CMSC 341
Priority Queues & Heaps

Based on slides from previous iterations of this course

Today's Topics

- Priority Queues
  - Abstract Data Type
- Implementations of Priority Queues:
  - Lists
  - BSTs
  - Heaps
- Heaps
  - Properties
  - Insertion
  - Deletion

Priority Queues

Priority Queue ADT

- A priority queue stores a collection of entries
- Typically, an entry is a pair

  `(key, value)`

  where the key indicates the priority
  - Smaller value, higher priority
- Keys in a priority queue can be arbitrary objects on which an order is defined

Priority Queue vs Queue

- Priority queue is a specific type of queue

- Queues are FIFO
  - The element in the queue for the longest time is the first one we take out
- Priority queues: most important, first out
  - The element in the priority queue with the highest priority is the first one we take out
  - Examples: emergency rooms, airline boarding

Implementing Priority Queues

- Priority queues are an Abstract Data Type
  - They are a concept, and hence there are many different ways to implement them
- Possible implementations include
  - A sorted list
  - An ordinary BST
  - A balanced BST
- Run time will vary based on implementation

Implementing a Priority Queue

Priority Queue: Sorted List

- We can implemented a priority queue with a sorted list (array, vector, etc.)

- Sorted by priority upon <u>insertion</u>
  - To find the highest priority, simply find the minimum, in O(1) time

    ```
    findMin() --> list.front()
    ```

  - Insertion can take O(n) time, however

Priority Queue: BST

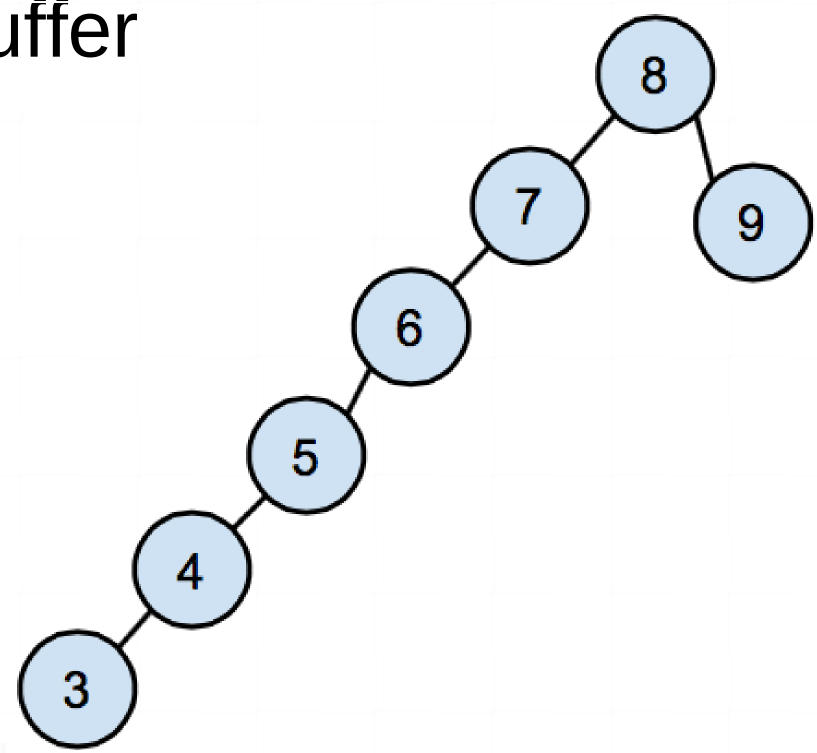
- A BST makes a bit more sense than a list
- Sorted like a regular BST upon insertion
  - To find the minimum, just go to the left

    call **`findMin()`**
    - And removal will be easy, because
      it will always be a leaf node!
  - Insertion should take no more than O(log n) time

    call **`Insert()`**

Priority Queue: BST Downsides

- Unfortunately, a BST Priority Queue can become unbalanced very easily, and the actual run time will suffer
    - If we have a low priority (high value) instance as our root, nearly everything will be to its left
- **`findMin()`** is now O(n) time ☹



