CMSC 341
Lecture 8
Introduction to Trees

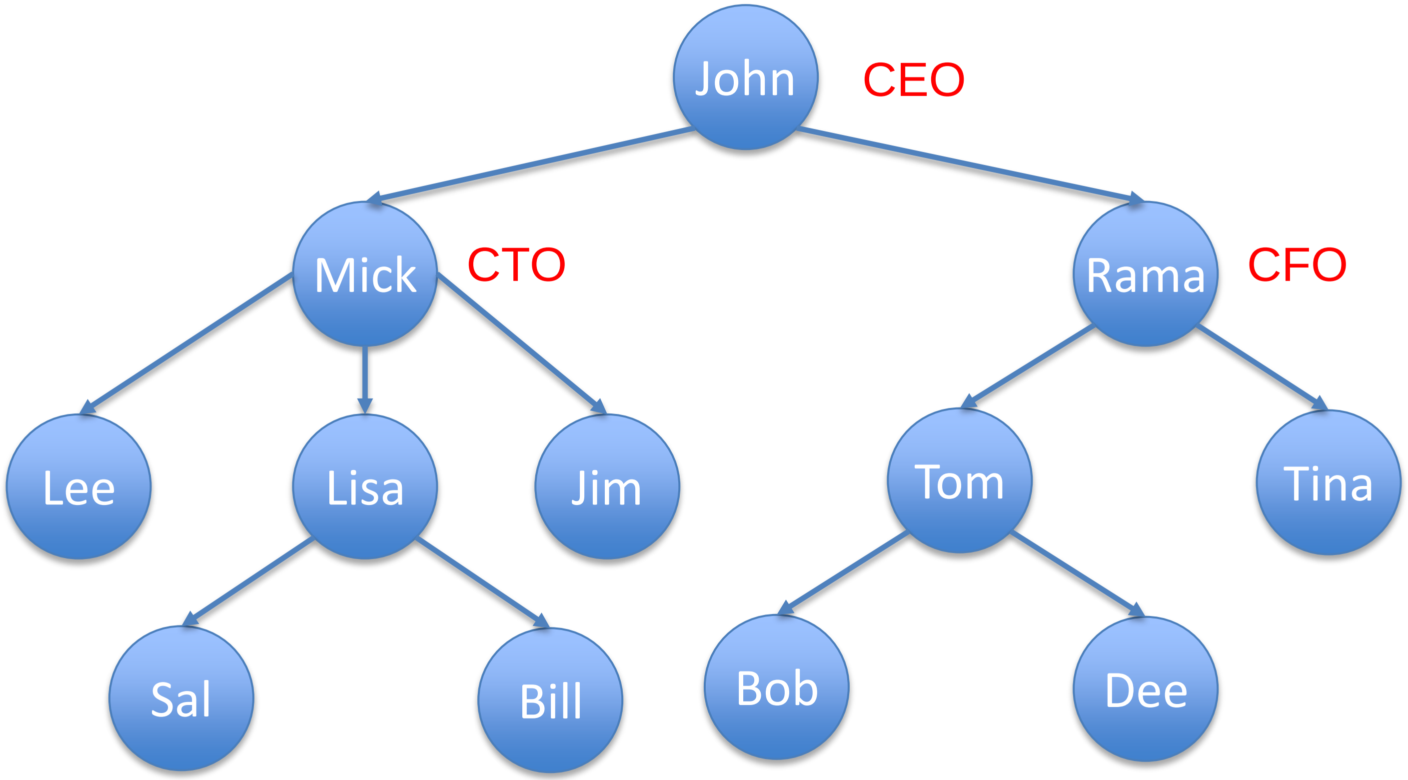
Prof. Gibson & Prof. Goodrich

Introduction to Trees

What is a Tree?

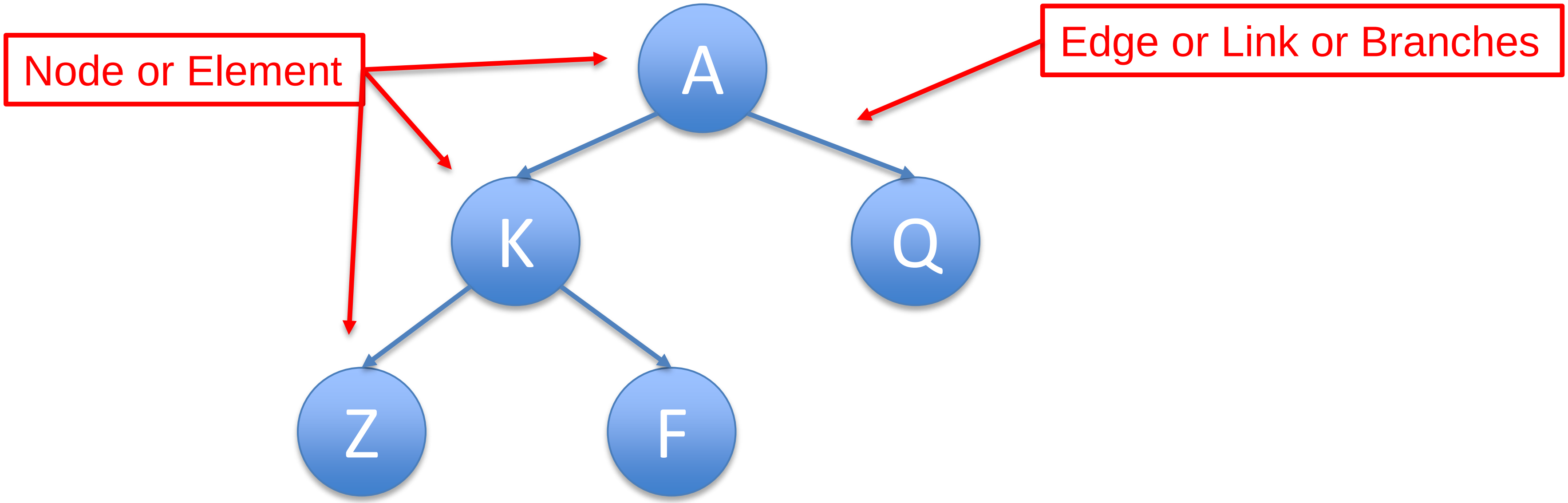
- In computer science, a *tree* is an abstract model of a hierarchical structure
- Applications:
  – Organization charts
  – File systems
  – Programming environments

Tree Example – Org Chart



What is a Tree?

- A *tree* is a collection of nodes (elements)

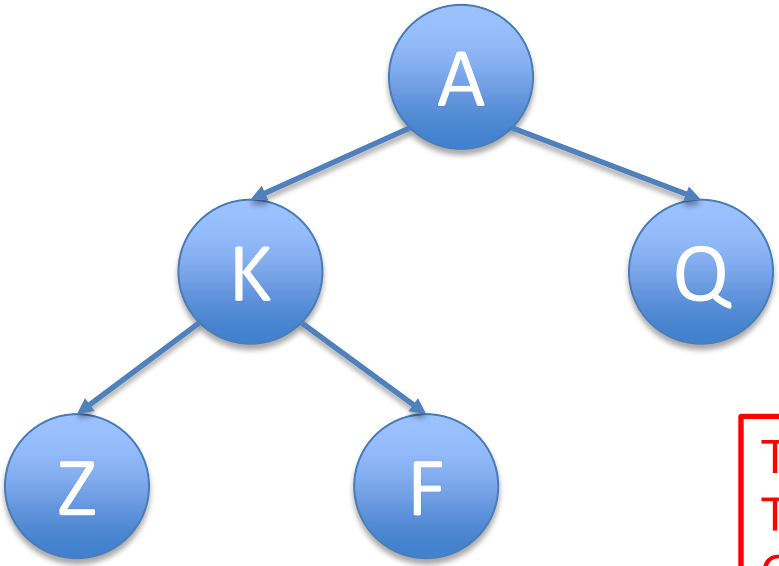


Tree Terminology

- There are two main ways that trees are described.
  1. Terms are related to "trees" such as *root*, *branches*, and *leaves*
  2. Terms are related to "ancestry" such as *parent*, *children*, *sibling*, *ancestors*, and *descendants*

What is a Tree?

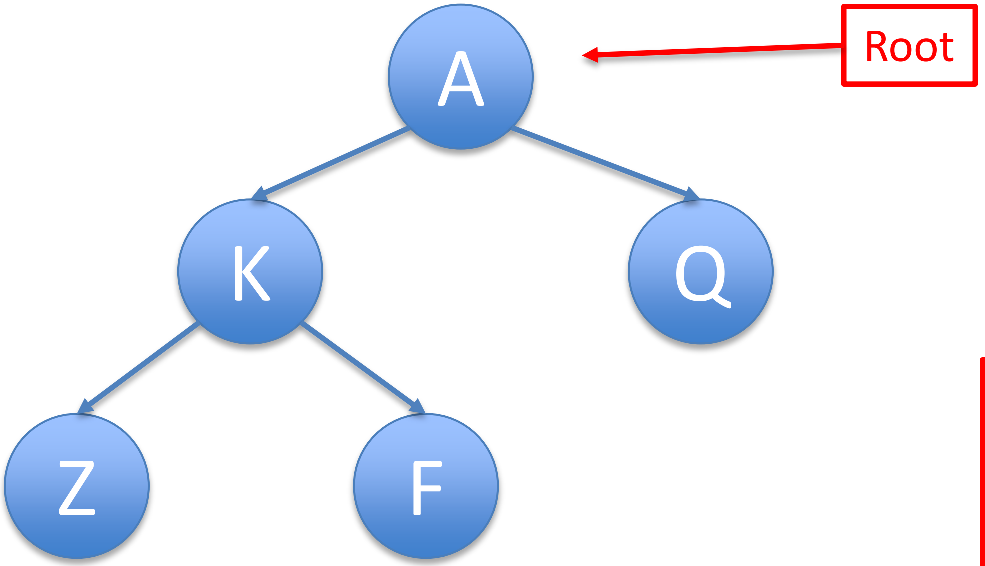
- Each node may have 0 or more ***children***



The children of A are K and Q.
The children of K are Z and F.
Q, Z, and F have no children.

What is a Tree?

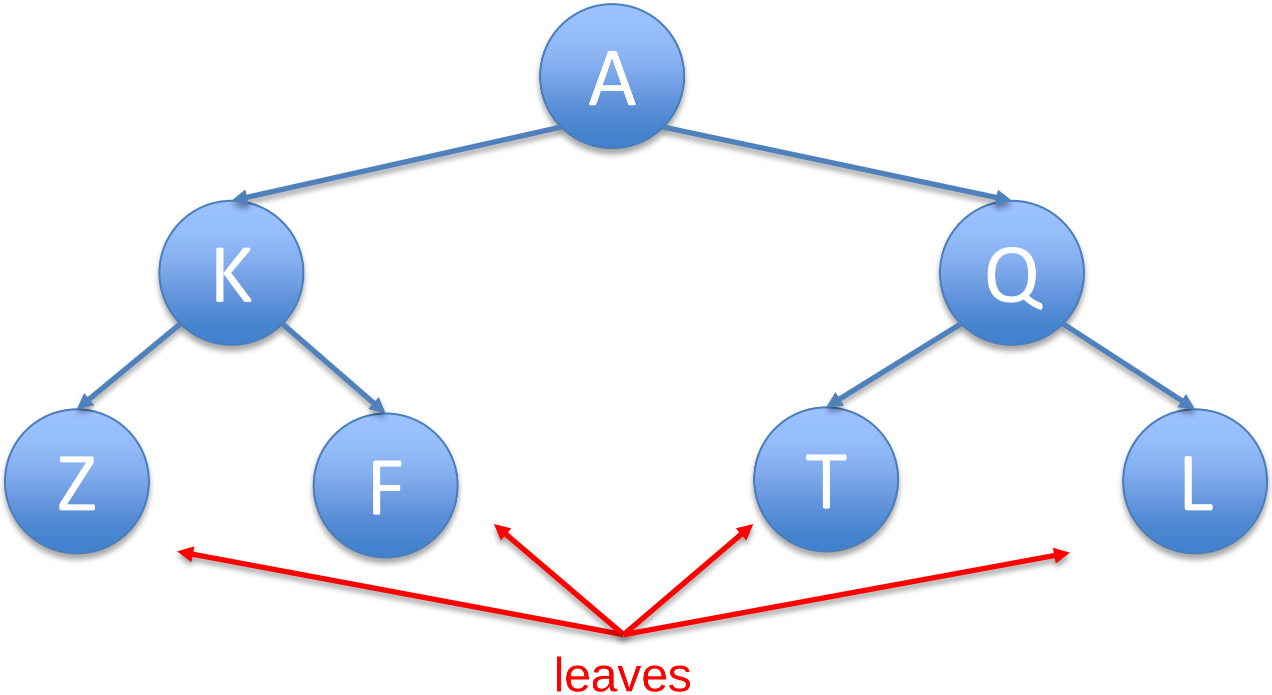
- Each node has **exactly one** *parent*
  - Except the starting / top node, called the *root*



The parent of K is A.
The parent of Q is A.
The parent of Z is K.
The parent of F is K.

