Statistical analysis

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Goddard Space Flight Center

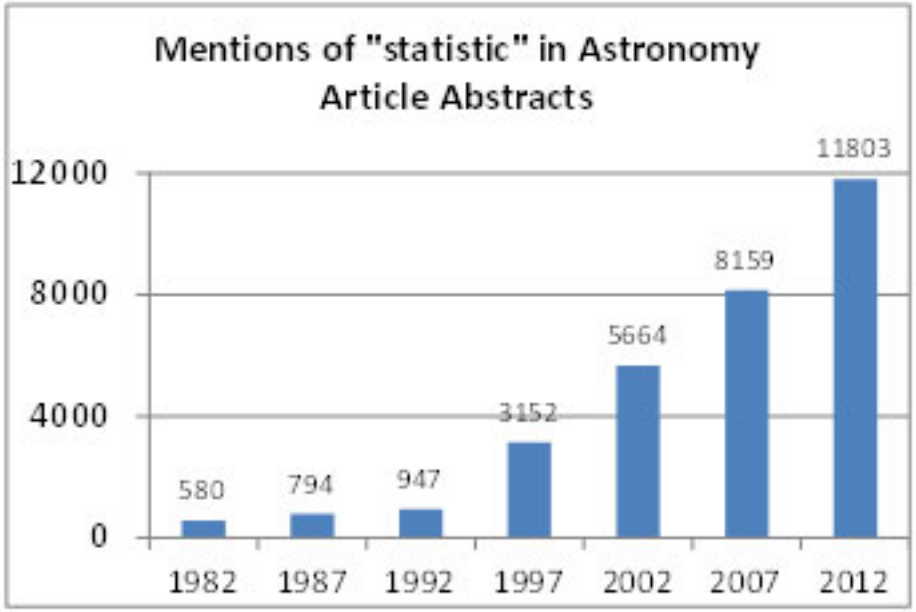
ASTR 288C, Lecture 8, 2017/10/23

Acknowledgement

Class material from:

- **Slides from Prof. M.A. Thomson (Univ. of Cambridge)**
  http://www.hep.phy.cam.ac.uk/~thomson/lectures/lectures.html
- **Prof. Sylvain Guiriec (George Washington Univ.)**

Why statistics?



Pierson et al. (2013)

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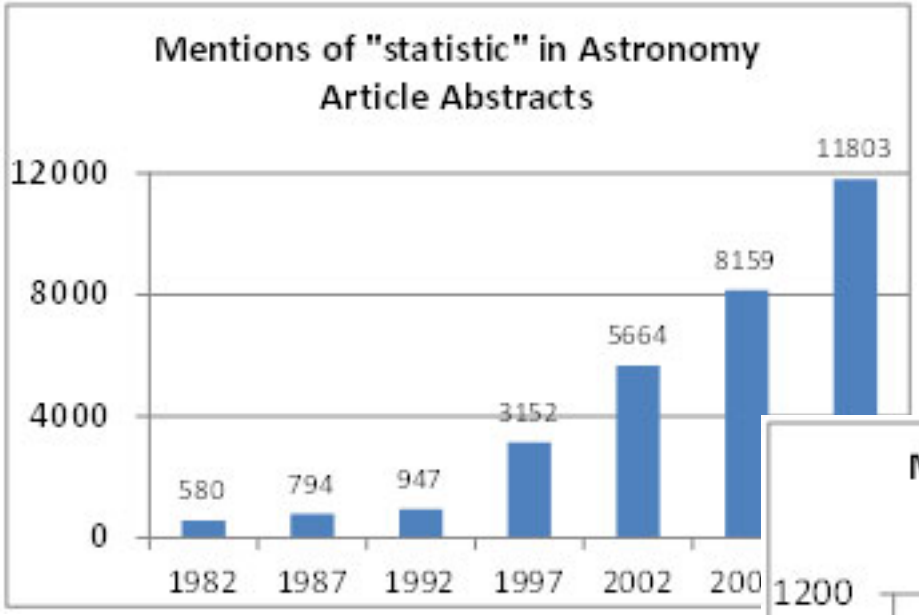

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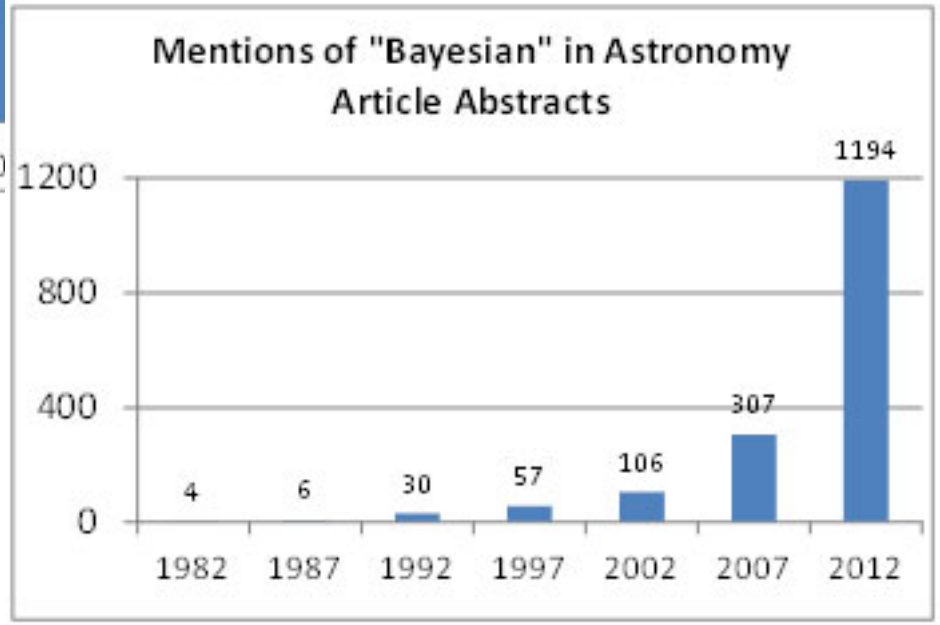


