpha(t)}{\Delta t}$$

$$= \lim_{\Delta x, \Delta y, \Delta t \to 0} \left[\frac{\tan^{-1}\left\{ \frac{(v_x(y+\Delta y) - v_x(y))\,\Delta t}{\Delta y} \right\} - 0}{\Delta t} \right]$$

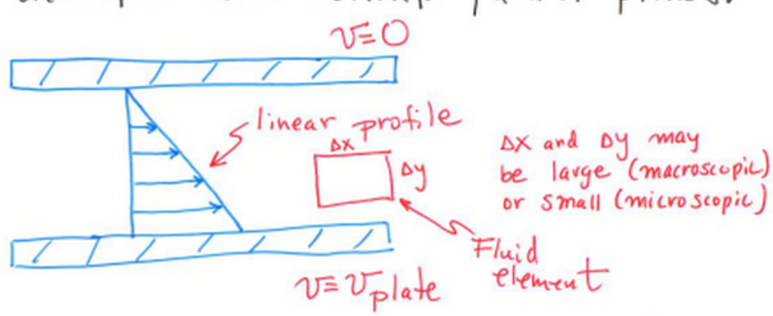
For small x, we have $\tan^{-1}(x) = x$ so that

$$\frac{d\alpha}{dt} = \frac{-d\gamma}{dt} = \frac{dv_x}{dy} = -\text{rate of strain}$$

Which also means

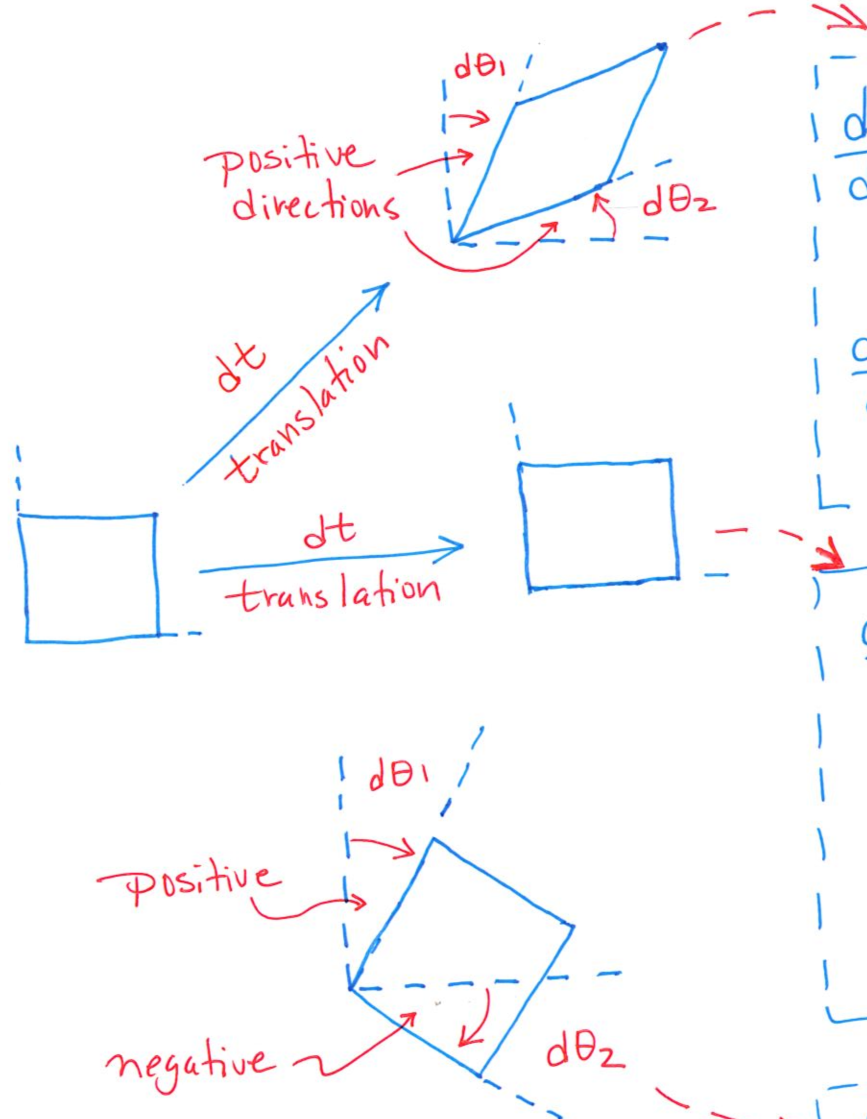
$$\mu = \text{shear stress}/\text{rate of strain}$$

Does $\tau_{yx} = \tau_{xy}$ apply? Consider steady state and fully developed flow between parallel plates.