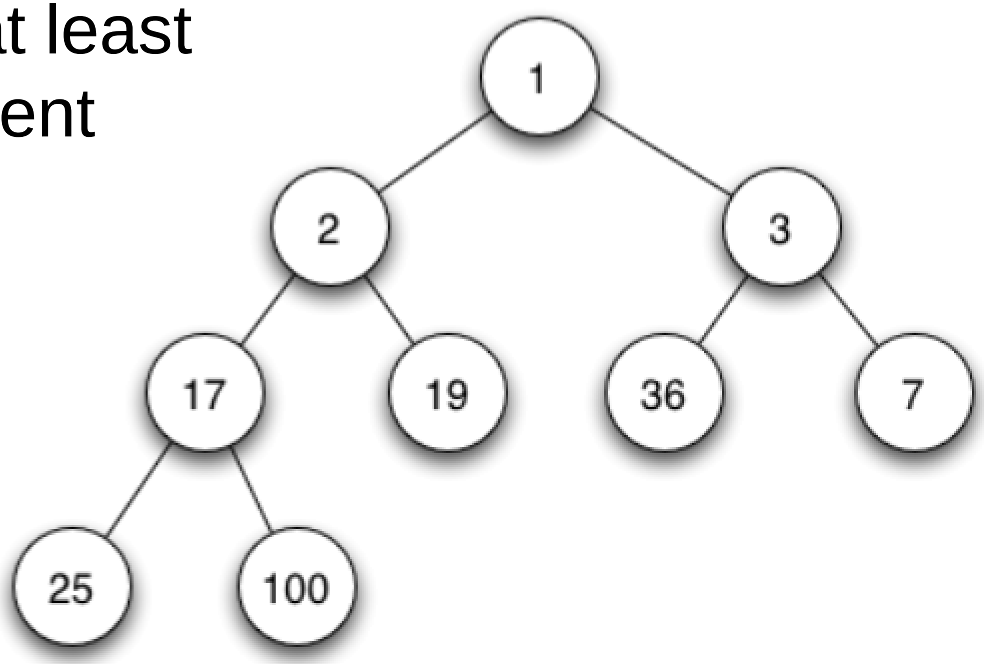
Priority Queue: Heap

- The most common way to implement a priority queue is using a heap
- A heap is a binary tree (<u>not</u> a BST!!!) that satisfies the "heap condition":
  - Nodes in the tree are sorted based in relation to their parent's value, such that if A is a parent node of B, then the key of node A is ordered with respect to the key of node B with the same ordering applying across the heap
- Additionally, the tree must be <u>complete</u>

Heaps

Min Binary Heap

- A **min binary heap** is a…
  - Complete binary tree
  - Neither child is smaller than the value in the parent
  - Both children are at least as large as the parent



- In other words, smaller items go above larger ones

Min Binary Heap

- This property is called a **partial ordering**
  - There is no set relation between siblings, cousins, etc – only that the values grow as we increase our distance from the root
- As a result of this partial ordering, every path from the root to a leaf visits nodes in a non-decreasing order

Min Binary Heap Performance

- Performance
  - (n is the number of elements in the heap)

- construction O(n)
- **`findMin()`** O(1)
- **`insert()`** O(lg n)
- **`deleteMin()`** O(lg n)

Min Binary Heap Performance

- Heap efficiency results, in part, from the implementation
  - Conceptually a complete binary tree
  - Can think of implementation in an array/vector (in level order) with the root at **index 1**