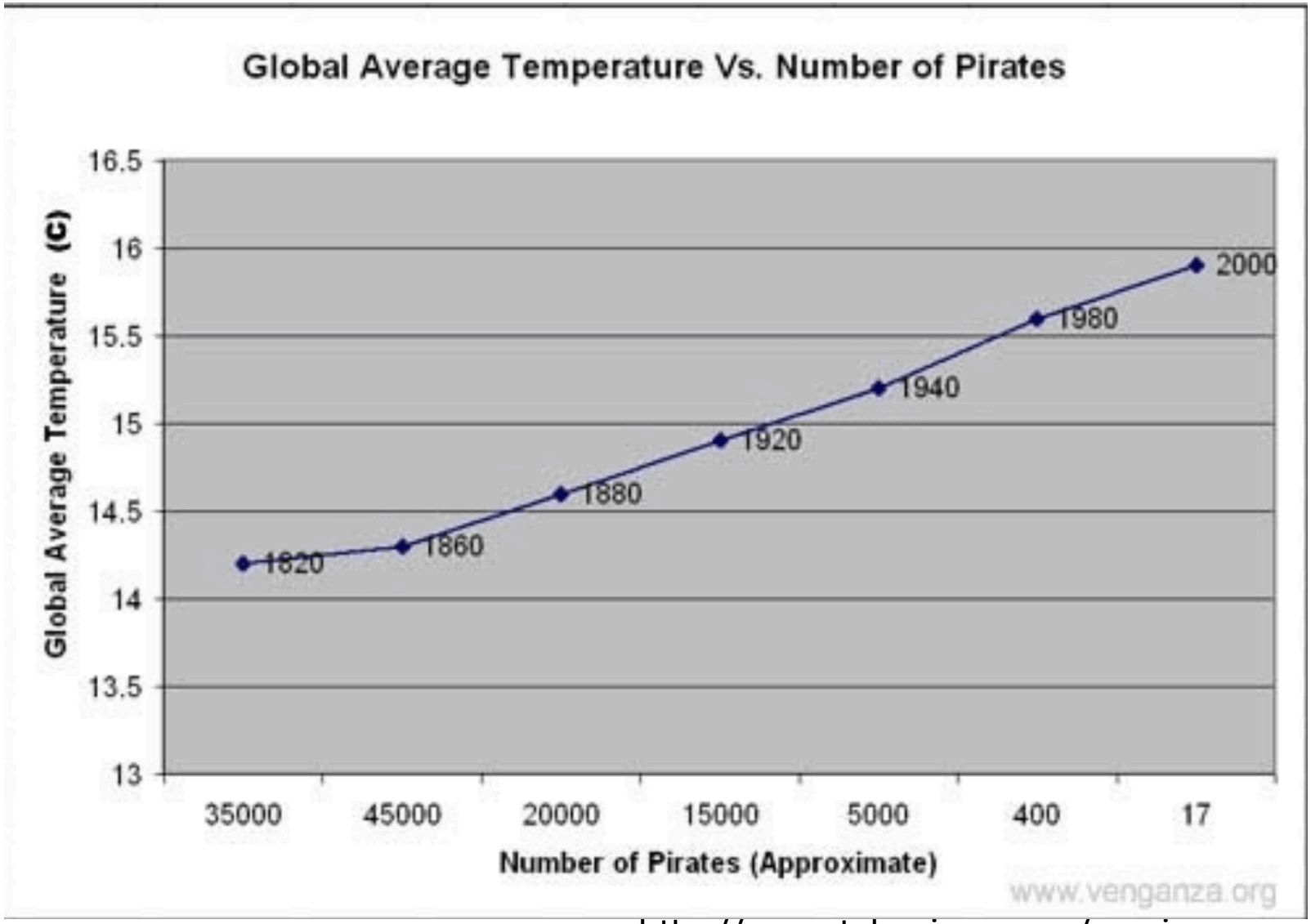
Most astronomers and physicists don't understand statistics

Suppose you derive a 90% confidence region on some parameter of interest. What does this mean ?

Does it tell you that there is a probability of 0.9 that the true value of the parameter lies in the range calculated ?

