THE MODEL ECONOMIES

An OLG model with capital: Consider the simple OLG model with fiat money. Suppose further that a production technology exists that produces $xk_t$ consumption goods at time $t+1$ if $k_t$ units of the consumption good are converted into capital goods at time $t$. The population grows at rate $n$. Each initial old person receives a stock of capital that will produce $xk_0$ goods in the first period.

An OLG model with illiquid capital and three-period-lived agents: Consider the above OLG model with capital with the following modifications: Individuals live for three periods. Capital produces the consumption good two periods after the initial investment. Capital depreciates completely in the production process. Assume that $x > n^2$. It is impossible to observe the capital created by others, and it is impossible to enforce repayment of IOUs.

An OLG model with capital and three-period-lived agents and uncertainty: Consider an economy similar to the model above, but which also features some uncertainty. There are a large number of entrepreneurs, each of whom is endowed with an idea for a project, but no resources to fund it. The return on the project is stochastic (uncertain). Each project requires an investment of $\mu k$ goods ($\mu > 1$) to get it up and running. Projects that succeed produce $x \mu k$ in the next period with probability $p$. Projects that fail produce 0 with probability $1 - p$. There are also investors in the economy. Each investor has $k$ goods to invest. In order to determine whether or not his investment has succeeded or failed, an investor incurs a cost of $\theta$ goods.

An OLG model with reserve requirements: Consider the OLG model with three-period lived agents and illiquid capital. Suppose that the money stock grows at rate $z$. The rate of return to capital satisfies $x > (n/z)^2$. The reserve requirement for banks is $\gamma < 1$ per good deposited. Let $h_t$ be the goods deposited in a bank by an individual. Assume that the quantity of goods deposited in banks is a positive function of the rate of return offered.

An OLG model with workers, entrepreneurs, and bankers: Consider an OLG model with two-period-lived agents. The stock of fiat money is constant. Let $Q_t$ be the real value of total inside money balances. Let $H_t$ be the real value of total fiat money balances. There are three types of agents: (1) Workers are endowed with (produce) the consumption good when young. They are unable to create capital and therefore must acquire money in order to consume when old. Workers receive different endowments, and therefore desire different money balances. Worker $i$ desires money balances worth $s_i$ units of the consumption good. Workers all “look” alike. Identity revelation (required to withdraw bank deposits, for example) costs a fee in the amount of $\phi$ goods. (2) Entrepreneurs have the same endowments and preferences as workers. In addition, they can create capital that produces $x$ goods in the next period for each good invested in the current period. Entrepreneurs cannot be located by workers. (3) Bankers receive no endowment and have no ability to create capital. They do have the ability to locate entrepreneurs and their identity is costlessly known to all.

An OLG model with rich and poor people and government debt: Consider an OLG model with two-period-lived agents. The economy is populated by two types of people: (1) rich people are endowed with $Y$ units of the consumption good when young, and (2) poor people are endowed with $y$ units of the consumption good when young. Of course, $Y > y$. The populations of both types of agents grow at the constant rate $n$. There is a standard linear capital technology: $k_t$ units of the consumption good invested at time $t$ produces $xk_t$ units of the consumption good at time $t+1$. Investment in capital requires a minimum investment $k^*$, and $Y > k^* > y$. Investment in capital cannot be observed by the government if done by an individual. The money stock grows at the constant rate $z$. 
1. The OLG model with capital and three-period-lived agents and uncertainty.
   a. In this particular model, as in the real world, we observe banks providing financial intermediation services instead of individuals. Explain why. (A formal argument with mathematical notation is not required. An intuitive explanation will suffice, but it must be precise and complete.)

2. The OLG model with workers, entrepreneurs, and bankers.
   Consider the following experiment: in period \( t \) there is a positive (permanent) productivity shock. I.e., the rate of return to capital rises to \( x' > x \).
   a. What is the average rate of return earned on deposits in this economy?
   b. What is the expression for \( GDP_{t+1} \) in this economy?
   c. Show that \( M1_t \) rises as a result of the shock. (Hint: \( M1_t = (1 + H_t/Q_t) M_t \).)
   d. Explain why output at time \( t \) and output at time \( t + 1 \) also rise as a result of the shock.
   e. You just demonstrated that increases in \( M1 \) coincide with and precede increases in output. We can therefore conclude that if the monetary authority intervenes to stabilize the money supply when a productivity shock occurs, output will be stabilized. Right (or wrong)? Explain.

   a. Explain thoroughly why the government should be interested in uncovering and shutting down insolvent banks?

4. Homework repeat. Consider the simple OLG model with three-period-lived agents. The rate of growth of the population is \( n \) and the rate of growth of the money supply is \( z \). Previously, we assumed that financial intermediation was costless. In reality however, it is not. Suppose that the intermediation of capital costs \( f \) units of the consumption good for each unit of capital intermediated (\( f < \sqrt{X} \)). Assume that these transaction costs occur when agents make withdrawals from banks (when they are middle-aged).
   a. What will be the equilibrium rate of return offered by intermediaries if they are the ones who bear the transaction costs?
   b. Under what conditions will fiat money be valued in this economy?

5. Government Debt.
   a. The U.S. government issues high-denomination bonds to finance its deficit. Why might it do this? In your response you should explain both the reason for issuing debt as well as the reason for issuing debt in high denominations only. (A formal argument with mathematical notation is not required. An intuitive explanation will suffice, but it must be precise and complete.)