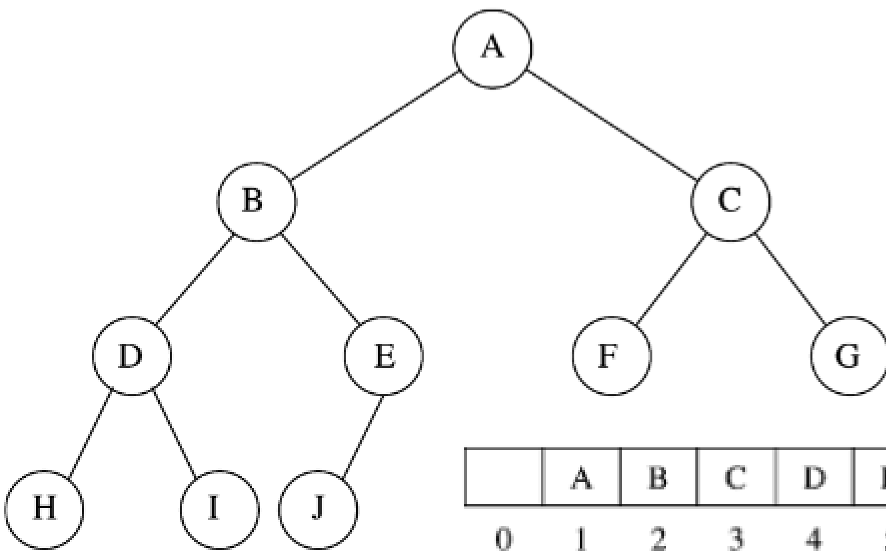
To Find	Formula
Parent index	`floor((index)/2)`
Left Child index	`2(index)`
Right Child index	`2(index) + 1`

Min Binary Heap Performance

- For a node at index i
  - its left child is at index 2i
  - its right child is at index 2i+1
  - its parent is at index $\lfloor i/2 \rfloor$
- No pointer storage
- Fast computation of 2i and $\lfloor i/2 \rfloor$ by bit shifting
  - $i << 1 = 2i$
  - $i >> 1 = \lfloor i/2 \rfloor$

Min Binary Heap: Exercises

- How to find the parent of E?
- The left child of D?
- The right child of A?

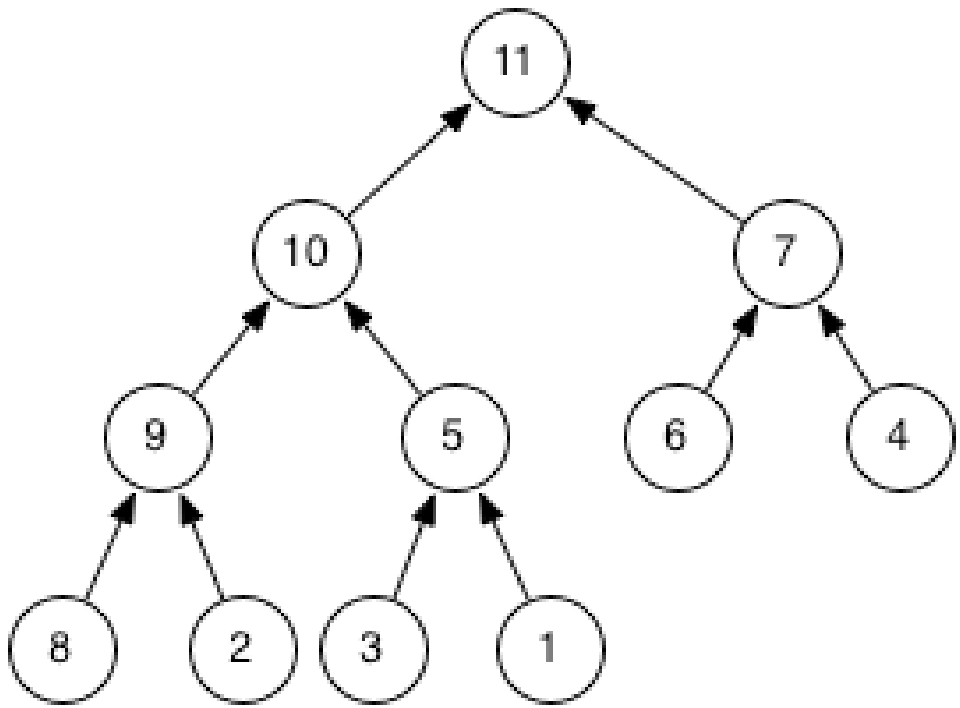


To Find	Formula
Parent index	`floor((index)/2)`
Left Child index	`2(index)`
Right Child index	`2(index) + 1`