Slides from Keith Arnaud
Source: http://slideplayer.com/slide/9361524/

Be careful with statistics analysis



http://www.tylervigen.com/spurious-correlations

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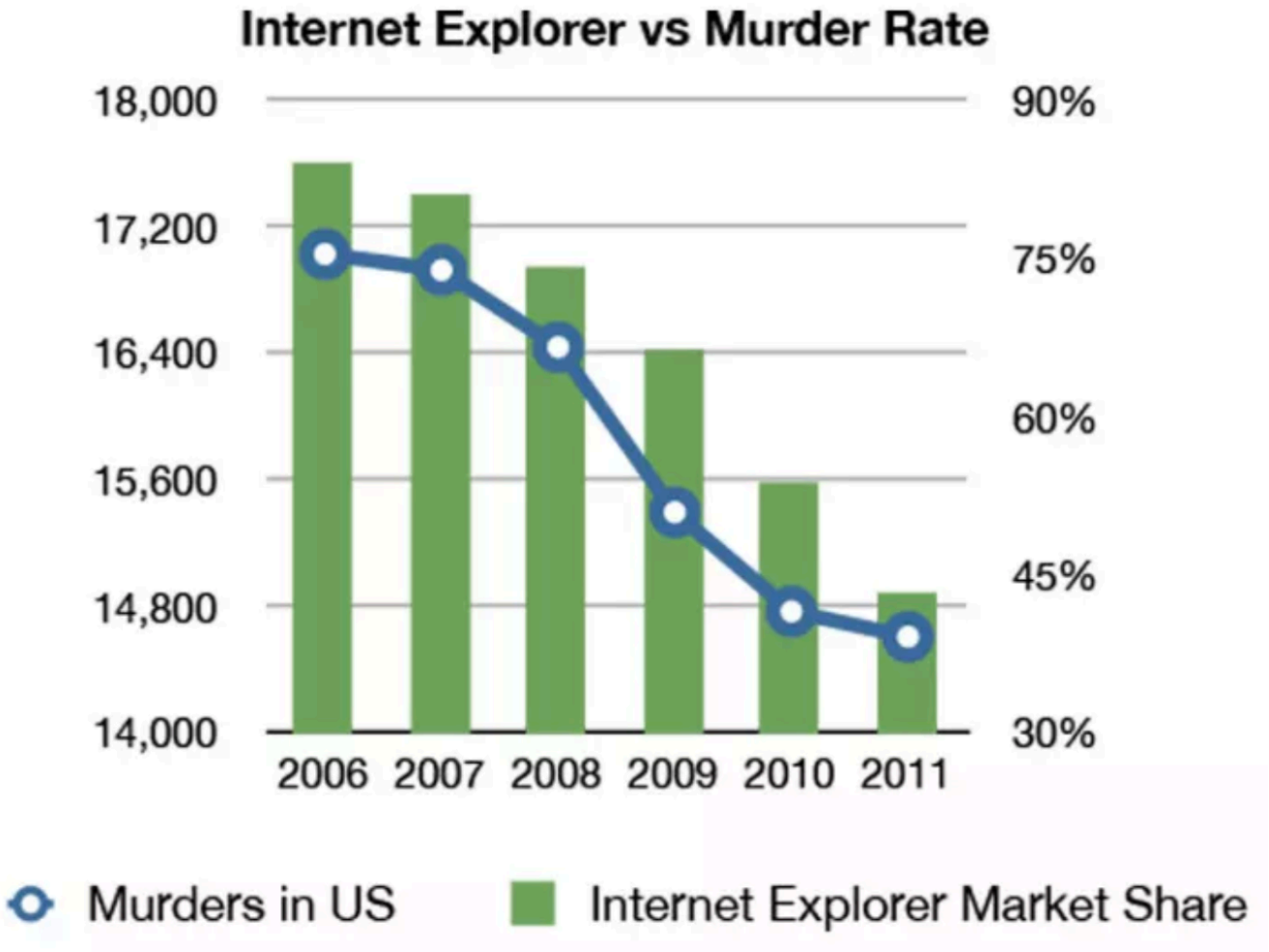

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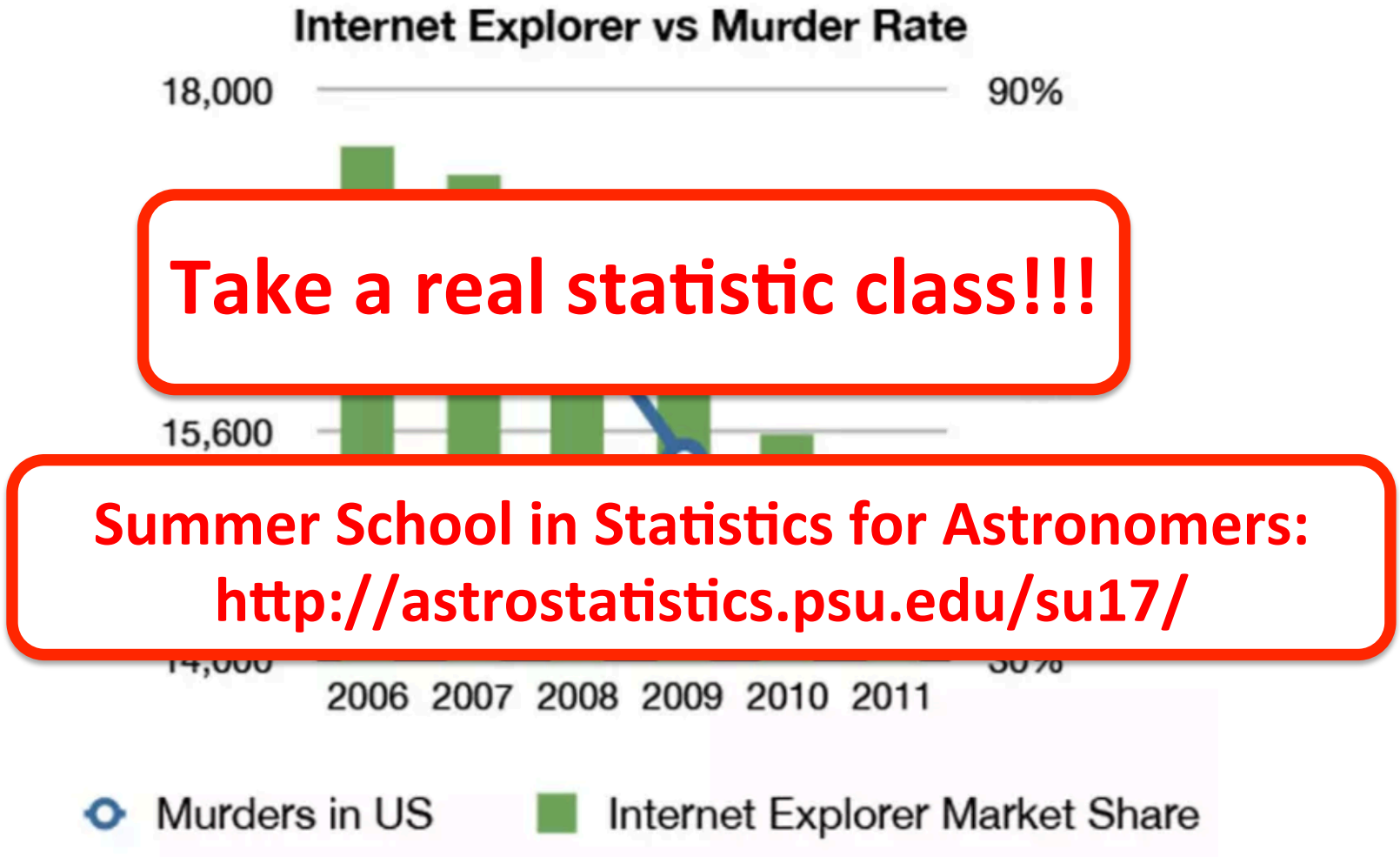


