Name: ____________________

**1.** (10 points) The algorithm P-Sum computes the sum of the elements of an array $L$ of length $n$. Determine the work, span, and parallelism of P-Sum. Justify your answers.

P-Sum($L$)

$n = L.length$
**if** $n == 1$
  **return** $L[1]$
$c = \lfloor n/2 \rfloor$
$x =$ **spawn** P-Sum($L[1\,..\,c]$)
$y =$ P-Sum($L[c+1\,..\,n]$)
**sync**
**return** $x + y$

**Solution:** The work $T_1$ is just the running-time of the algorithm with no parallelism. Since at each level the algorithm makes two recursive calls plus some constant-time work, $T_1$ satisfies

$$T_1(n) = 2T(n/2) + \Theta(1).$$

You can solve this directly or use the Master Theorem (case 1). In either case, you find that $T_1(n) = \Theta(n)$.

The span $T_\infty$ is the running-time assuming an arbitrarily large number of processors. Since the recursive calls are now run in parallel, their running-time is $T_\infty(n/2)$, *not* $2T_\infty(n/2)$, so $T_\infty$ satisfies

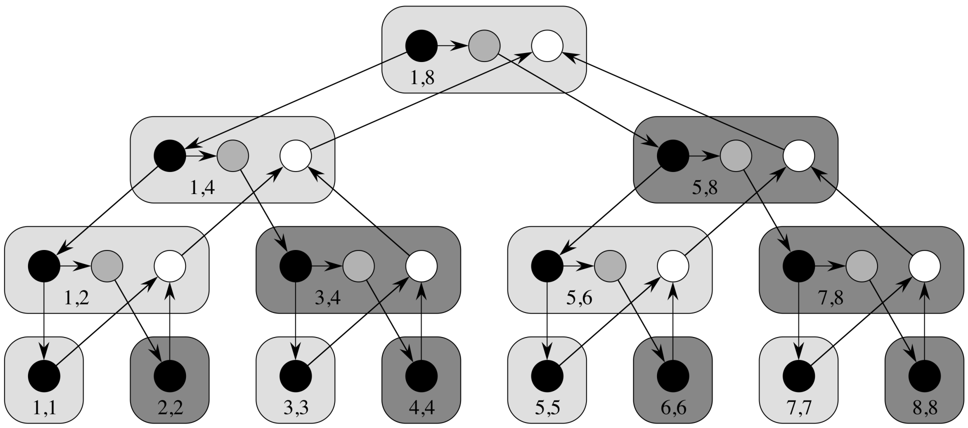
$$T_\infty(n) = T_\infty(n/2) + \Theta(1),$$

and case 2 of the Master Theorem tells us that $T_\infty(n) = \Theta(\lg n)$.

The parallelism is just $T_1(n)/T_\infty(n) = \Theta(n/\lg n)$.

(continued on other side)

**2.** (10 points) The algorithm MAT-VEC-MAIN-LOOP computes the product of a matrix $A$ with a vector $x$, writing the result to a vector $y$. Consider the following DAG for MAT-VEC-MAIN-LOOP applied to an 8-by-8 matrix $A$ and 8-long vectors $x$ and $y$:




Determine the work and span from the DAG. What is the *parallel slackness* when $P = 2$? Justify your answers.

**Solution:** The work is just the number of strands (circles), which is 29. The span is the length (in strands) of the longest path through the DAG, which is 10. The parallel slackness with $P = 2$ is

$$T_1/(PT_\infty) = 29/(2 \cdot 10) = 1.45.$$