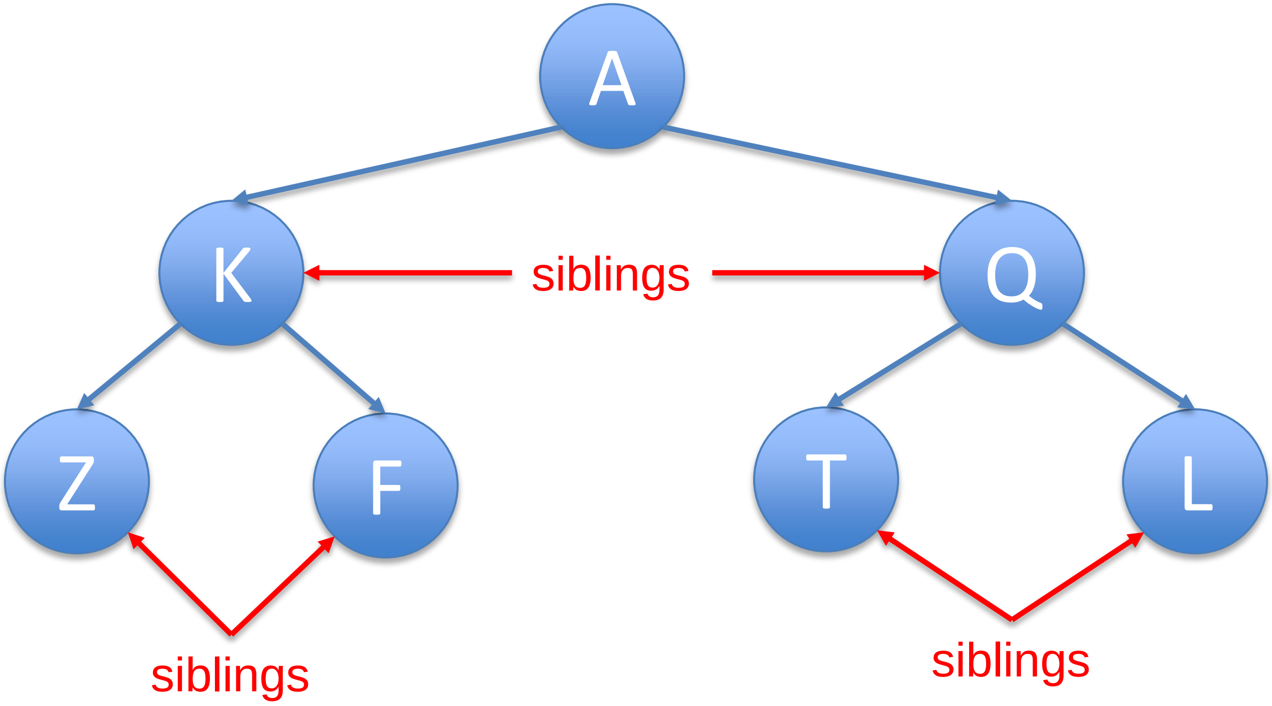
What is a Tree?

- Nodes with no children are called ***leaves***
- Which are leaves?



What is a Tree?

- Nodes with same parent are ***siblings***
- *Which are siblings?*

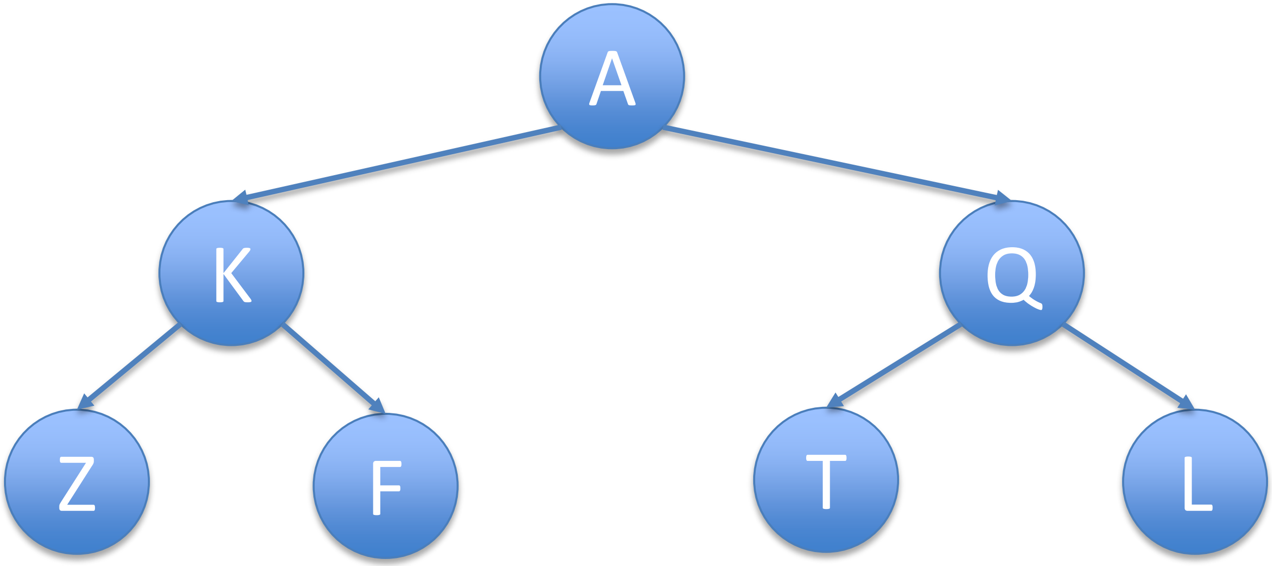


What is a Tree?

- If there is a path between node A and node Z:

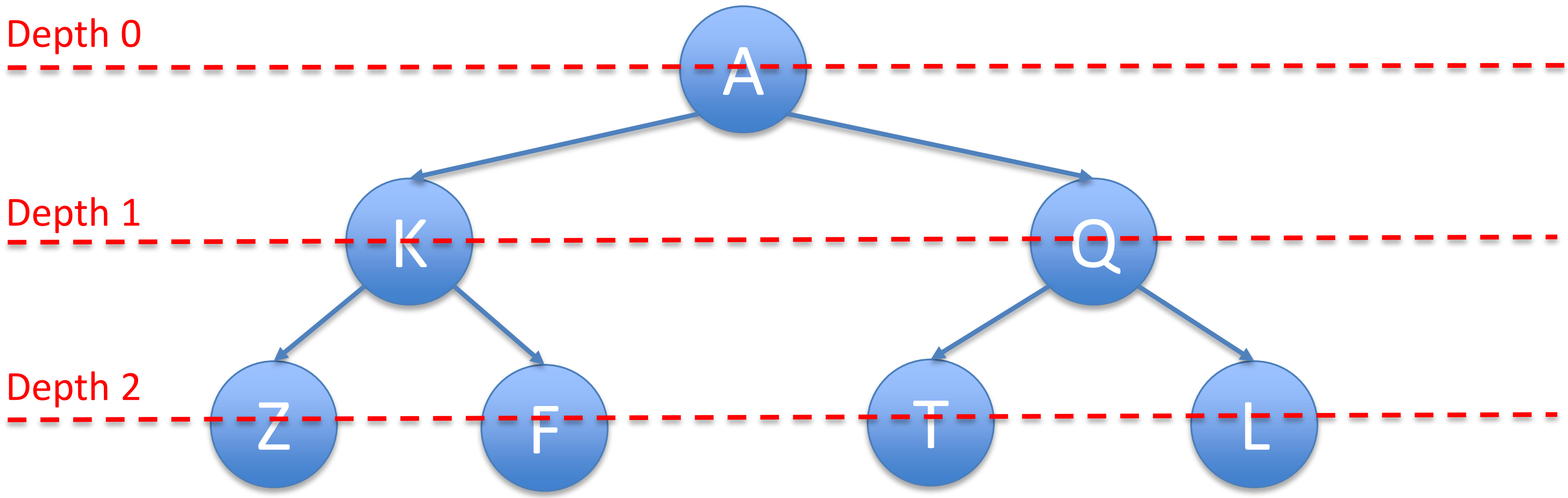
  Z is a *descendant* of A

  A is an *ancestor* of Z



What is a Tree?

- *Depth* of a node: The number of ancestors excluding itself.



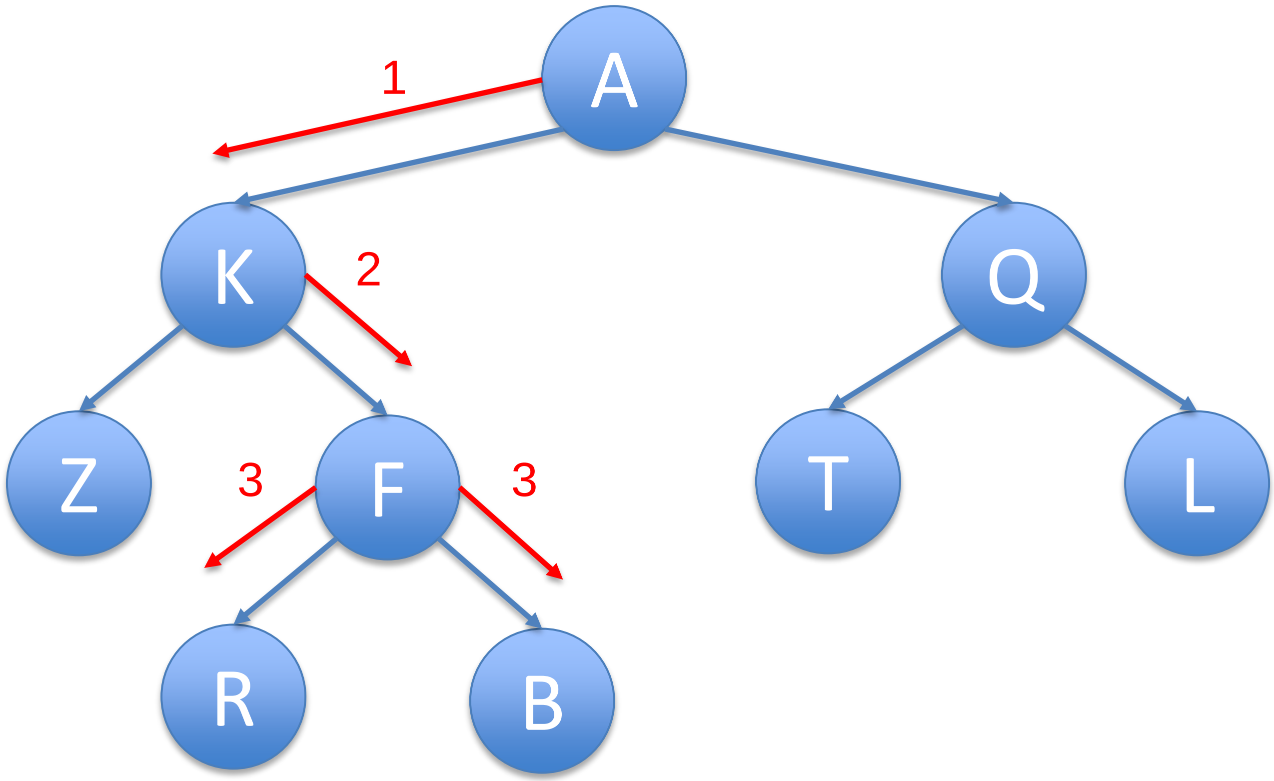
Count number of edges between root and node for depth

What is a Tree?