0	1	2	3	4	5	6	7	8	9	10	11	12	13
	A	B	C	D	E	F	G	H	I	J			

Convert a Heap to an Array

- Level-order traversal



11	10	7	9	5	6	4	8	2	3	1

Building a Heap

Insert Operation

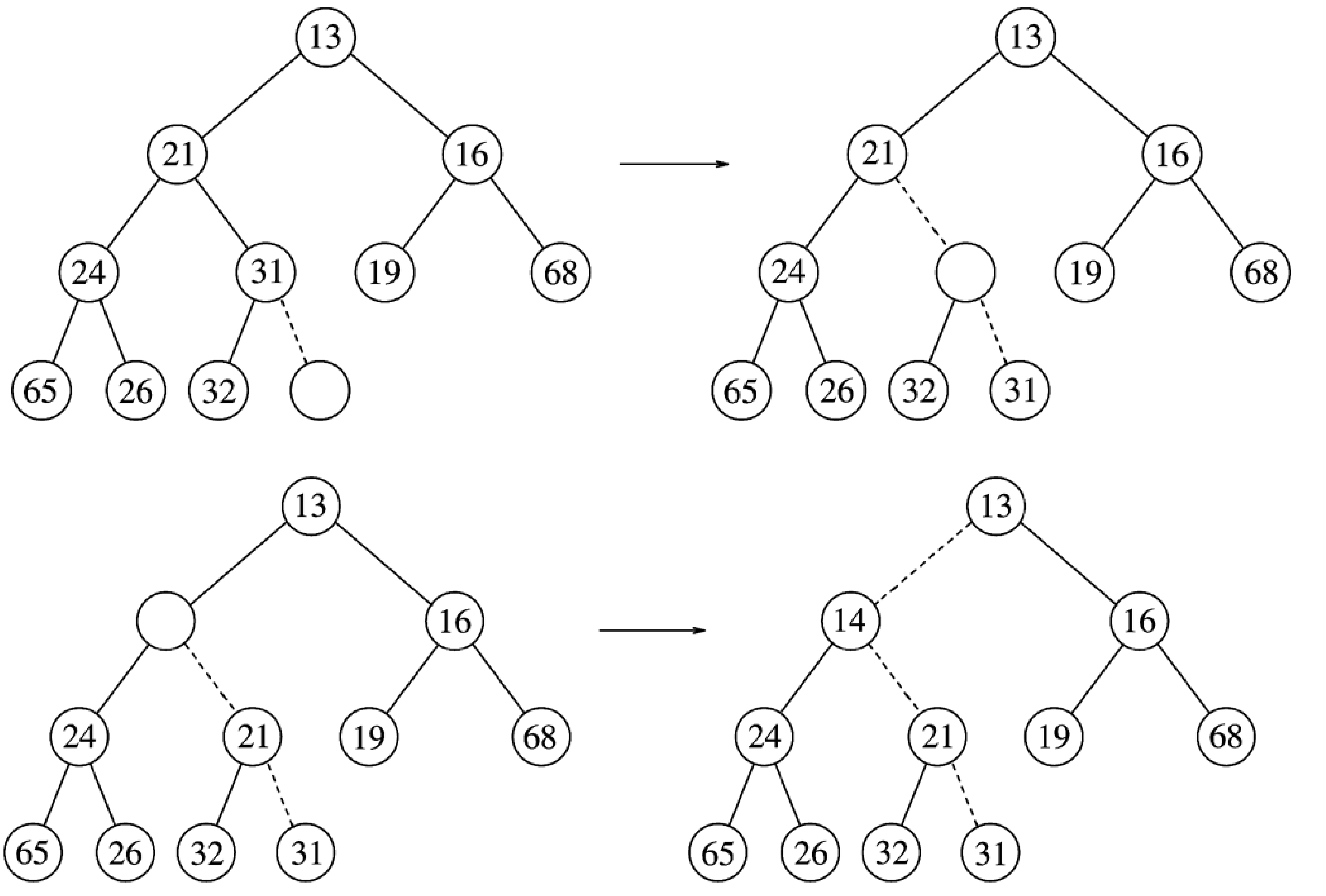
- Must maintain
  - Heap shape:
    - Easy, just insert new element at “the end” of the array
  - Min heap order:
    1. Could be wrong after insertion if new element is smaller than its ancestors
    2. Continuously swap the new element with its parent until parent is not greater than it
    - Called sift up or percolate up
- Performance of insert is O(lg n) in the worst case because the height of a CBT is O(lg n)