Take a real statistic class!!!

Summer School in Statistics for Astronomers: http://astrostatistics.psu.edu/su17/

http://www.tylervigen.com/spurious-correlations

Basic statistical terms

- Mean (average): $\mu \equiv \langle x \rangle = \int x P(x) \mathrm{d}x$
- Mean of squares: $\langle x^2 \rangle = \int x^2 P(x) \mathrm{d}x$
- Variance: $Var(x) \equiv \sigma^2 \equiv \langle (x-\mu)^2 \rangle = \int (x-\mu)^2 P(x) \mathrm{d}x$

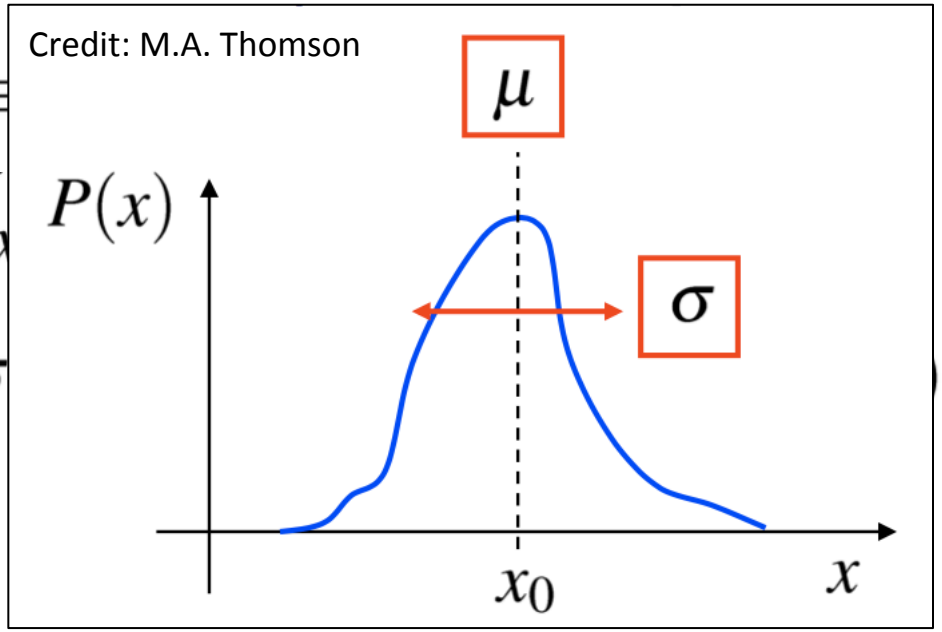
$$= \langle x^2 \rangle - \mu^2$$

- Standard deviation σ = sqrt(variance)
- Degrees of freedom

  = number of data − number of variables

Basic statistical term

- Mean (average): $\mu \equiv$
- Mean of squares: $\langle x$
- Variance: $Var(x) \equiv \sigma$ $\quad \ldots \, dx$



- Standard deviation σ = sqrt(variance)
- Degrees of freedom

  = number of data − number of variables

Poisson distribution

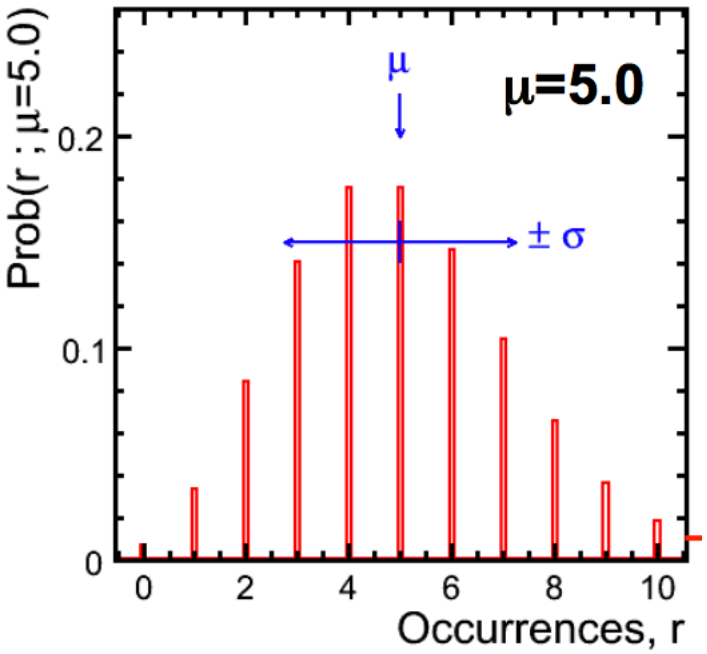
- Most of the high-energy instruments simply count photons → Poisson distribution
- Poisson distribution:

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$\langle x \rangle = \mu$$

$$\sigma^2 = \mu$$

μ: Average of the distribution
σ: Standard deviation



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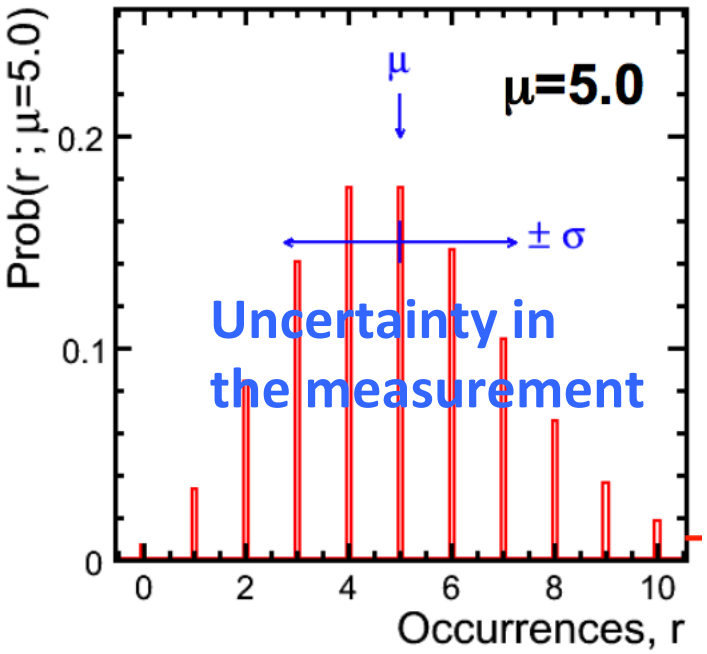


Gaussian distribution

- When the number of data (μ) is large,

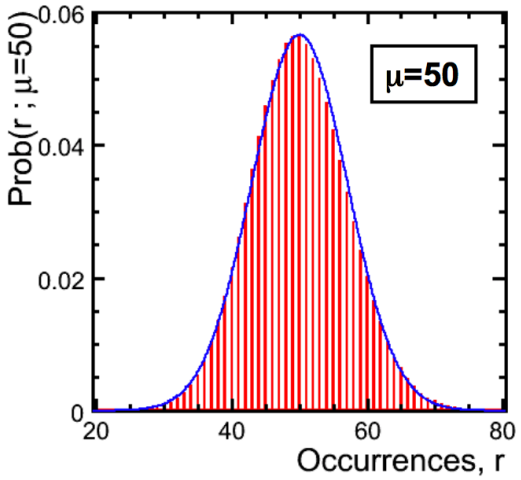
Poisson distribution → **Gaussian distribution**

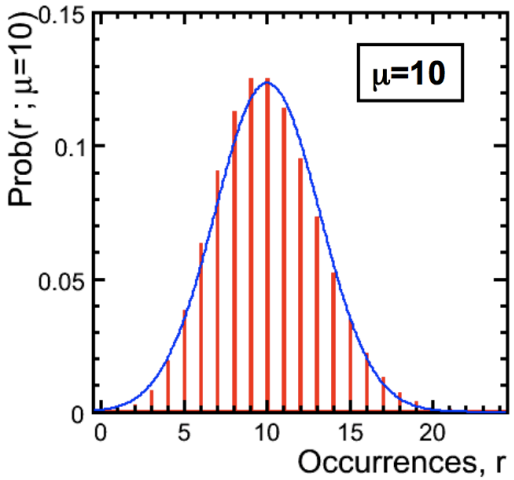
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\langle x \rangle = \mu$$