- *Height* of a tree: Number of edges between root and farthest leaf

What is the height of this tree?

Height = 3

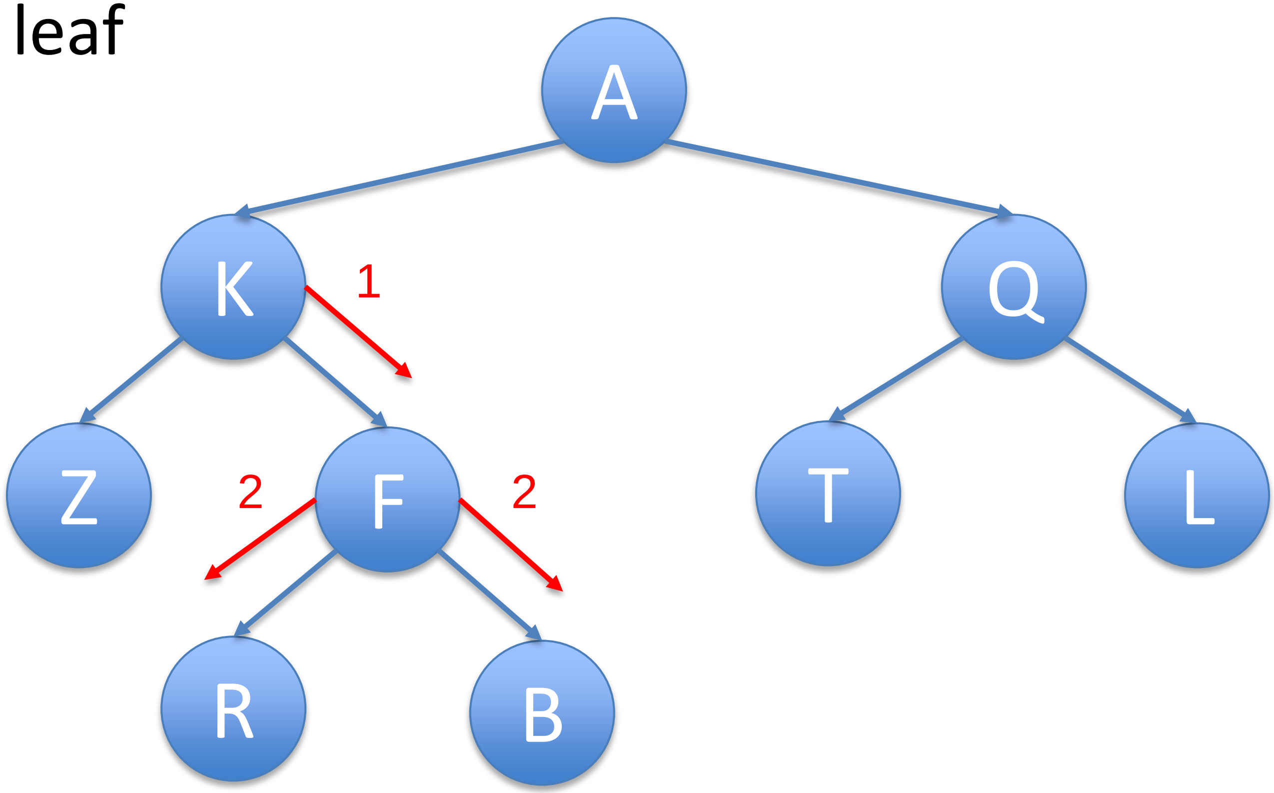


What is a Tree?

- *Height* of a node: Number of edges between node and deepest leaf

What is the height of node K?

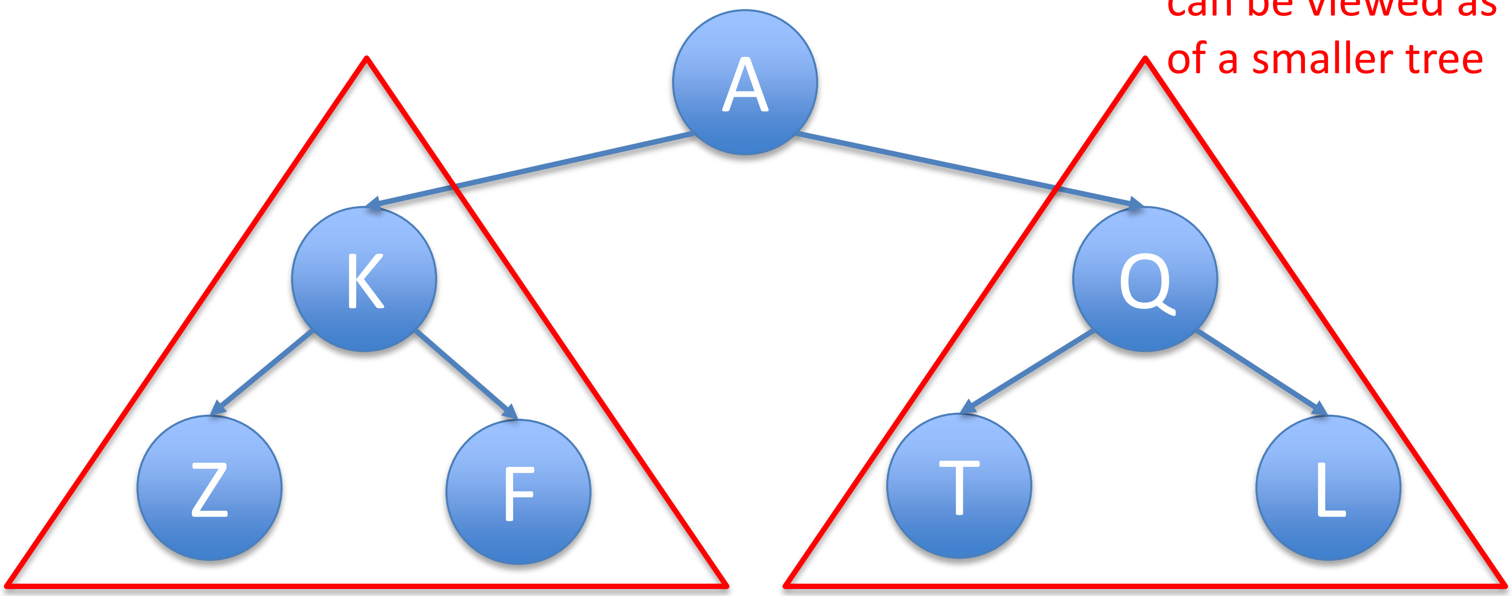
Height = 2



What is a Tree?

- *Subtree*: A tree that consists of a child and the child's descendants

Considered **recursive** because each sub-tree can be viewed as the root of a smaller tree

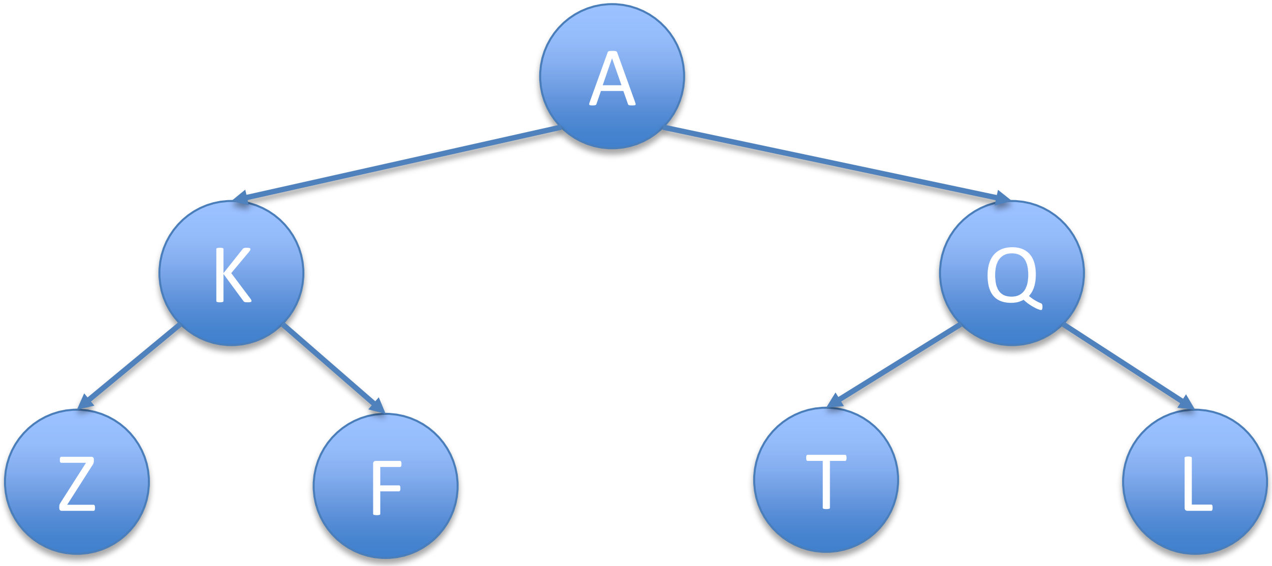


Subtree 1
Includes K, Z, and F

Subtree 2
Includes Q, T, and L

Tree Terminology Practice

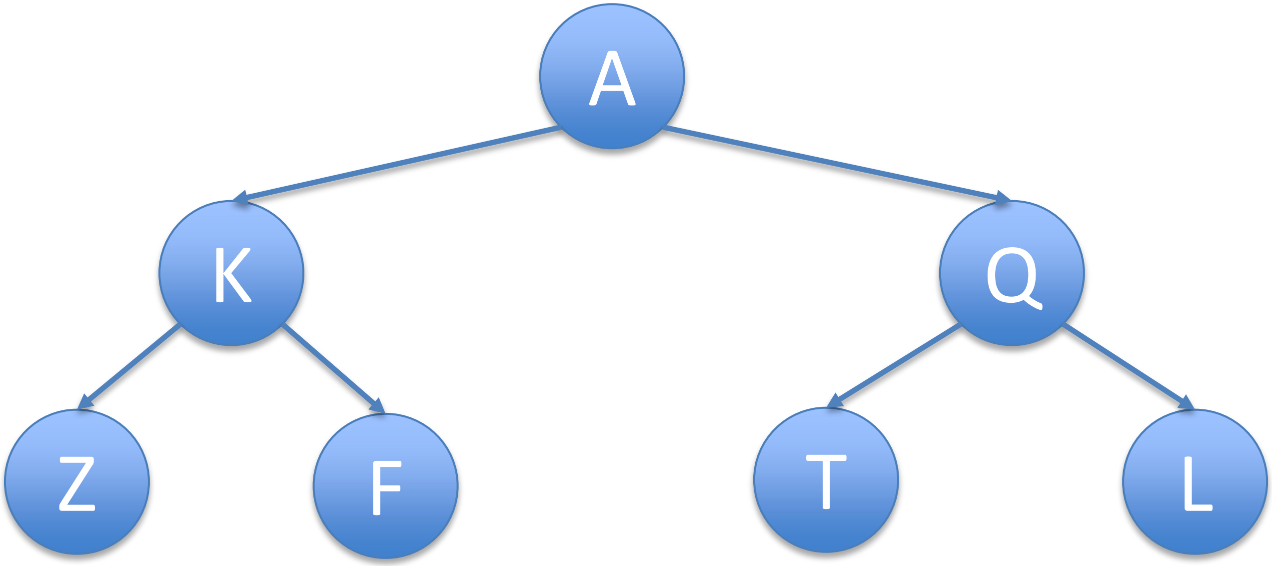
1. How could we describe Z?