Insert Code

```
void insert( const Comparable & x )
{
  if( currentSize == array.size( ) - 1 )
      array.resize( array.size( ) * 2 );

  // Percolate up
  int hole = ++currentSize;
  for( ; hole > 1 && x < array[ hole / 2 ];
       hole /= 2 )
  {
     // swap, from child to parent
     array[ hole ] = array[ hole / 2 ];
  }
  array[ hole ] = x;
}
```

Insert Example: 14



Delete Operation

- Steps
  - Remove min element (the root)
  - Maintain heap shape
  - Maintain min heap order
- To maintain heap shape, actual node removed is “last one” in the array
  - Replace root value with value from last node and delete last node
  - Sift-down the new root value
    - Continually exchange value with the smaller child until no child is smaller.

Delete Code

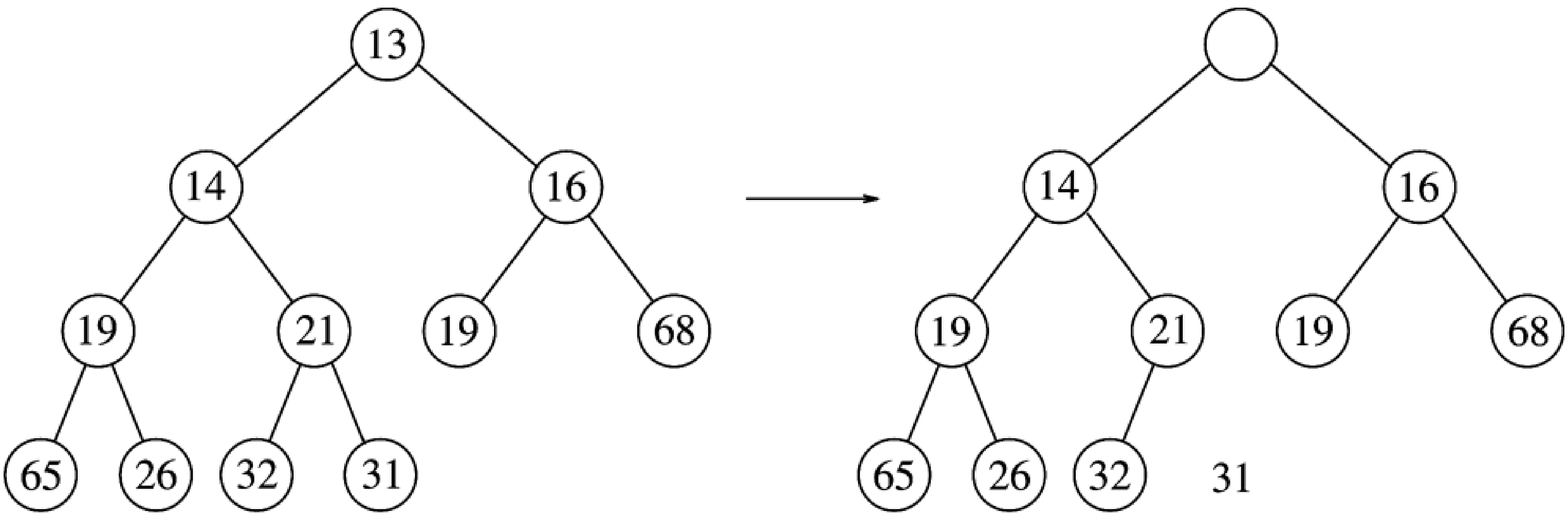
```
void deleteMin( )
{
  if( isEmpty( ) )
  {
    throw UnderflowException( );
  }
  array[ 1 ] = array[currentSize--];
  percolateDown( 1 );
}
```