We know in this a fluid element regardless of size does not accelerated either linearly or angularly. Therefore τ_{yx} and τ_{xy} are independent of position and $\tau_{yx} = \tau_{xy}$. A more complex argument can be used to show $\tau_{yx} = \tau_{xy}$ even for unsteady flow.

More generally, we have the following



$$\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} > 0$$
(rate of strain $\neq 0$)

$$\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} = 0$$
(no rotation)

$$\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} = 0$$
(rate of strain $= 0$)

$$\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} = 0$$
(no rotation)

$$\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} = 0$$
(rate of strain $= 0$)

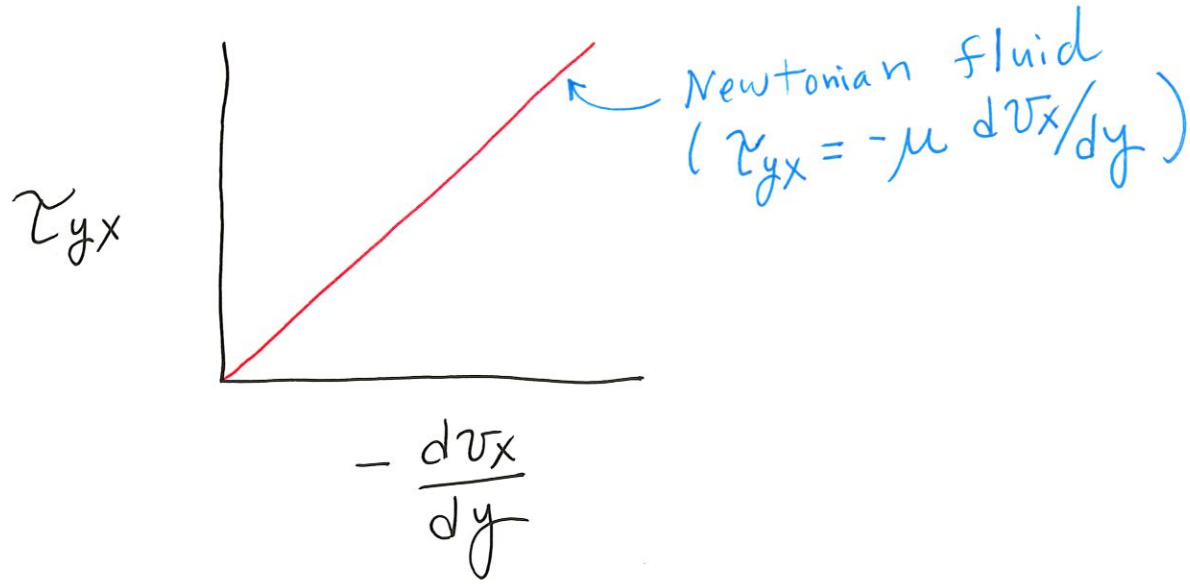
$$\frac{d\theta_1}{dt} - \frac{d\theta}{dt} > 0$$
(rotation)

Using arguments similar to before, and assuming $\frac{\partial v_x}{dy} \neq 0$ and $\frac{\partial v_y}{dx} \neq 0$ we get:

Total rate of strain $= -\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} = -\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{dx}$

Rate of rotation (equal to average of the rotation rates of the two sides): $\Omega = \frac{1}{2}\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right)$
$= \frac{1}{2}\left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right)$