Z is a node, a leaf, a sibling of F and a child of K

Tree Terminology Practice

2. How could we describe the relationship between T and L?



T is a sibling of L and they are both leaves

Tree Terminology Summary

- A *tree* is a collection of nodes(elements)
- Each node may have 0 or more *children*
  - (Unlike a list, which has 0 or 1 successors)
- Each node has *exactly one parent*
  - Except the starting / top node, called the *root*
- Links from a node to its successors are called *edges* or *branches*
- Nodes with same parent are *siblings*
- Nodes with no children are called *leaves*

Types of Trees

Types of Trees

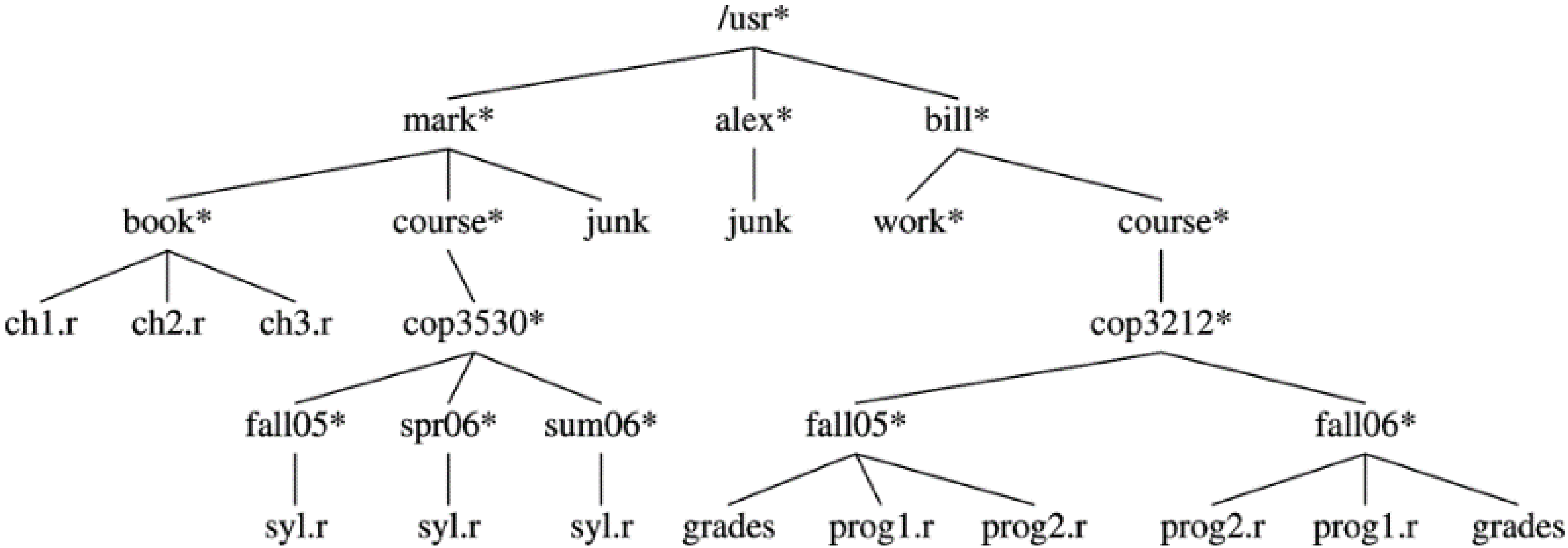
- Regular Tree
- Regular Binary Tree
- Binary Search Tree (BST)

All regular binary trees are also regular trees.

All binary search trees (BST) are also regular binary trees.

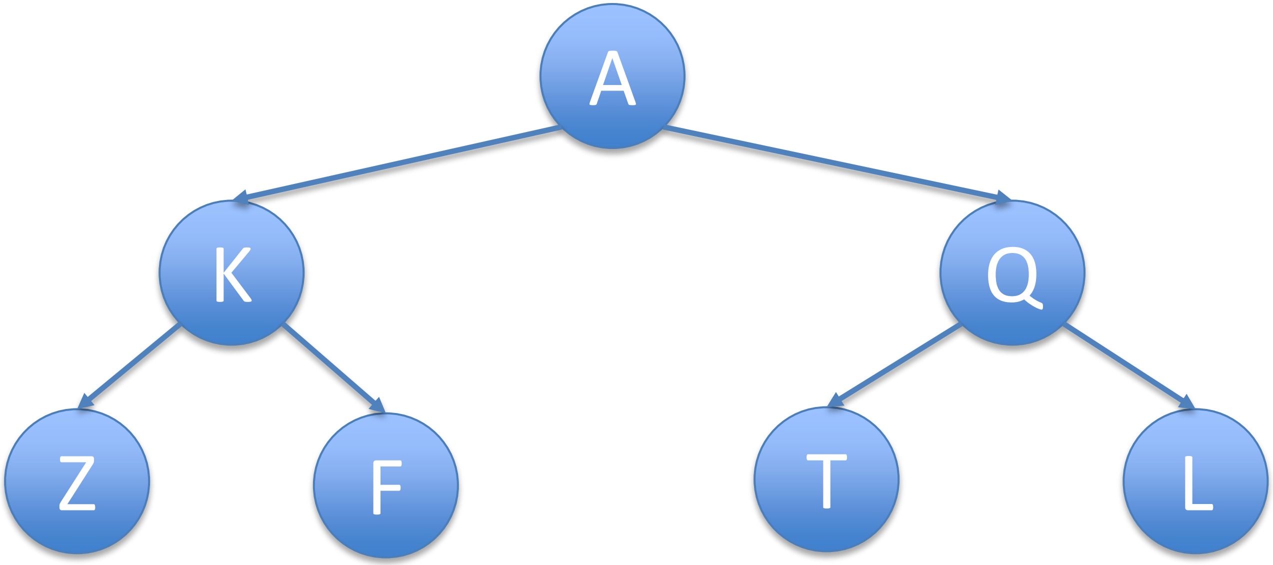
Regular (Non-binary) Tree

- Many links to many children



Regular Binary Tree

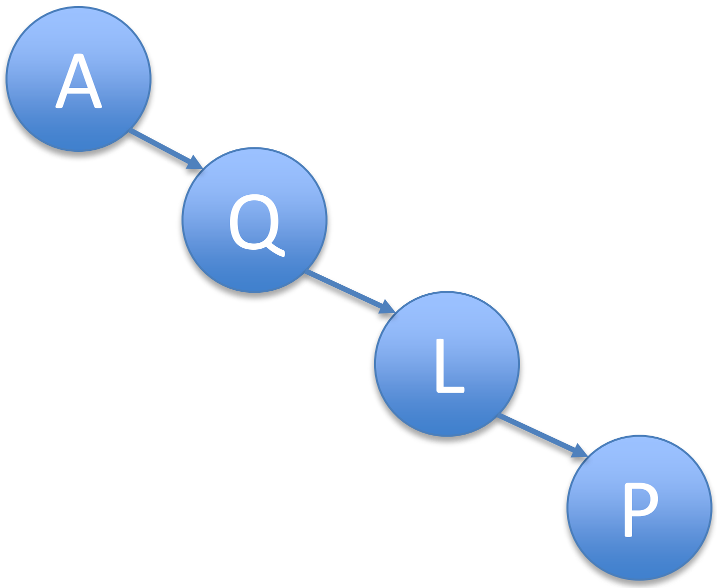
- No node can have more than two children.



Average depth is $O(\sqrt{n})$

Regular Binary Tree

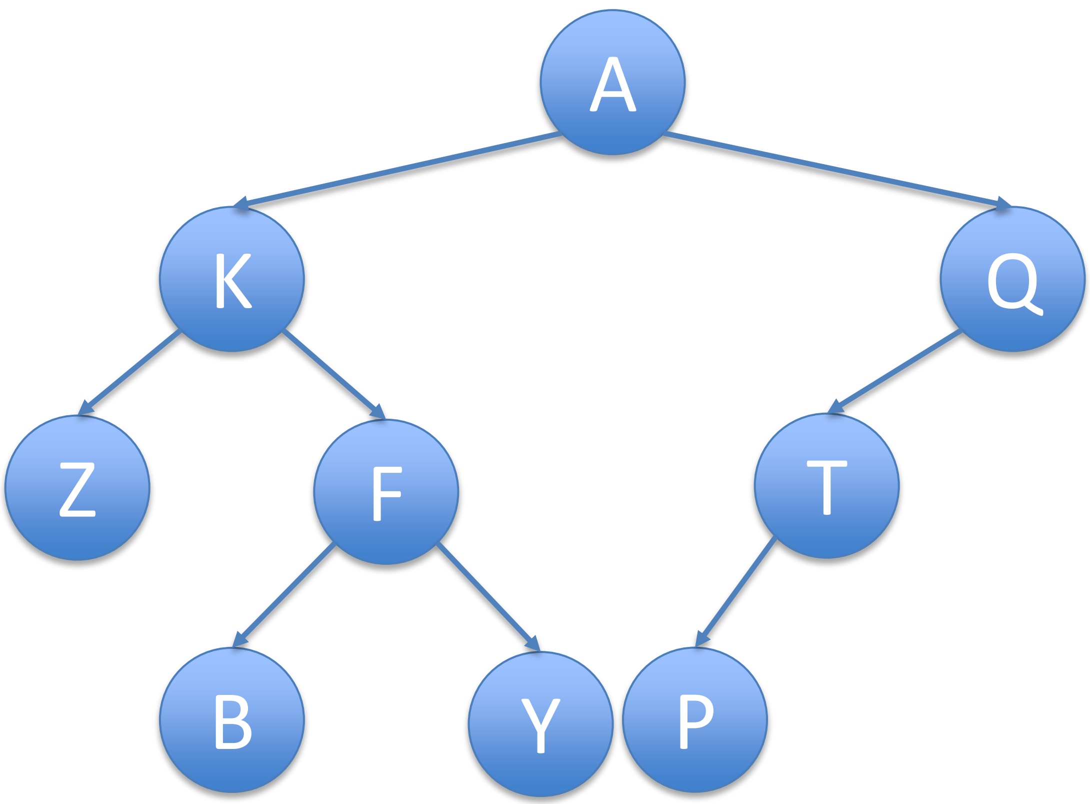
- No node can have more than two children.



