Price Surprises

1. Consider the Lucas model. Suppose that inflation is nonrandom, with \( z_t = z \) and \( M_t = zM_{t-1} \).
   a. Derive an expression for the price on island \( i \) at time \( t \).
   b. Derive an expression for the rate of return to work.
   c. Money is superneutral if changes in money growth have no real effects. Is money superneutral in the Lucas model? Explain with reference to your expression in part (c) for the rate of return to work.

2. Consider the Lucas model. Suppose that inflation is random. Let monetary policy be described by:
   \[
   M_t = \begin{cases} 
   M_{t-1} & \text{with probability } \theta \\
   3M_{t-1} & \text{with probability } 1 - \theta
   \end{cases}
   \]
   and suppose that the number of young individuals born on island \( i \) in period \( t \), \( N^i_t \), is random according to the following specification:
   \[
   N^i_t = \begin{cases} 
   \frac{3}{4}N & \text{with probability } 0.5 \\
   \frac{1}{4}N & \text{with probability } 0.5
   \end{cases}
   \]
   a. How many states of the world are the agents able to distinguish when there is limited information (they do not know the value of \( z_t \) at time \( t \) or the number of young on their island)? Explain and show your work.
   b. Plot the inflation and output combinations produced by the economy. What do you see?
   c. Suppose the government decides to print money to stimulate output in every period. According to the Lucas model, what will happen? Explain - clearly and carefully.

Capital

3. Consider a standard OLG model with capital.
   a. Suppose that \( f(k_t) = xk_t \), and that capital is the only asset. Specify the households’ first-period, second-period, and lifetime budget constraints.
   b. Consider an economy with capital and fiat money. What is the ‘Tobin effect?’
   c. Consider an economy with capital and fiat money. Explain how an increase in inflation may lead to an increase in output. (Hint: use the principle of rate-of-return equality and assume that the returns to capital are diminishing.)

Liquidity and Financial Intermediation

4. Consider the OLG model with illiquid capital and three-period-lived agents. Suppose that the population grows at rate \( n \) and the fiat money stock is constant.
   a. Suppose there is a single intermediary. What is the minimum one-period rate of return that the intermediary must offer in order to attract depositors? Explain.
   b. What rate of return will be offered on deposits if there is a large number of intermediaries competing for deposits? Explain.

5. Consider the OLG model with capital and uncertainty.
   a. In this model, investors can choose to invest in many projects, or in a small number of projects. What tradeoff do investors face when designing their investment strategies? Explain.
   b. In this particular model, as in the real world, we observe banks providing financial intermediation services instead of individuals. Explain why. (A formal argument with mathematical notation is not required. An intuitive explanation will suffice, but it must be precise and complete.)