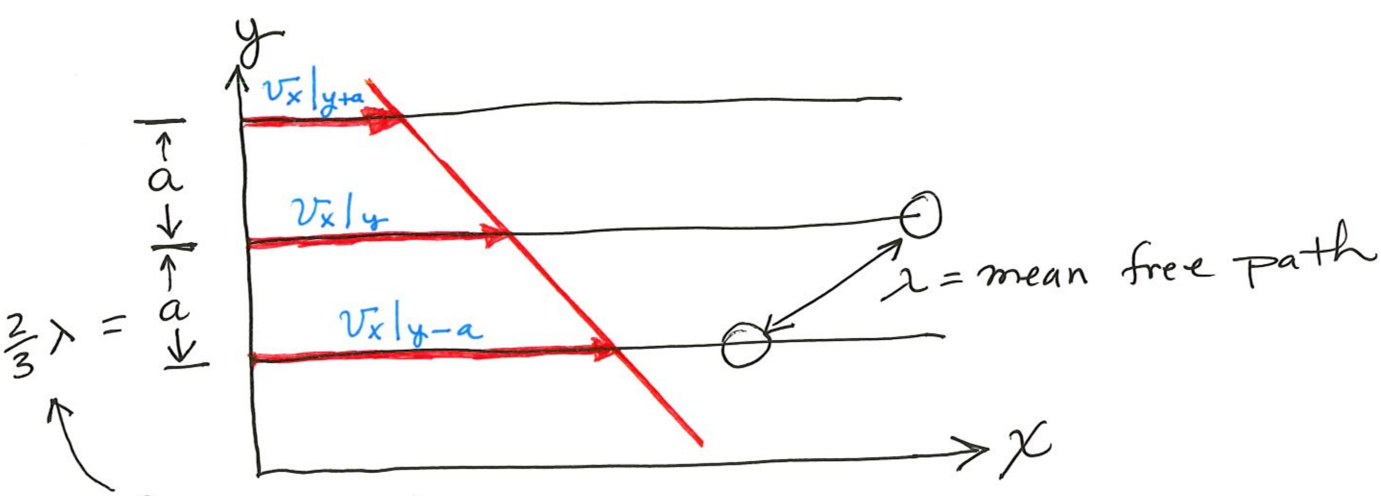
Types of fluids



Theories of viscosity: Kinetic theory of gases.

1. Maxwell Boltzman distribution of velocities
2. Molecules do not interact until they collide, then they collide ellastically

Potential energy diagram ?



average length parallel to y axis travelled between collisions

Kinetic theory results:

Boltzmann constant $= R/N_{av}$

Average molecular speed: $\bar{u} = \sqrt{\dfrac{8kT}{\pi m}}$

molecule mass

Frequency of bombardment per unit area: $Z = \frac{1}{2}\bar{u}\,n$

number density of molecules

Mean free path: $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n}$

diameter of molecule

Molecules arriving at a plane have on average had their last collision at a distance $2/3$ of the mean free path from the wall (denote as "a")

$$a = 2/3\,\lambda$$

By definition (using momentum flux definition), and assuming that molecules arriving at plane y have x-direction momentum representative of the region they last collided:

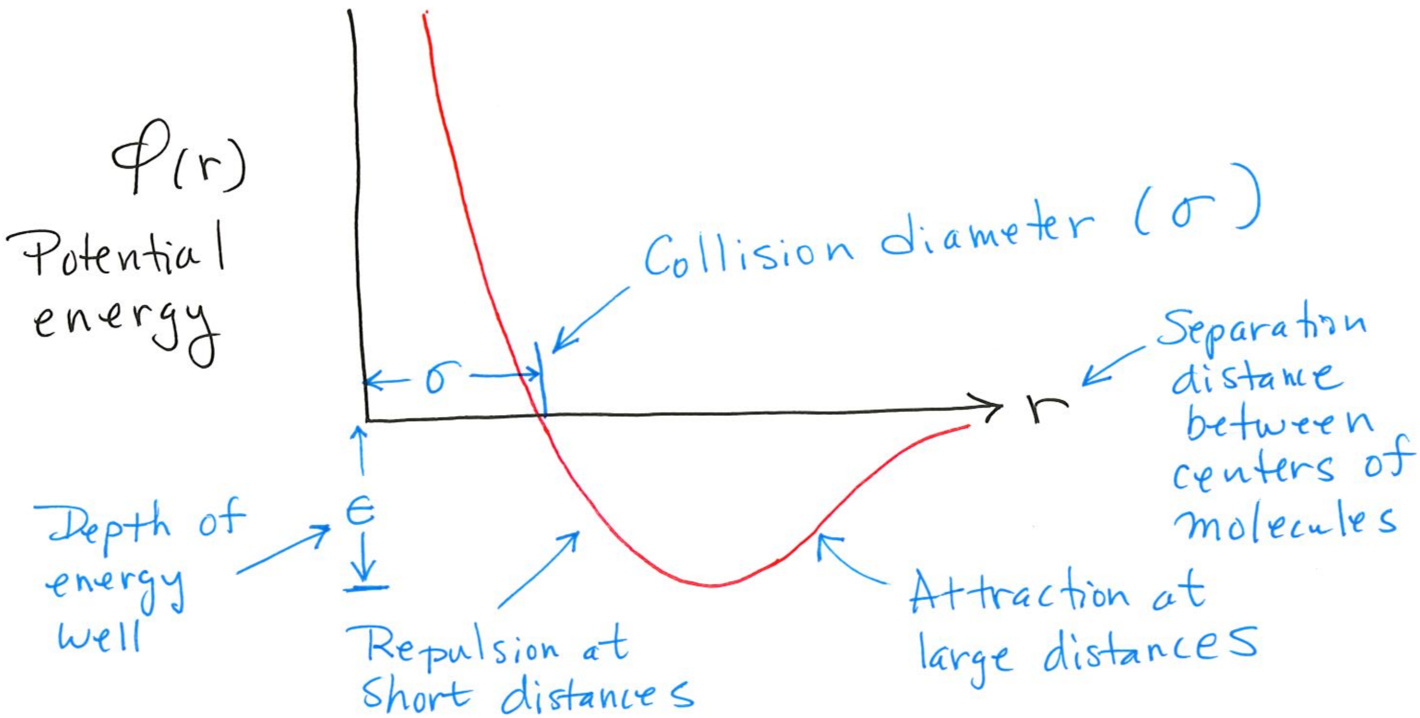
$$\tau_{yx} = Z\left(m v_x\big|_{y-a}\right) - Z\left(m v_x\big|_{y+a}\right)$$