Worst scenario depth is $O(n-1)$

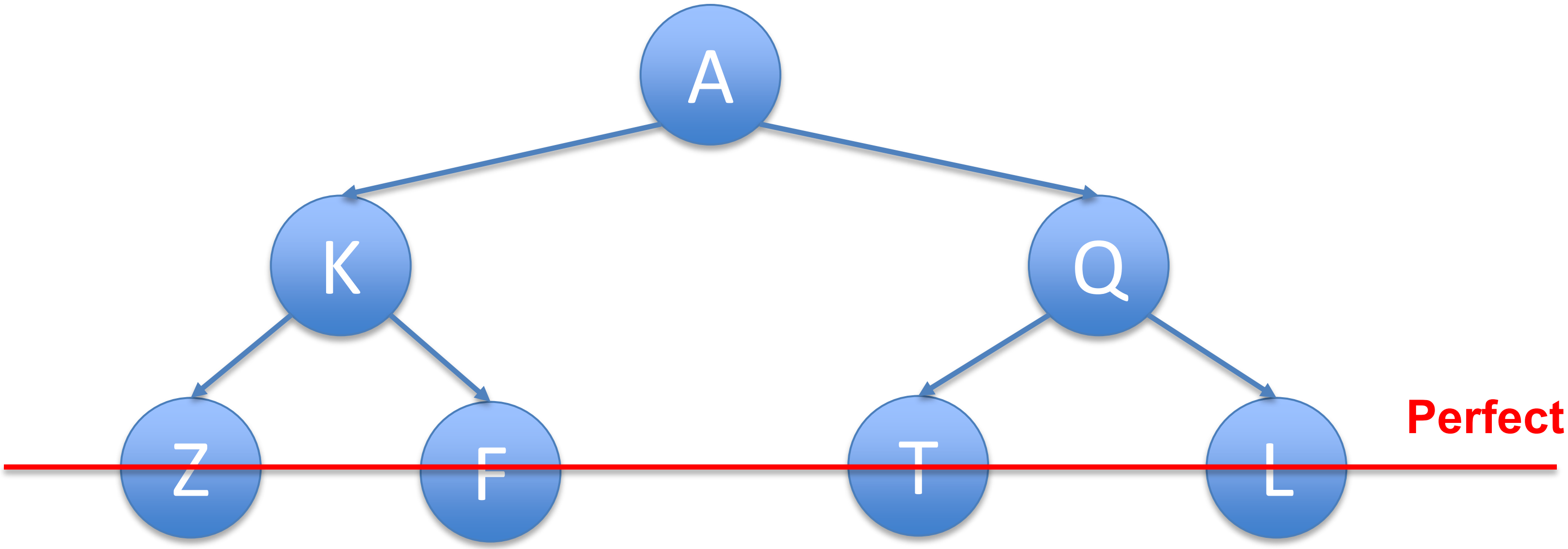
Binary Search Tree (BST)

- Has at **MOST** two children



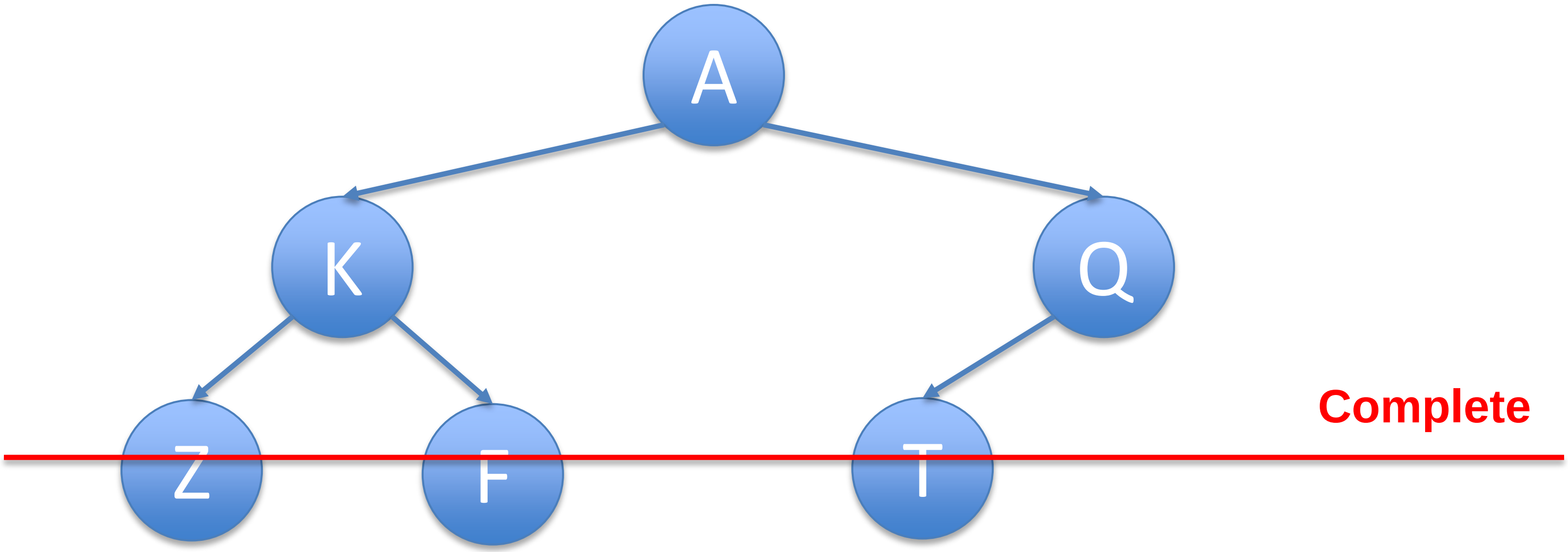
Perfect Binary Tree

- A binary tree is <u>perfect</u> if all leaves are at the same level



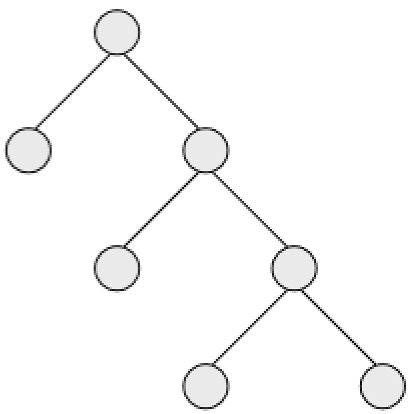
Complete Binary Tree

- A binary tree is <u>*complete*</u> if:
  - All leaves are at level h or level h-1 (for some h)
  - All leaves are as far to the left as possible

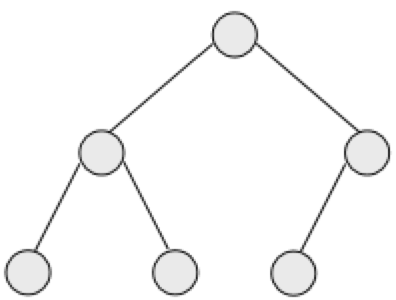


Complete & Full Binary Trees

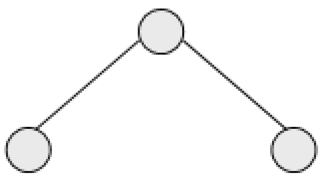
- Is each tree full, complete, neither, or both?



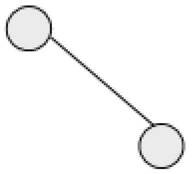
Full, but not complete



Complete, but not full



Full and complete (“perfect”)

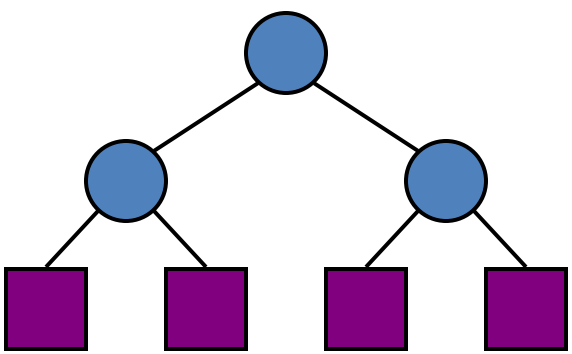


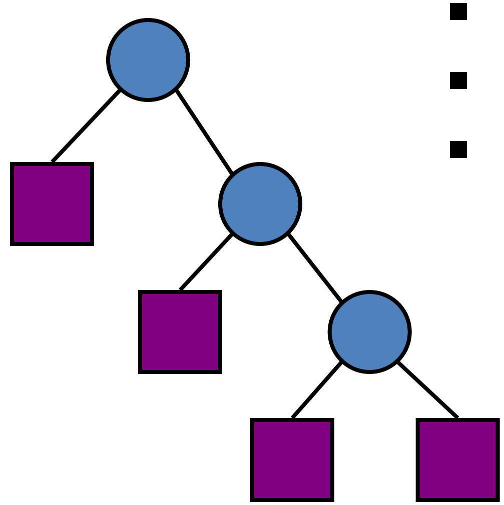
Neither full nor complete

Properties of Proper Binary Trees

- Notation
  - n number of nodes
  - e number of external nodes
  - i number of internal nodes
  - h height

- Properties:
  - $e = i + 1$
  - $n = 2e - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $e \leq 2^h$
  - $h \geq \log_2 e$
  - $h \geq \log_2 (n + 1) - 1$





Binary Search Tree (BST)

- A **binary search tree (BST)** or **ordered binary tree** is a type of binary tree where the nodes are arranged in order:
  - For each node, all elements in its left subtree are less than or equal to the node (<=)
  - All the elements in its right subtree are greater than the node (>)

BSTs Next Class!

Other Binary Tree Information

- Trees are ***SHALLOW*** – they can hold many nodes with very few levels
- A height of 20 can hold 1,048,575 nodes
- 2^{height} -1 = How many TOTAL nodes can be held by this tree
  - Can also be expressed as $2^{(depth+1)}$ - 1

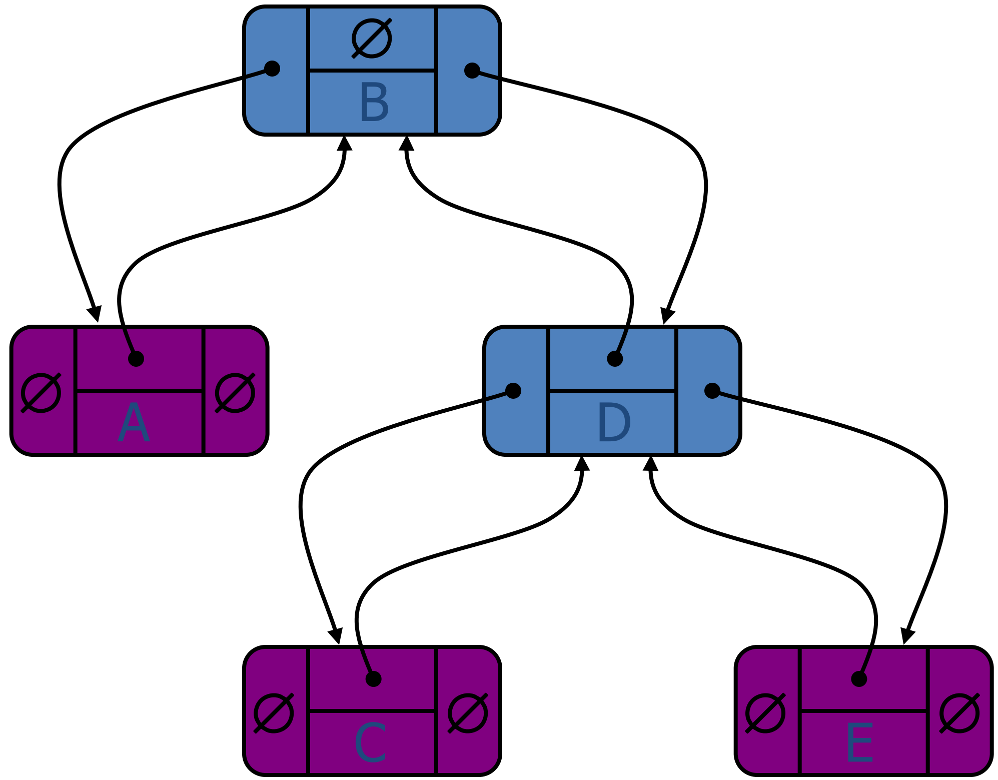
Tree Implementations

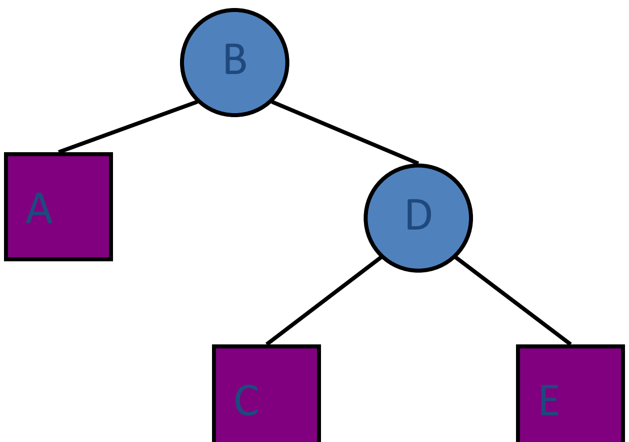
Tree Implementation

- There are two ways to construct trees
  - Linked Lists
    - Use links to connect to the other nodes in the tree
  - Array (K-ary)
    - Can only use if we know the MAXIMUM number of children allowed

Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT





K-ary Trees (also called M-ary)

- “k” is the number of children (links)
- Built as an array of nodes
- Will only work if we know the MAXIMUM number of children
- Empty spots in the array to denote a missing node
- Useful in coding since we can dictate the number of nodes we want
  - Also since there is a formula to calculate the node’s kids
- Child and grandchild index and corresponding items can be found in constant time.