Percolate Down Function

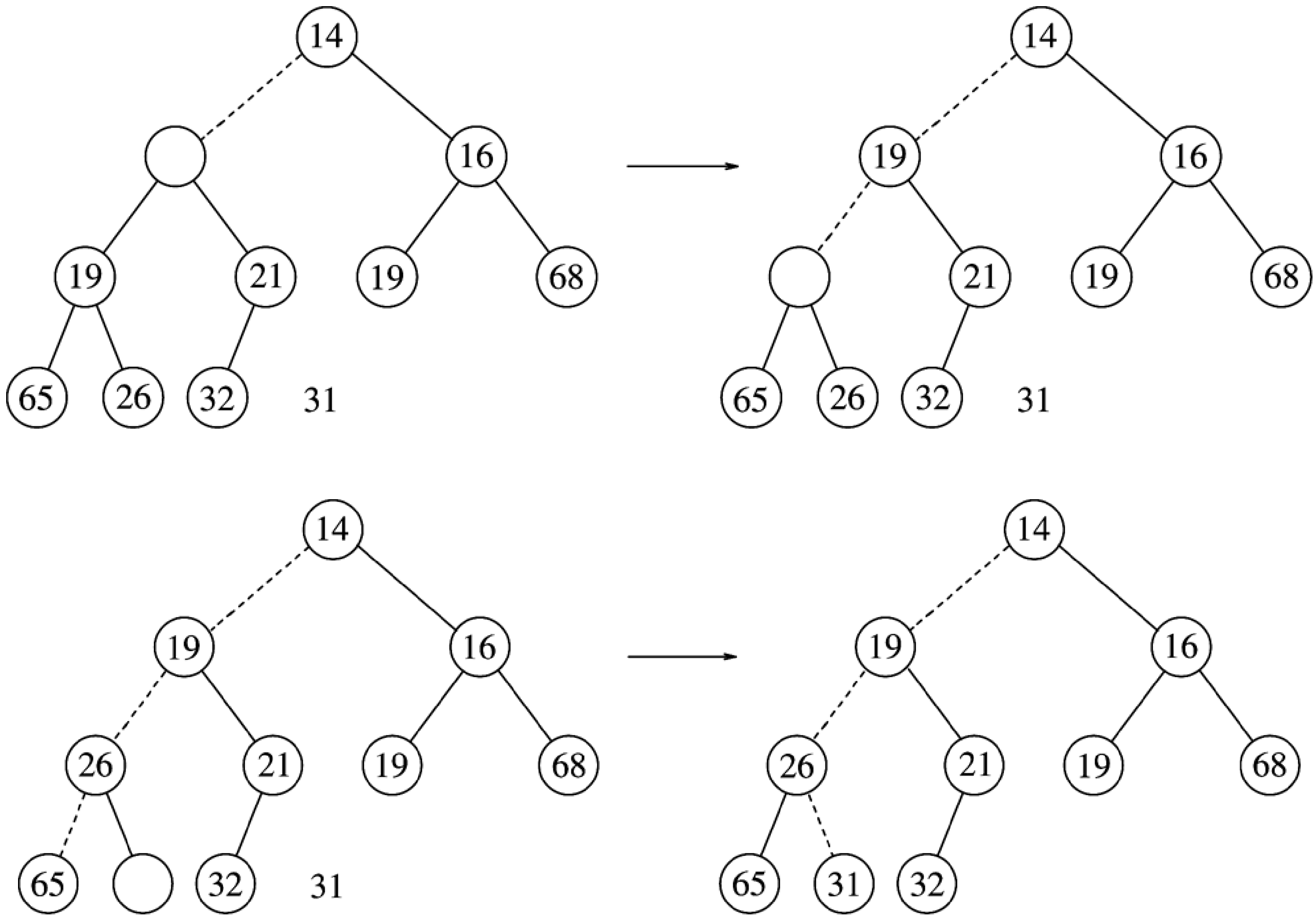
```
void percolateDown( int hole )
{
  int child;
  Comparable tmp = array[ hole ];

  for( ; hole * 2 <= currentSize; hole = child )
  {
    child = hole * 2;
    if( child != currentSize && array[child+1]<array[child ])
      { child++; }
    if( array[ child ] < tmp )
      { array[ hole ] = array[ child ]; }
    else { break; }
  }
  array[ hole ] = tmp;
}
```

Example: Delete Min



Example: Delete Min



Announcements

- Homework 4
  - Due Thursday, October 26th at 8:59:59 PM
- Project 3
  - Due Tuesday, October 31st at 8:59:59 PM

- Next Time:
  - Priority Queues and Heaps (part 2)