Part 10a: Analogies between transport modes

Thermal energy eqn.:

$$\frac{DT}{Dt} = \underbrace{\frac{K}{\rho C_P}}_{\text{Thermal diffusivity}} \nabla^2 T$$

Thermal conductivity → K

Species continuity eqn.:

$$\frac{D\rho_A}{Dt} = \underbrace{\mathcal{D}_{AB}}_{\text{Mass diffusivity}} \nabla^2 \rho_A$$

Momentum transport eqn.:

$$\frac{D\vec{v}}{Dt} = \underbrace{\frac{\mu}{\rho}}_{\text{Momentum diffusivity}} \nabla^2 \vec{v} + \vec{\nabla} P/\rho$$

or

$$\frac{D|\vec{\omega}|}{Dt} = \frac{\mu}{\rho} \nabla^2 |\vec{\omega}|$$

Dimensionless numbers:

Heat transfer	Mass transfer
$Nu_H = \frac{hD}{K}$ (heat trans. coef. → h)	$Nu_M = Sh = \frac{kD}{\mathcal{D}_{AD}}$ (mass trans. coef. → k)
$Pr = \frac{C_P \mu}{K} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$	$Sc = \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$

Note: $\dot{m} = k A \Delta \rho_A$ (like $\dot{Q} = h A \Delta T$)

If boundary conditions are the same, then the solutions are completely analogous for heat and mass transfer

Example: For laminar boundary layer with fixed temperature on plate, the heat transfer result is:

$$Nu_{loc} = \tfrac{1}{3}\, Re_x^{1/2}\, Pr^{1/3}$$

$h_{loc}x/k$

$\frac{\rho v x}{\mu}$

μ/ρ

$\frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$

$k/\rho C_p$

For mass transfer with $x_{A,i}$ fixed on plate

$$Sh_{loc} = \tfrac{1}{3}\, Re_x^{1/2}\, Sc^{1/3}$$

$\frac{k_{loc}\, x}{\mathcal{D}_{AB}}$

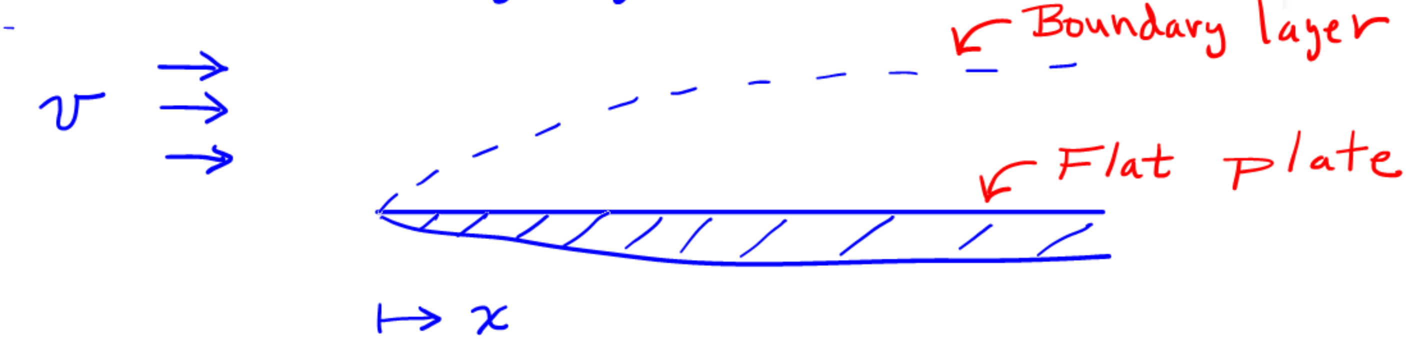
$\frac{\rho v x}{\mu}$

μ/ρ

$\frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$

$\mathcal{D}_{AB}$

Laminar boundary layer theory:



Empirical correlations can also be extended using this analogy:

"j factor" for heat transfer

$$j_H = \frac{Nu}{Re\,Pr} \cdot Pr^{2/3} = St_H\, Pr^{2/3} = f(Re)$$

$$j_D = \frac{Sh}{Re\,Sc} \cdot Sc^{2/3} = St_D\, Sc^{2/3} = f(Re)$$

"j factor" for mass transfer

$St_D = \dfrac{k}{v_R}$ — k: mass transfer coefficient; v_R: velocity

General result:

$$j_H = j_D = f(Re) = f/2$$

very reliable

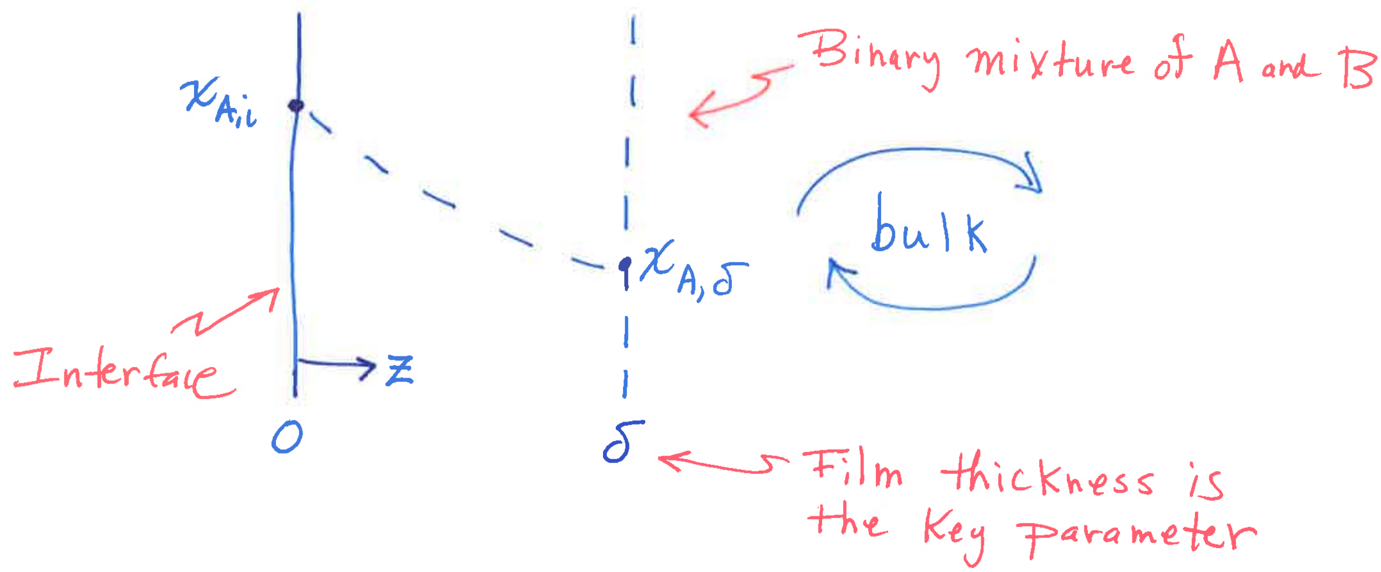
less reliable (works best if no form drag)

The above equalities are "exact" for a laminar boundary layer.

Part 10 b

Models for binary mass transfer

Consider the "film" model for binary mass transfer in a gas (convenient to employ different models for gas or liquid):



Material balance at steady state:

$$\frac{dN_A}{dz} = \frac{dN_B}{dz} = \frac{dN_T}{dz} = 0 \,; \quad N_T = N_A + N_B$$

Fick's law:

$$-c\,\mathcal{D}_{AB}\frac{dx_A}{dz} = J_A^* = N_A - c_A v^* \quad (v^* = N_T/c)$$

$$= N_A - x_A N_T$$

$c = P/RT$
$= \text{constant for ideal gas}$

Previous result leads to:

$$\frac{d x_A}{dz} = \frac{x_A N_T}{c \mathcal{D}_{AB}} - \frac{N_A}{c \mathcal{D}_{AB}}$$

Solution is:

constant of integration

$$x_A = K \exp(\lambda z) + N_A / N_T$$

$$\lambda = \frac{N_T}{c \mathcal{D}_{AB}} = v^* / \mathcal{D}_{AB}$$

Determine K:

at $z = 0$; $x_{A,i} = K + N_A / N_T$

at $z = \delta$; $x_{A,b} = K \exp(\lambda \delta) + N_A / N_T$

Solve for K: $K = \Delta x_A (1 - \exp(\lambda \delta))^{-1}$, $\Delta x_A = x_{A,i} - x_{A,b}$

x_A profile: $$x_A = \frac{\Delta x_A \exp(\lambda z)}{1 - \exp(\lambda \delta)} + N_A / N_T$$

Substitute into Fick's Law: $J^*_{A,i} = c \mathcal{D}_{AB} \nabla x_A |_i$

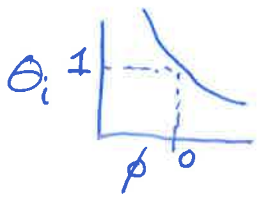
$\lambda \delta = \phi = N_T \delta / (c \mathcal{D}_{AB}) = \dfrac{v^*}{k_{J,X}}$ "stefan flow"

$$J^*_{A,i} = \underbrace{c \frac{\mathcal{D}_{AB}}{\delta}}_{k^*_{J,X}} \underbrace{\frac{\lambda \delta}{\exp(\lambda \delta) - 1}}_{\theta_i} \Delta x_A$$

low flux mass transfer coefficient

flux correction factor

Inverse of Staton number based on stefan flow velocity



Note that $k^{\bullet}_{J,X} = k_{J,X} \theta_i$ flux corrected coefficient