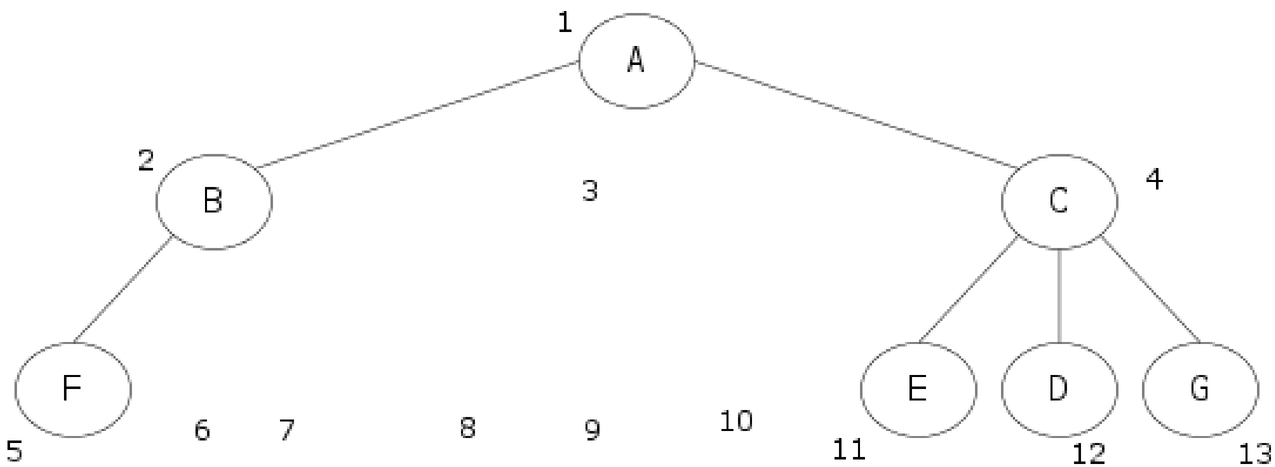
K-ary Trees

- A k-ary tree is a tree in which the children of a node appear at distinct index positions in 0..k-1
- This means the maximum number of children for a node is k

K-ary Trees

- Some k-ary trees have special names
  - 2-ary trees are called **binary trees**
  - 3-ary trees are called **trinary trees** or **ternary trees**
  - 1-ary trees are called **lists**

Array Representation Of A Tree

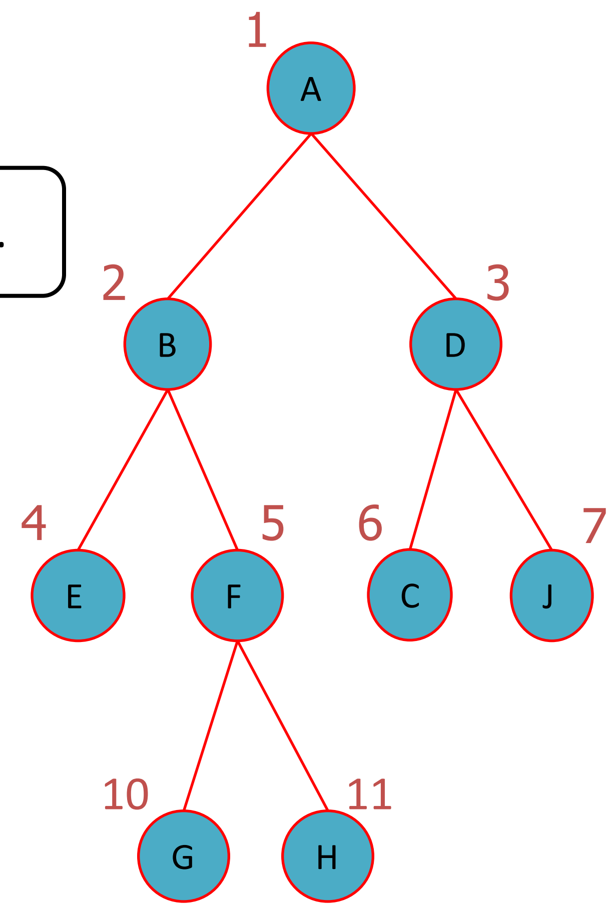


1	A
2	B
3	
4	C
5	F
6	
7	
8	
9	
10	
11	E
12	D
13	G

Array-Based Representation of Binary Trees

- Nodes are stored in an array A

- Node v is stored at A[rank(v)]
  - rank(root) = 1
  - if node is the left child of parent(node), rank(node) = 2 · rank(parent(node))
  - if node is the right child of parent(node), rank(node) = 2 · rank(parent(node)) + 1



Tree Traversals

Traversals of Binary Trees

- To iterate over and process the nodes of a tree
  - We *walk* the tree and *visit* the nodes in order
  - This process is called ***tree traversal***
- Three kinds of binary tree traversal:
  - ***Pre***order
  - ***In***order
  - ***Post***order

Traversals of Binary Trees

- *Preorder*: Visit root, traverse left, traverse right
- *Inorder*: Traverse left, visit root, traverse right
- *Postorder*: Traverse left, traverse right, visit root

Algorithm for Preorder Traversal

1. `if` the tree is empty
2. Return

   `else`

3. Visit the root.
4. Preorder traverse the left subtree.
5. Preorder traverse the right subtree.

Algorithm for Inorder Traversal

1. `if` the tree is empty
2. Return

   `else`

3. Inorder traverse the left subtree.
4. Visit the root.
5. Inorder traverse the right subtree.

Algorithm for Postorder Traversal

1. `if` the tree is empty
2. Return

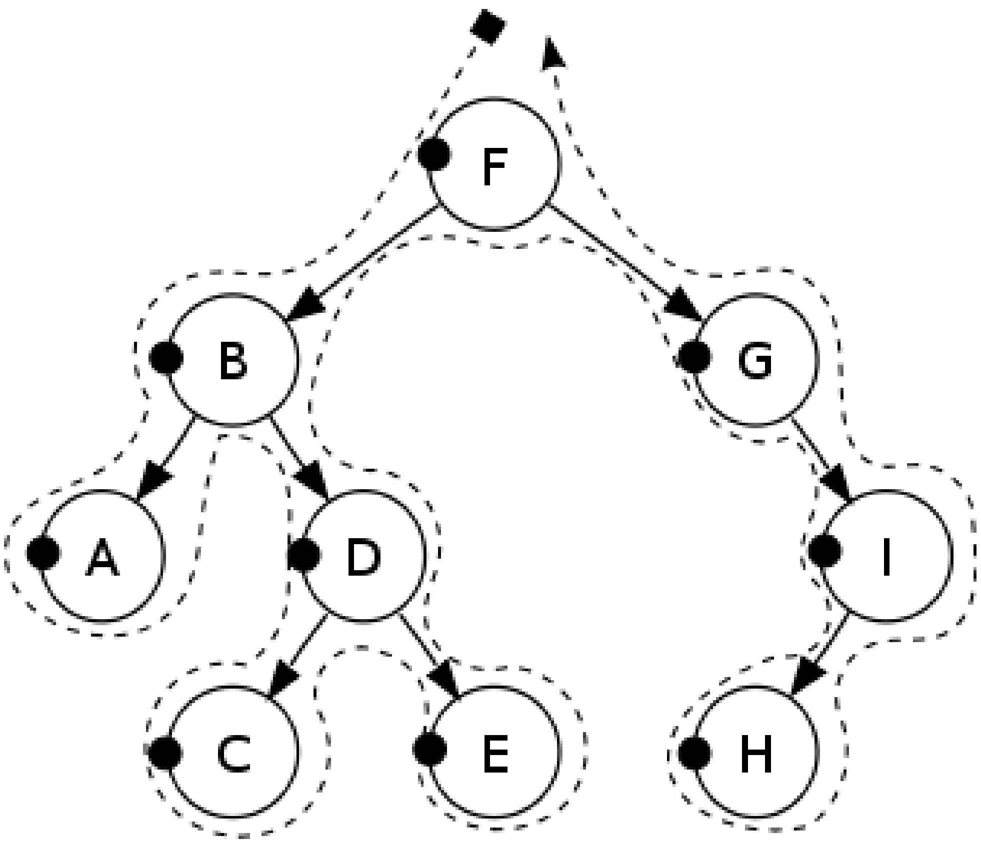
   `else`

3. Postorder traverse the left subtree.
4. Postorder traverse the right subtree.
5. Visit the root.

Preorder Traversals

Preorder:
F, B, A, D, C, E, G, I, H

Display a node's data as soon as you see it



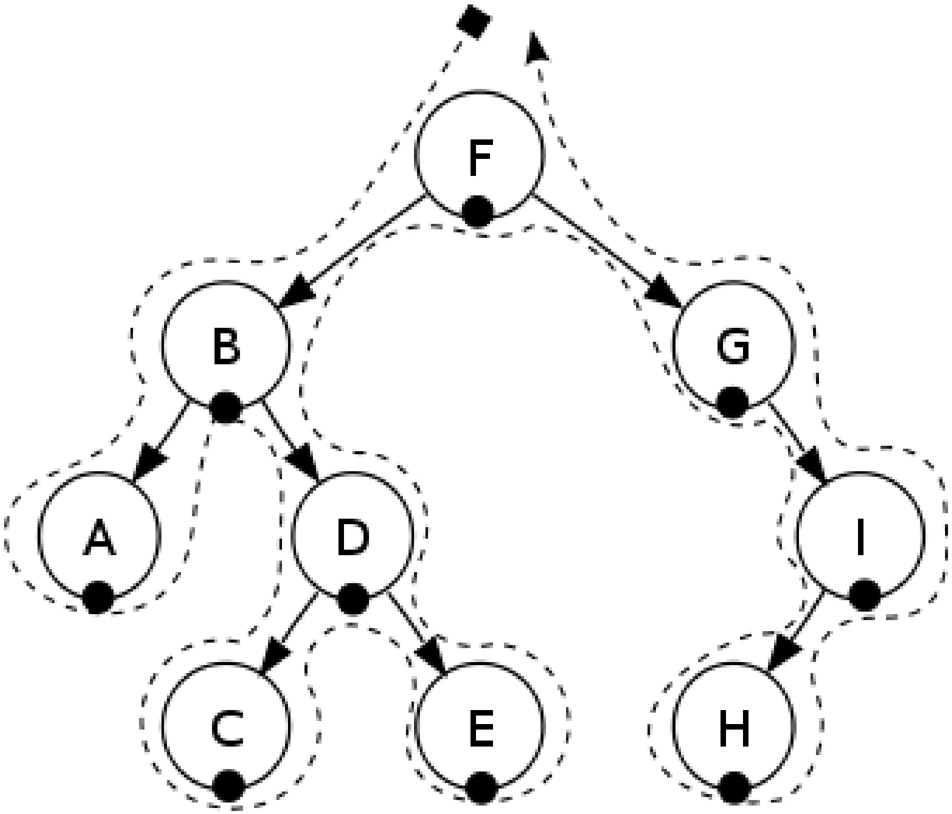
1. Display the data part of root element (or current element)
2. Traverse the left subtree by recursively calling the pre-order function.
3. Traverse the right subtree by recursively calling the pre-order function.