$$\langle (x-\mu)^2 \rangle = \sigma^2$$

$$P(x;\mu) \approx k e^{-\frac{(x-\mu)^2}{2\mu}}$$





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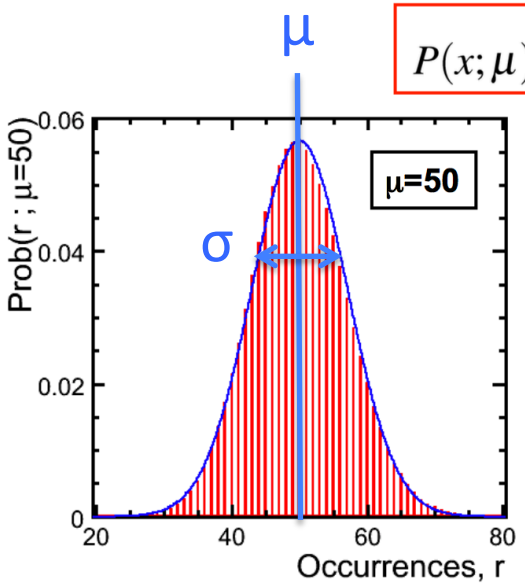
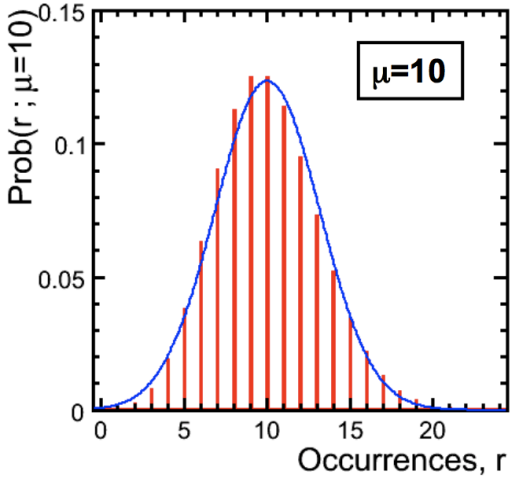
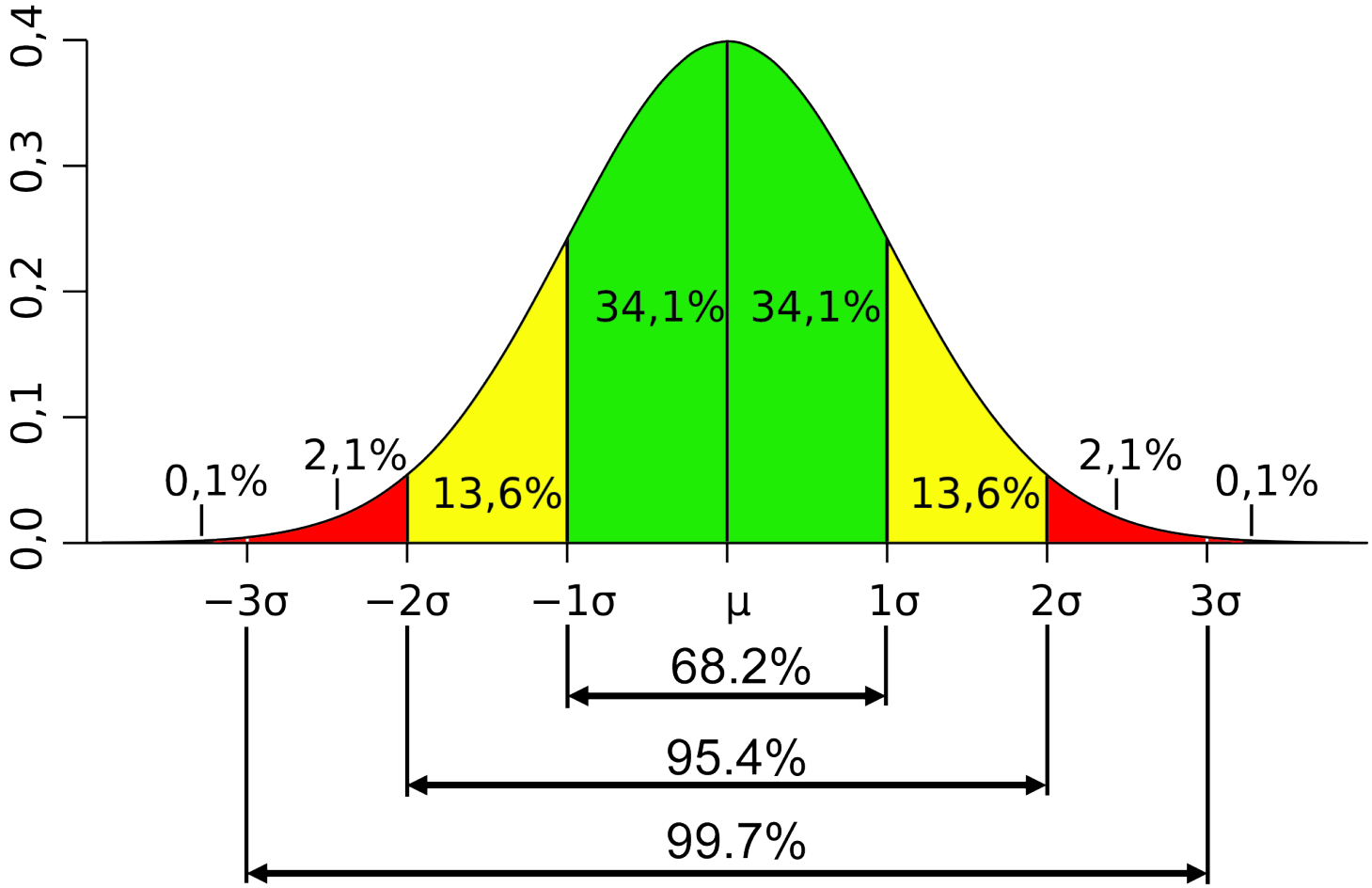