but
$$v_x\big|_{y-a} = v_x\big|_y - 2/3\,\lambda\,dv_x/dy$$
$$v_x\big|_{y+a} = v_x\big|_y + 2/3\,\lambda\,dv_x/dy$$

so that
$$\tau_{yx} = -\underbrace{1/3\,n\,m\,\bar{u}\,\lambda}_{\mu}\,dv_x/dy$$

$$\frac{2}{3}\,\frac{1}{\pi^{3/2}}\,\frac{\sqrt{mkT}}{d^2} \leftarrow \mu$$

More rigorous theory



Leonard Jones (6-12) potential

$$\varphi(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

Leads to Chapman - Enskog theory

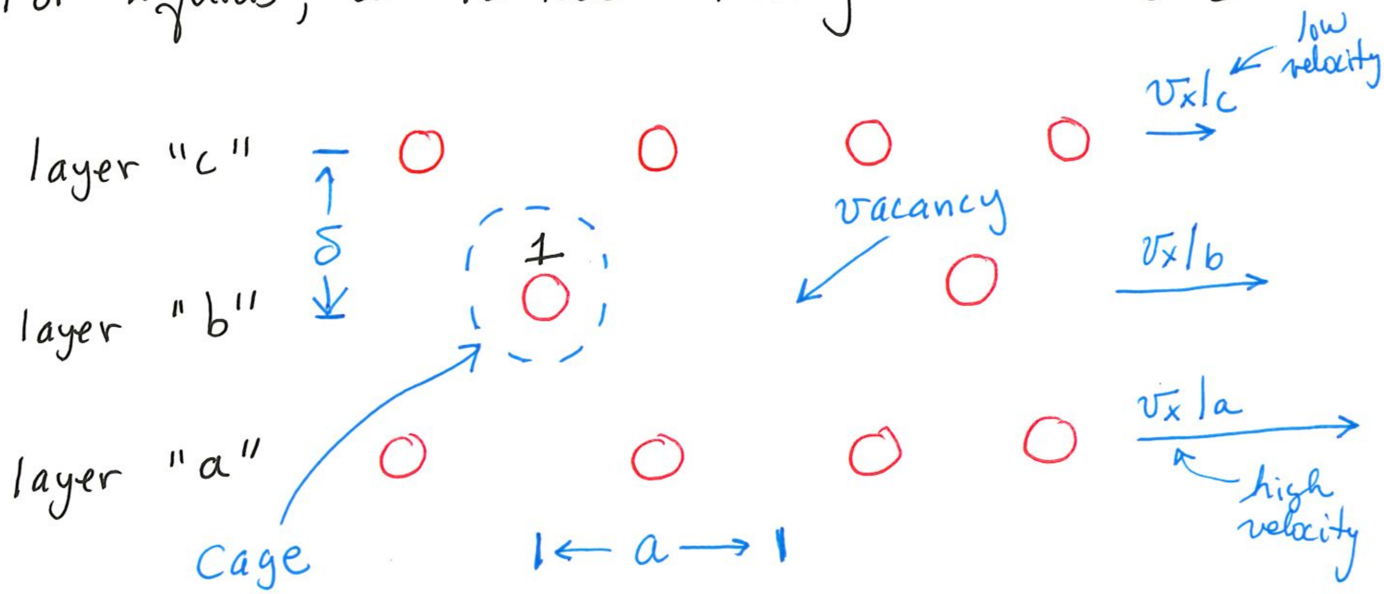
$$\underset{g/cm\,s}{\mu} = 2.669 \times 10^{-5}\, \frac{\sqrt{MT}}{\sigma^2\,\Omega}$$