From: https://en.wikipedia.org/wiki/Tree_traversal

Inorder Traversals

Inorder:
A, B, C, D, E, F, G, H, I

Display the nodes in order (sort of from left to right, with the lower nodes first)

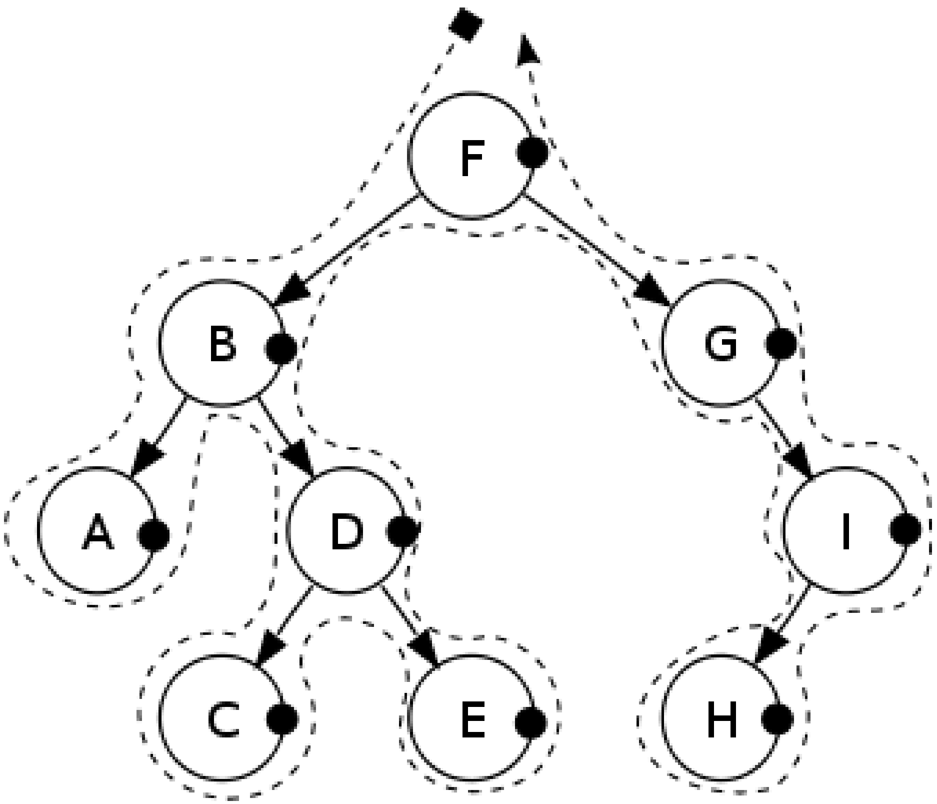


1. Traverse the left subtree by recursively calling the in-order function
2. Display the data part of root element (or current element)
3. Traverse the right subtree by recursively calling the in-order function

Postorder Traversals

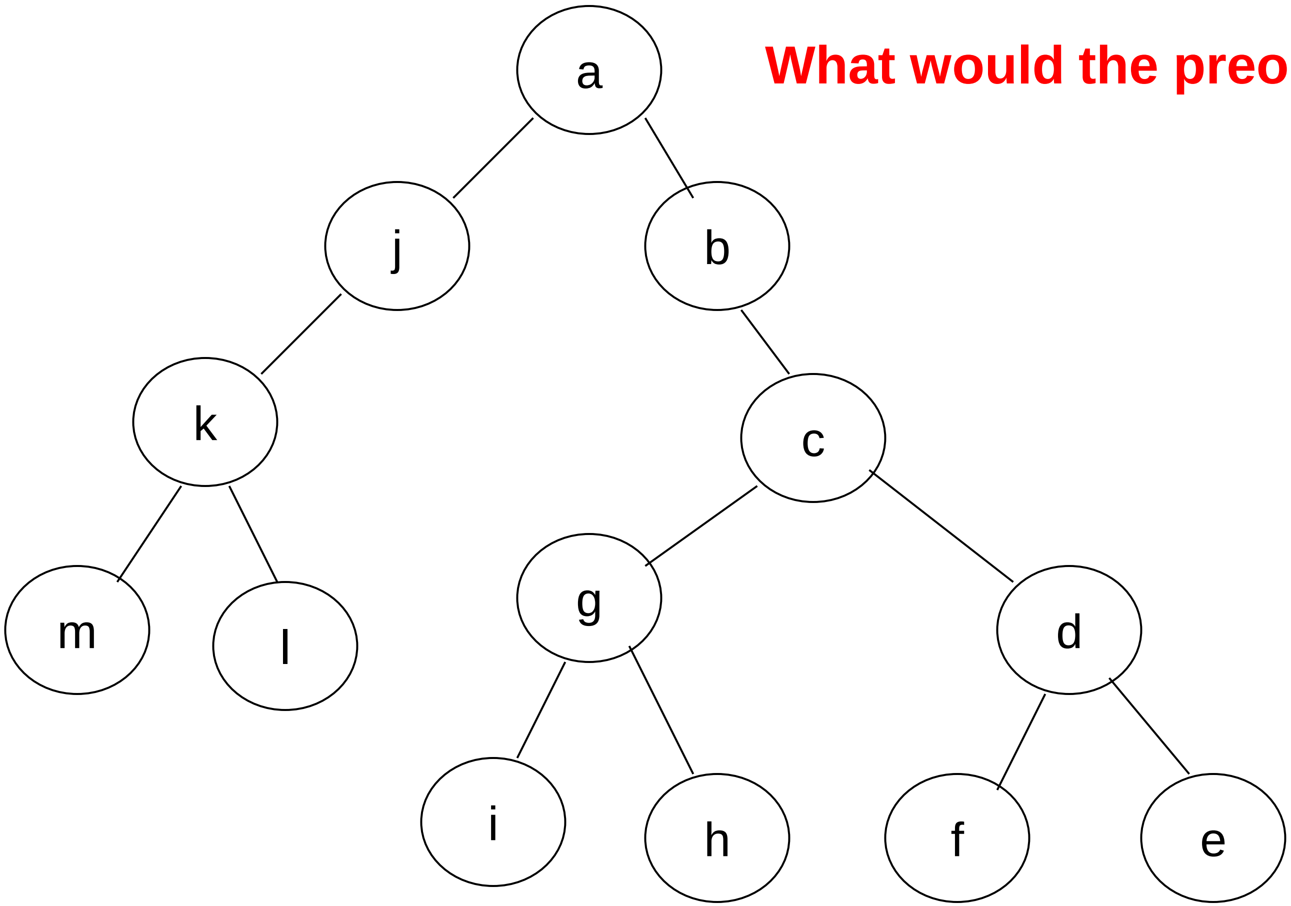
Postorder:
A, C, E, D, B, H, I, G, F

Display a node's data the last time you see it



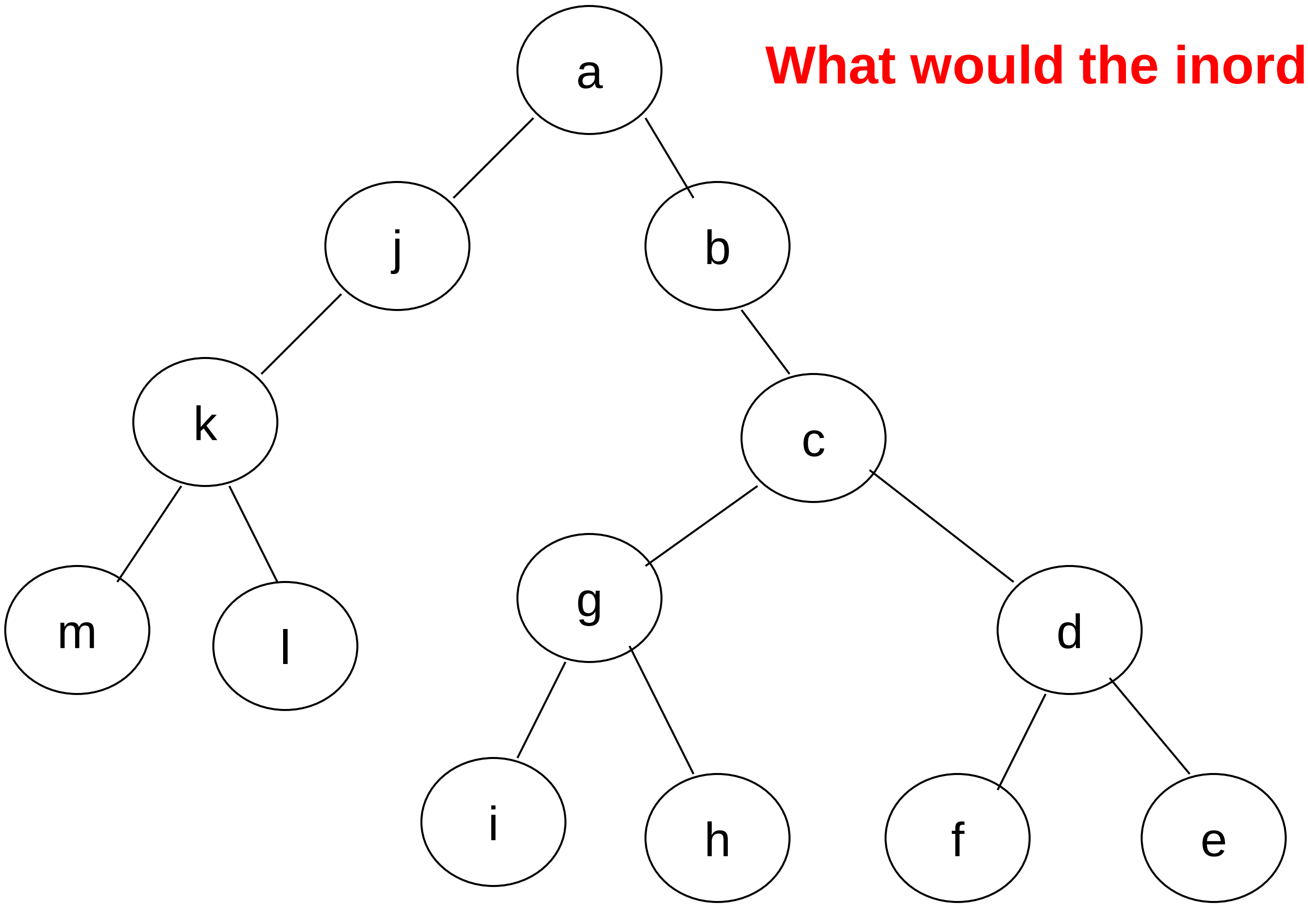
1. Traverse the left subtree by recursively calling the post-order function.
2. Traverse the right subtree by recursively calling the post-order function.
3. Display the data part of root element (or current element).