Detection significance vs false-detection rate



Source: https://kanbanize.com/blog/normal-gaussian-distribution-over-cycle-time/

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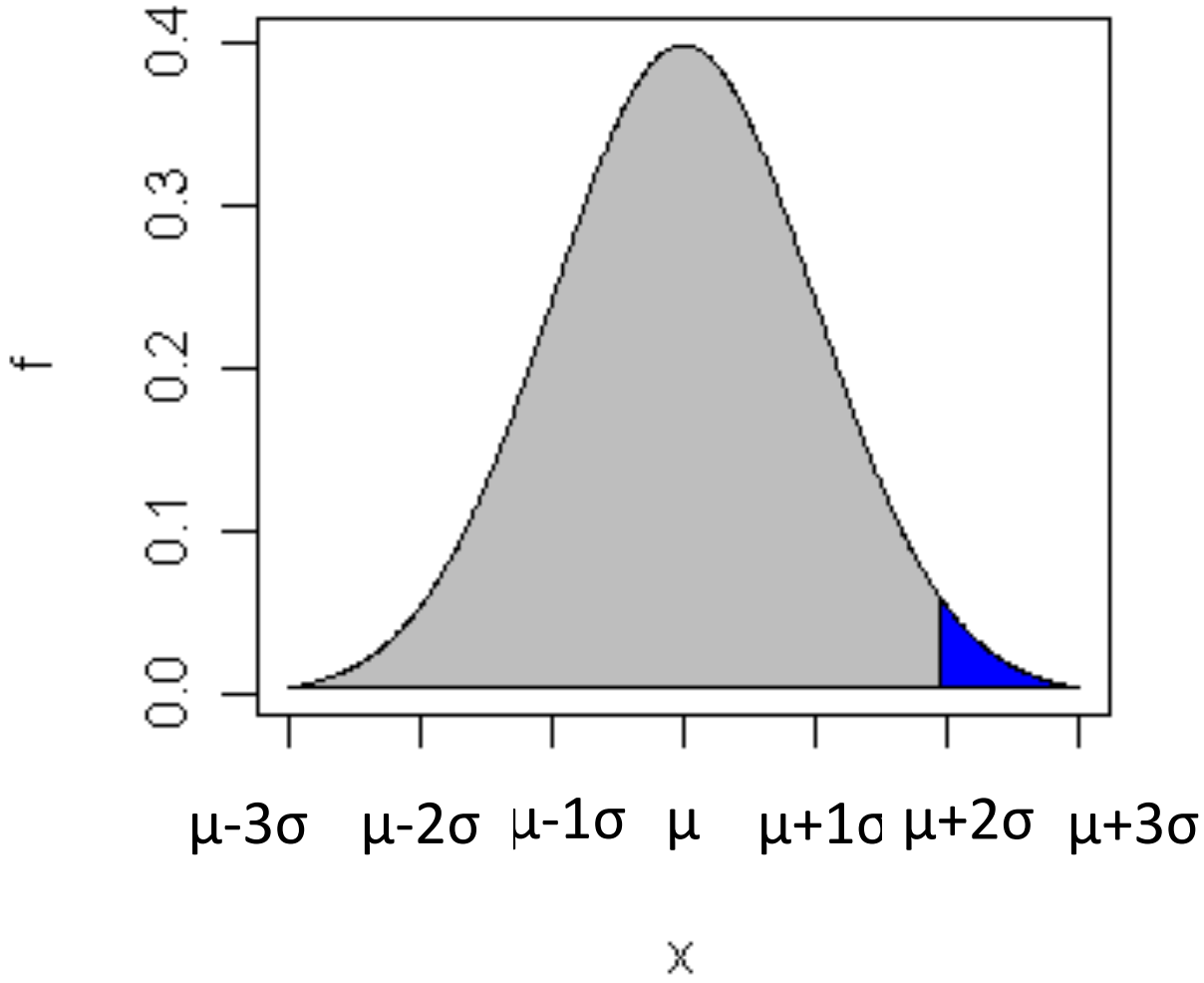

Source: http://www.statsref.com/HTML/index.html?normal_distribution.html

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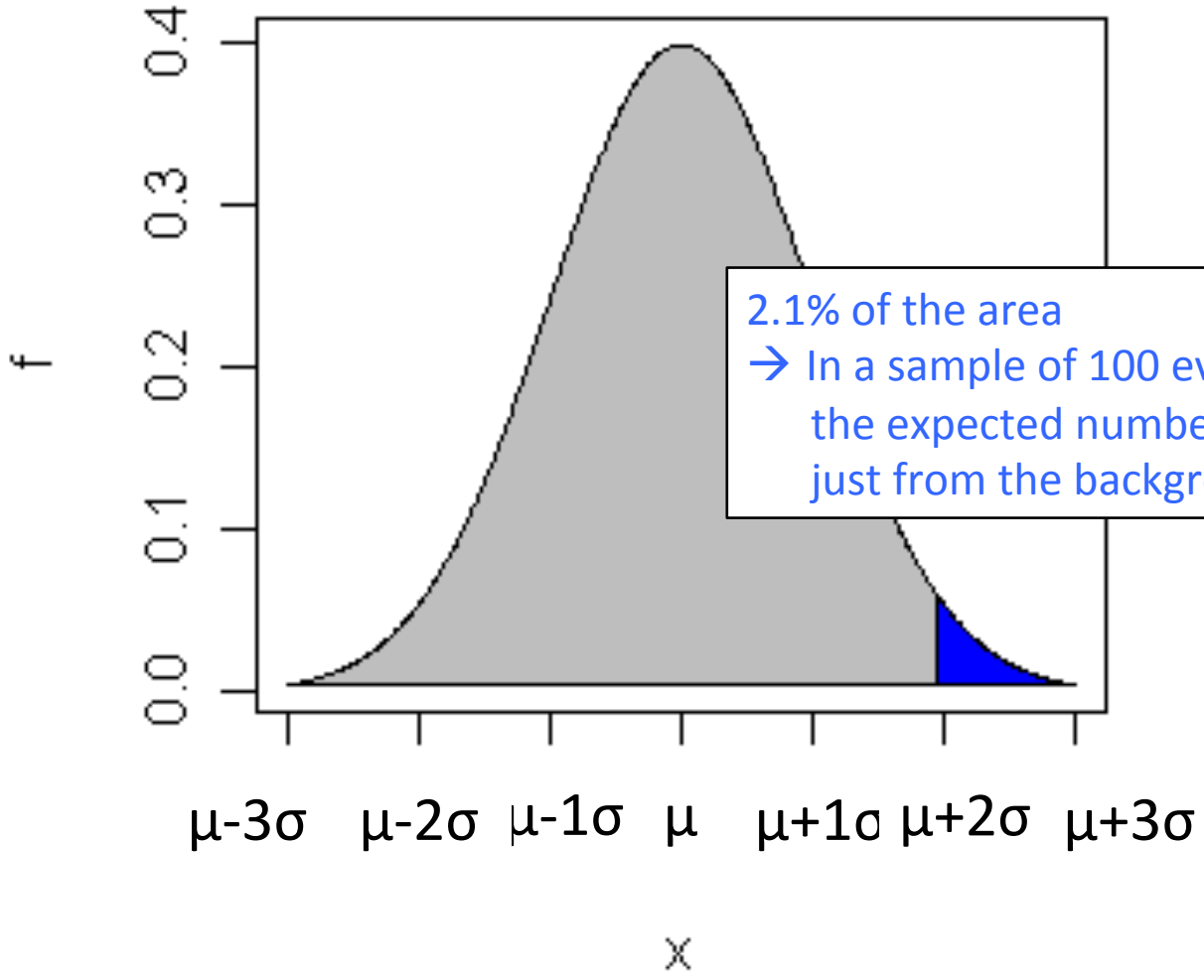


