1. Show that the Cobb-Douglas Production Function, \( Y = AK^\alpha L^{1-\alpha} \), satisfies the first two properties of a neoclassical production function, and that its cross-partial is positive.

2. Suppose that private saving is described by the following specification:

\[
S = \bar{S} + s(Y - T),
\]

where \( Y - T \) is disposable income, \( \bar{S} \) is autonomous saving, and \( s \) is the marginal propensity to save (a positive number between zero and one).

Suppose that the investment function is given by:

\[
I = T - \gamma r,
\]

where \( r \) is the real interest rate and \( \gamma \) is a positive number measuring the responsiveness of investment demand to changes in the real interest rate.

a. What does the above specification of the savings function imply for the interest elasticity of private saving?

b. Discuss graphically and analytically the effect of a tax reduction program on private saving, national saving, investment, and the real interest rate. Also indicate how your answer depends on the value of \( s \).

c. Discuss graphically and analytically the impact of a technological innovation on private saving, national saving, investment, and the real interest rate.

3. Theoretically, the removal of interest rate ceilings leaves the equilibrium volume of saving and investment unchanged if private saving is interest rate inelastic. The weak empirical effect of financial deregulation (financial liberalization) on saving and investment is usually considered evidence supporting the view that the private saving function is interest rate inelastic. Suppose that financial liberalization entails both the removal of interest rate ceilings and the removal of restrictions on consumer credit (relaxation of borrowing constraints on consumers). What kind of the private saving function is consistent with the empirical lack of correlation between observed saving and the observed interest rate under these circumstances?

4. Consider a mandatory, fully funded pension system in which individuals pay an amount \( F \) when working and receive an amount \( (1 + r)F \) when retired. Using the two-period framework discussed in class, analyze the effect of the introduction of this pension scheme on the consumption and saving behavior of individuals. The article “Pension Reform in Developing Countries” states that mandatory, fully funded pension schemes may lead to higher saving by making people more aware of the need to save for the future, and through forced saving. Relate this statement to your previous analysis.

5. The article “Fitting the Theory to the Facts” discusses the evidence on Ricardian equivalence. This evidence mainly consists of the correlation between changes in government saving rates (budget deficits) and changes in private saving rates. Explain why this type of evidence may be misleading. (Hint: Look carefully at the Ricardian equivalence statement.)

6. In lecture, we noted that many economists have argued that more favorable tax treatment of interest income would increase saving. This statement motivated us to derive the saving function. Our analysis implied that unless the substitution effect dominates the income effect, increases in the interest rate are unlikely to bring about substantial increases in saving. Empirical evidence (for example, Mankiw (1981), Hansen and Singleton (1983), Hall (1988), Campbell and Mankiw (1989)) suggests that consumption growth (and hence saving) is relatively unresponsive to changes in the interest rate. At first blush
then, theory combined with evidence may lead you to believe that favorable tax treatment of interest income may not increase saving. A more careful analysis may change your mind. In general, tax policy designed to increase saving does not simply involve changes in the interest rate. It also typically entails a change in composition between taxes on interest income and other taxes that leaves government revenue unchanged. The following setup examines just such a scenario:

Suppose that the government initially raises revenue only by taxing interest income. Thus, the individual’s budget constraint is $C_1 + C_2/[1 + (1 - \tau)r] \leq Y_1 + Y_2/[1 + (1 - \tau)r]$, where $\tau$ is the tax rate. The government’s revenue is zero in period 1 and $\tau r(Y_1 - C_1^0)$ in period 2, where $C_1^0$ is the individual’s choice of $C_1$ given this tax rate. Now suppose the government eliminates the taxation of interest income and instead institutes lump-sum taxes of amounts $T_1$ and $T_2$ in the two periods; thus the individual’s budget constraint is now $C_1 + C_2/(1 + r) \leq (Y_1 - T_1) + (Y_2 - T_2)/(1 + r)$.

a. What condition must the new taxes satisfy so that the change does not affect the present value of government revenues?
b. If the new taxes satisfy the condition in part (a), is the old consumption bundle, $(C_1^0, C_2^0)$, not affordable, just affordable, or affordable with room to spare?
c. If the new taxes satisfy the condition in part (a), does first-period consumption rise, fall, or stay the same?

7. Consider a small open neoclassical economy.

a. Discuss the effect of an increase in investment demand on the equilibrium level of investment, national saving, and net exports.
b. Compare your answer in part (a) to the case of a closed economy.
c. Discuss the effect of an increase in the domestic supply of loanable funds on the equilibrium level of investment, national saving, and net exports.
d. Compare your answer in part (c) to the case of a closed economy.
e. Feldstein and Horioka (1980)\(^1\) report strong empirical evidence that national saving and investment are strongly positively correlated in most countries. Which of the two economic theories, small open economy or closed economy, is inconsistent with this empirical finding?

8. Suppose that a number of foreign countries begin to subsidize investment by instituting an investment tax credit. Show how this change in foreign policy affects private saving, investment, and the trade balance in the home country, a small open economy. What happens to international capital flows?

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