Tree Traversal Example



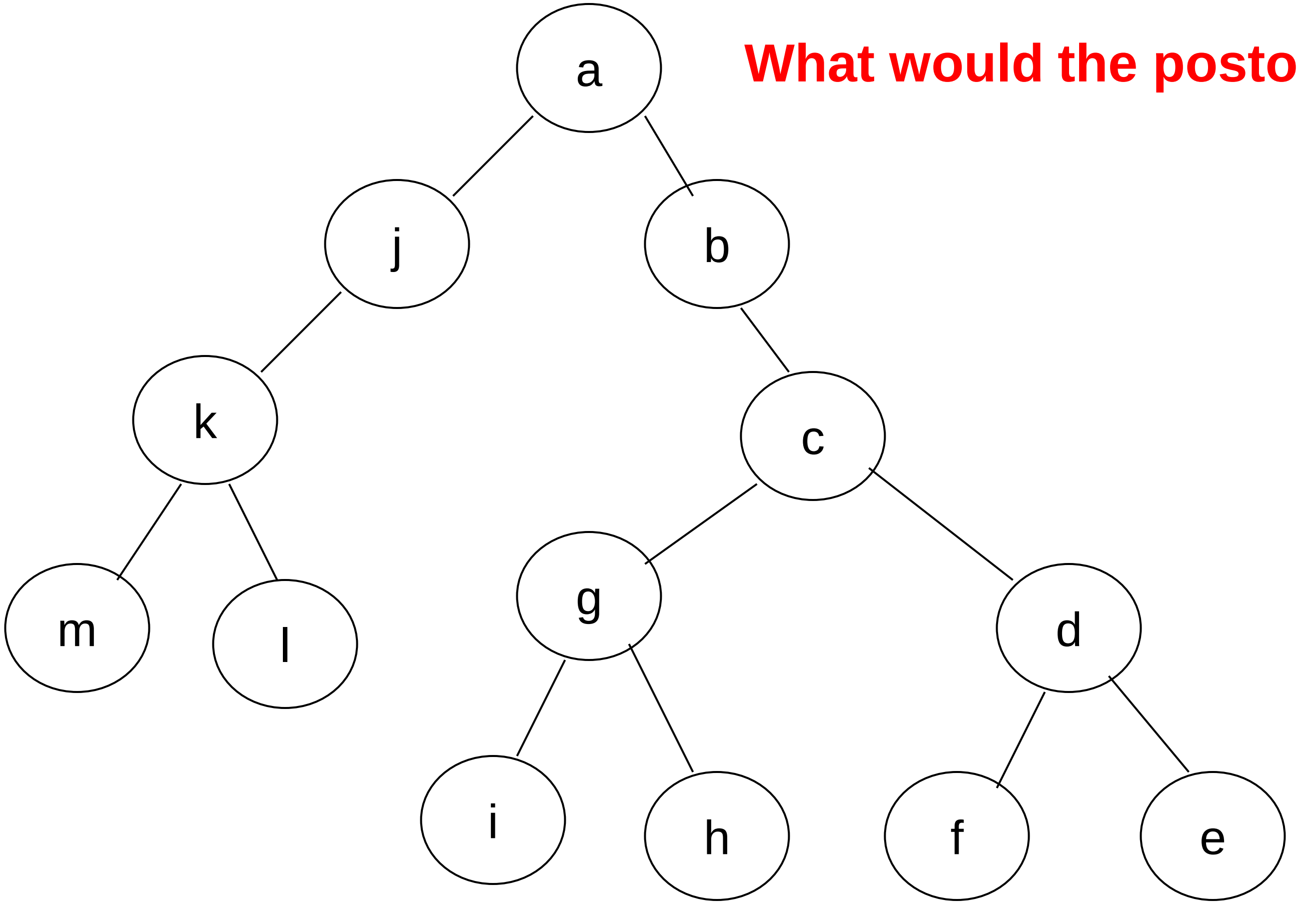
What would the preorder traversal look like?

Tree Traversal Example



What would the inorder traversal look like?

Tree Traversal Example



What would the postorder traversal look like?

Preorder Traversals

```
preorder (Node t)
    if (t == null)
        return;

    visit (t.value());
    preorder (t.lchild());
    preorder (t.rchild());

} // preorder
```

Preorder

N L R

Inorder Traversals

```
inorder (Node t)
   if (t == null)
      return;

   inorder (t.lchild());
   visit (t.value());
   inorder (t.rchild());

} // inorder
```

Inorder

L N R

Postorder Traversals

```
postorder (Node t)
    if (t == null)
        return;

    postorder (t.lchild());
    postorder (t.rchild());
    visit (t.value());

} // postorder
```

Postorder

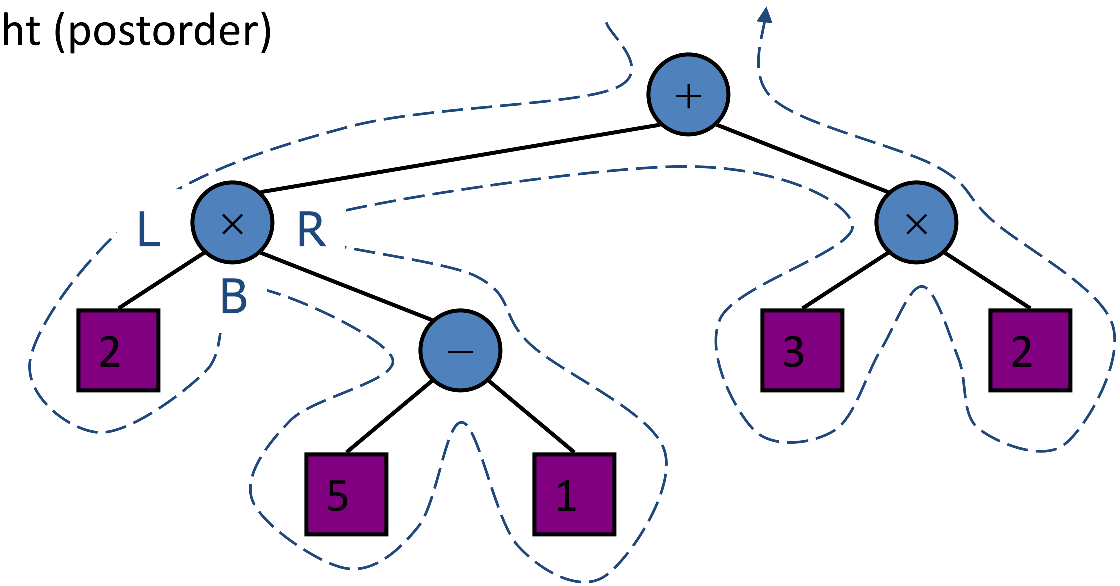
L R N

Another Tree Traversal

- A *level-order* walk effectively performs a breadth-first search over the entire tree
- Nodes are traversed level by level
  - Root node is visited first
  - Followed by its direct child nodes
  - Followed by its grandchild nodes
  - Until all nodes in the tree have been traversed

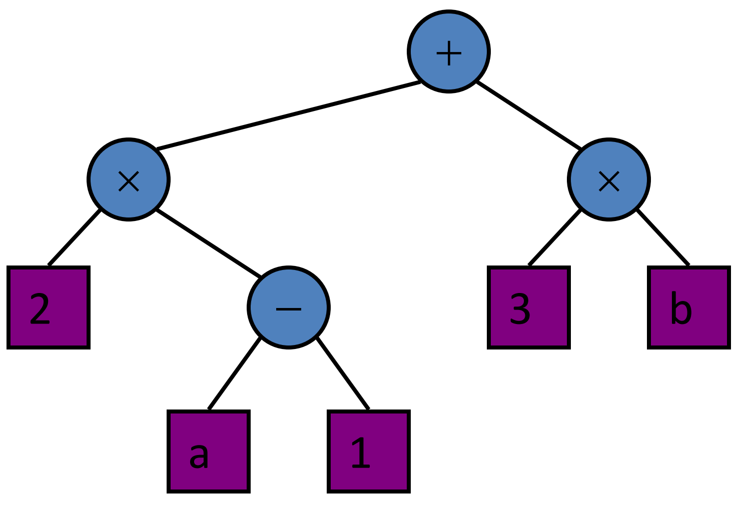
Euler Tour Traversal

- Generic traversal of a binary tree
- Includes a special cases the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Arithmetic Expression Tree

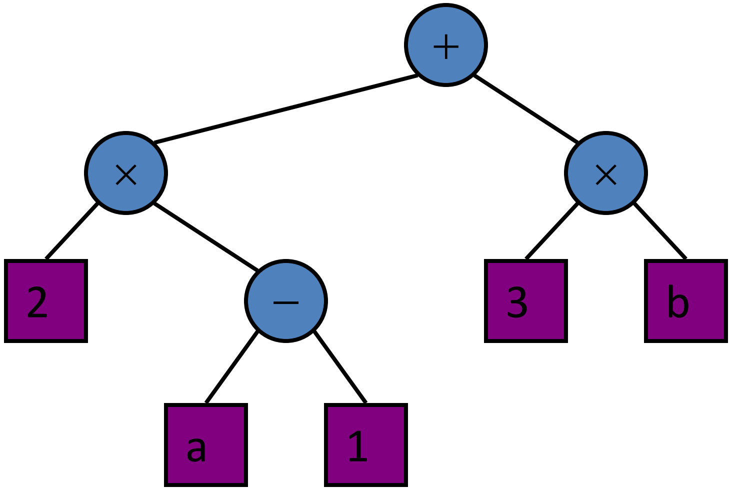
- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print “(“ before traversing left subtree
  - print “)“ after traversing right subtree

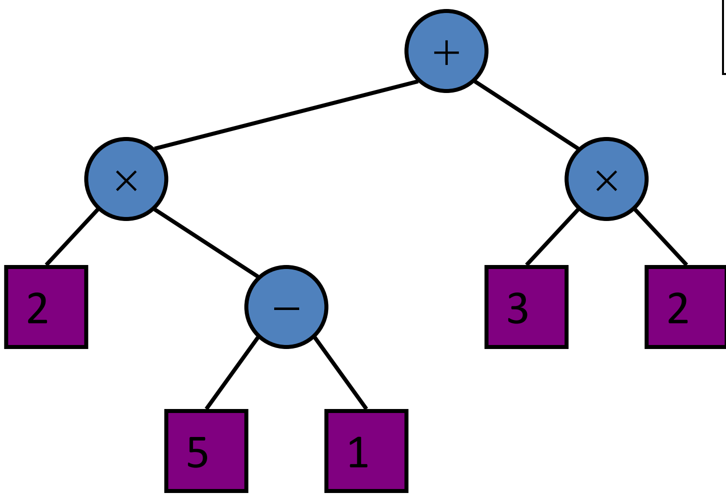
Algorithm *printExpression*(*v*)
  if ¬*v.isExternal*()
    print(“(”)
    inOrder(*v.left*())
  print(*v.element*())
  if ¬*v.isExternal*()
    inOrder(*v.right*())
    print (“)”)



$$((2 \times (a - 1)) + (3 \times b))$$

Evaluate Arithmetic Expressions

- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees



Algorithm *evalExpr*(*v*)
  if *v.isExternal*()
    return *v.element*()
  else
    x ← *evalExpr*(*v.left*())
    y ← *evalExpr*(*v.right*())
    ◊ ← operator stored at *v*
    return *x* ◊ *y*

Tree Functions

Binary Tree Functions

Node Setup

`void insert( x )`	--> Insert x
`void remove( x )`	--> Remove x
`boolean contains( x )`	--> Return true if x is present
`Comparable findMin( )`	--> Return smallest item
`Comparable findMax( )`	--> Return largest item
`boolean isEmpty( )`	--> Return true if empty; else false
`void makeEmpty( )`	--> Remove all items
`void printTree( )`	--> Print tree in sorted order

Generic Struct for Binary Tree

```
private struct BinaryNode
{

    Comparable element; // Data in the node
    BinaryNode *left;   // Left child
    BinaryNode *right;  // Right child

    // Constructors
    BinaryNode(const Comparable & theElement,
               BinaryNode *lt, BinaryNode *rt )
    {
        element  = theElement;
        left     = lt;
        right    = rt;
    }
}
```

Questions about Trees?

Announcements

- Homework 3 will be out tomorrow
  - Due Thursday, October 5th at 8:59:59 PM
- Project 2 is out
  - Due Tuesday, October 10th at 8:59:59 PM
- Next Time:
  - Project 2