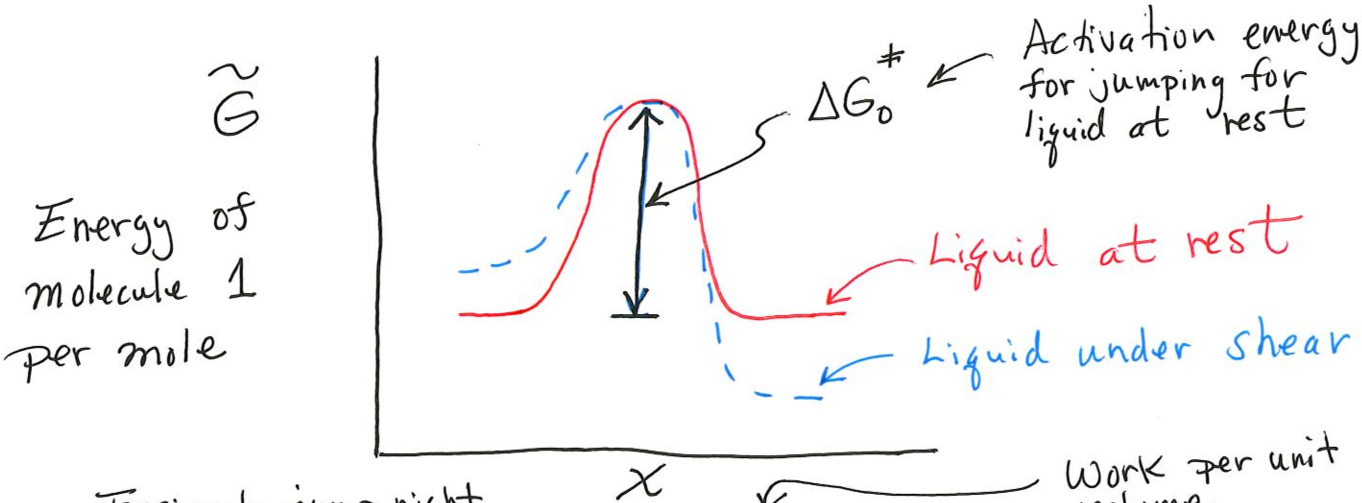
Molecular weight

Å

Where Ω = function of kT/ϵ

For Liquids, a "lattice" theory can be used





Easier to jump right and harder to jump left

Work per unit volume

$$\Delta \tilde{G}^{\ddagger} = \Delta \tilde{G}_0^{\ddagger} \pm \overbrace{\underbrace{\frac{a}{2}}_{\text{Distance from start to top of barrier}} \underbrace{\frac{\tau_{yx}}{\delta}}_{\text{shear force per area}/\delta\ =\ \text{Shear force per unit volume}}}^{\text{Work per unit volume}} \tilde{V}$$

Volume per mo

Frequency of forward and backward jumps

$$k_f = \frac{kT}{h} \exp\left(-\Delta G_0^{\ddagger}/RT\right) \exp\left(\frac{a\,\tau_{yx}\,\tilde{V}}{2\,\delta\,RT}\right)$$

$$k_b = \frac{kT}{h} \exp\left(-\Delta G_0^{\ddagger}/RT\right) \exp\left(-\frac{a\,\tau_{yx}\,\tilde{V}}{2\,\delta\,RT}\right)$$

The net velocity by which layer b slips ahead of layer c is given by

$$v_x\big|_b - v_x\big|_c = a\,(k_f - k_b)$$

Divide by δ:

$$\frac{dv_x}{dy} = \frac{a}{\delta}\,(k_f - k_b)$$

$\exp(x) \approx 1 + x$

Substitute, assuming $\dfrac{a\,\tau_{yx}\,\tilde{V}}{2\,\delta\,RT}$ is small

$$\tau_{yx} = -\underbrace{\left(\frac{\delta}{a}\right)^2 \frac{N_{av}\,h}{\tilde{V}} \exp\!\left(\frac{\Delta G_0^{\ddagger}}{RT}\right)}_{\mu} \frac{dv_x}{dy}$$

Approximately: $\Delta G_0^{\ddagger} \approx 0.4\ \Delta\tilde{U}_{vaporization}$

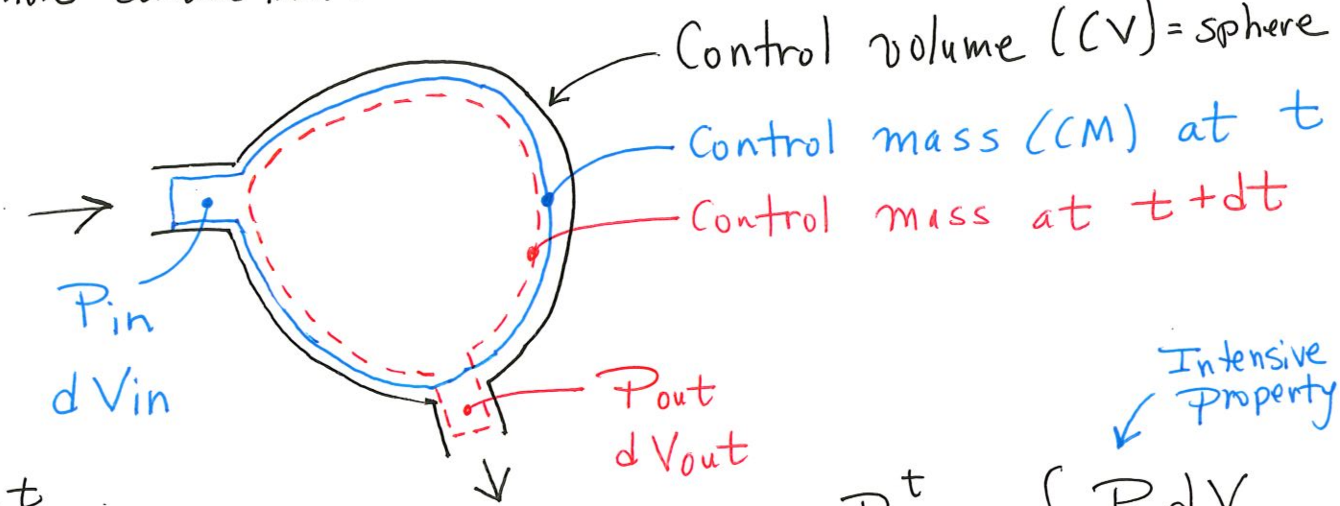
Also, Trouton's Rule: $\Delta\tilde{U}_{vaporization} \approx 9.4\,R\,T_b$

Normal boiling point

The Reynolds Transport Theorem

Convert conservation law for control mass (closed) to control volume (open) system

Ignore conduction:



P^t = any extensive property; $\quad P^t_{CV} = \int_{CV} P\, dV$ (Intensive property)

$$P^t_{CM} = \int_{CM} P\, dV$$

$$P^t_{CM}\Big|_t = P^t_{CV}\Big|_t + P_{in}\, dV_{in}$$

$$P^t_{CM}\Big|_{t+dt} = P^t_{CV}\Big|_{t+dt} + P_{out}\, dV_{out}$$

Therefore:

$$\frac{dP^t_{CM}}{dt} = \underbrace{\underbrace{\frac{dP^t_{CV}}{dt}}_{\text{Accumulation in CV}} \underbrace{- P_{in}\left(\frac{dV}{dt}\right)_{in}}_{\text{Inflow to CV}} + \underbrace{P_{out}\left(\frac{dV}{dt}\right)_{out}}_{\text{Outflow from CV}}}_{\text{Define as "Production" in CV}}$$

In general, if $\frac{dP^t_{CM}}{dt} = B$ for a control mass, then for a control volume production of $P^t_{CV} = B$

Examples ?

Including conduction (when P = momentum/volume):

Include conduction in the "B"

Include conduction in the "ins - outs"