2.1% of the area
→ In a sample of 100 events,
the expected number of 2σ detection
just from the background is 2.1 times.

Source: http://www.statsref.com/HTML/index.html?normal_distribution.html

Data fitting

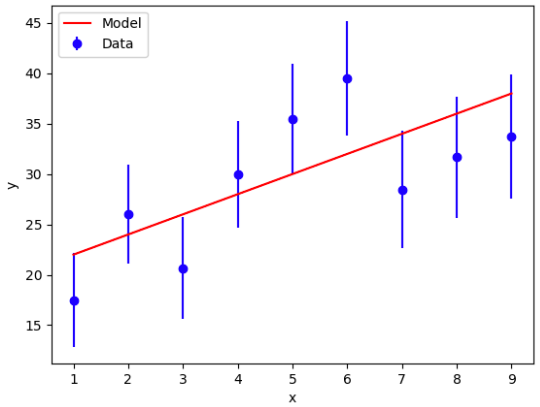
Maximum Likelihood as estimation for data with Gaussian distribution

The model parameters that have the best-chance to explain the observed data

- The likelihood function:

$$L(\theta) = \prod_{i=1}^{n} p(x_i; \theta)$$

The probability of measuring x_i, given a model θ



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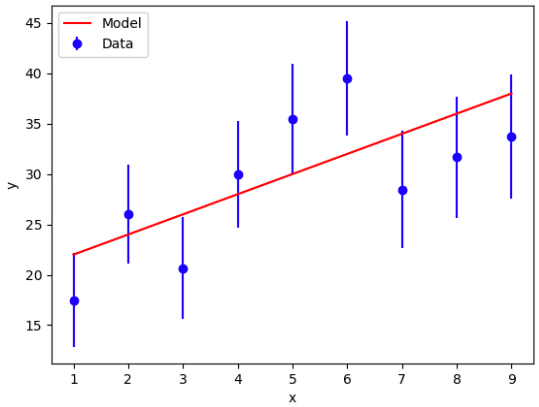
- If all your data have Gaussian distribution

$$L = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i}} e^{\frac{x_i - \mu_i}{2\sigma_i}}$$

$$ln(L) = -\frac{N}{2} ln(2\pi\sigma_i^2) - \sum_{i=1}^{N} \frac{(x_i - \mu_i)^2}{2\sigma_i^2}$$



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Only related to data, not model
→ don't need for maximizing L when finding the right model

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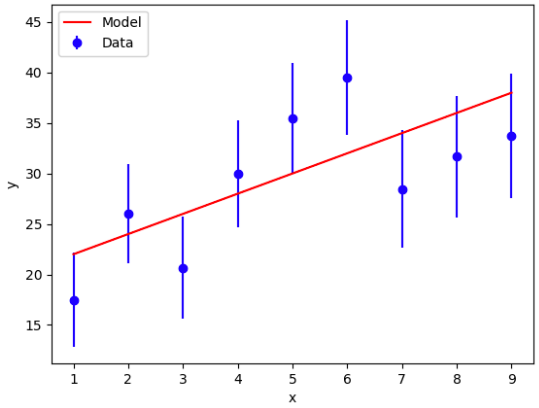

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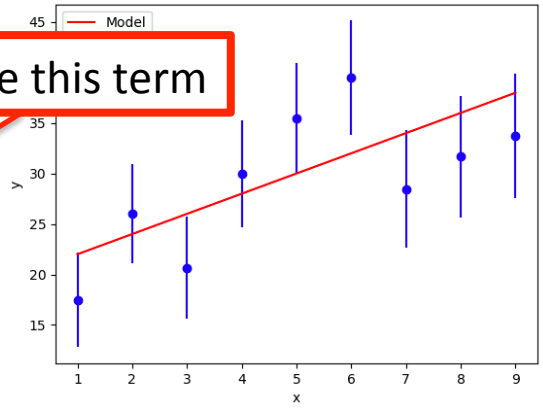
Maximize this term



Data fitting

Maximum Likelihood as estimation for data with Gaussian distribution

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Maximize this term

$$-\sum_{i=1}^{N}\frac{(x_i-\mu_i)^2}{2\sigma_i^2}$$

Minimize this term

$$\sum_{i=1}^{N}\frac{(x_i-\mu_i)^2}{\sigma_i^2}$$

Data fitting

Maximum Likelihood as estimation for data with Gaussian distribution

The model parameters that have the best-chance to explain the observed data

Maximize this term → $-\sum_{i=1}^{N} \frac{(x_i - \mu_i)^2}{2\sigma_i^2}$

Minimize this term → $\sum_{i=1}^{N} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \equiv \chi^2$

Data fitting

Maximum Likelihood as estimation for data with Gaussian distribution

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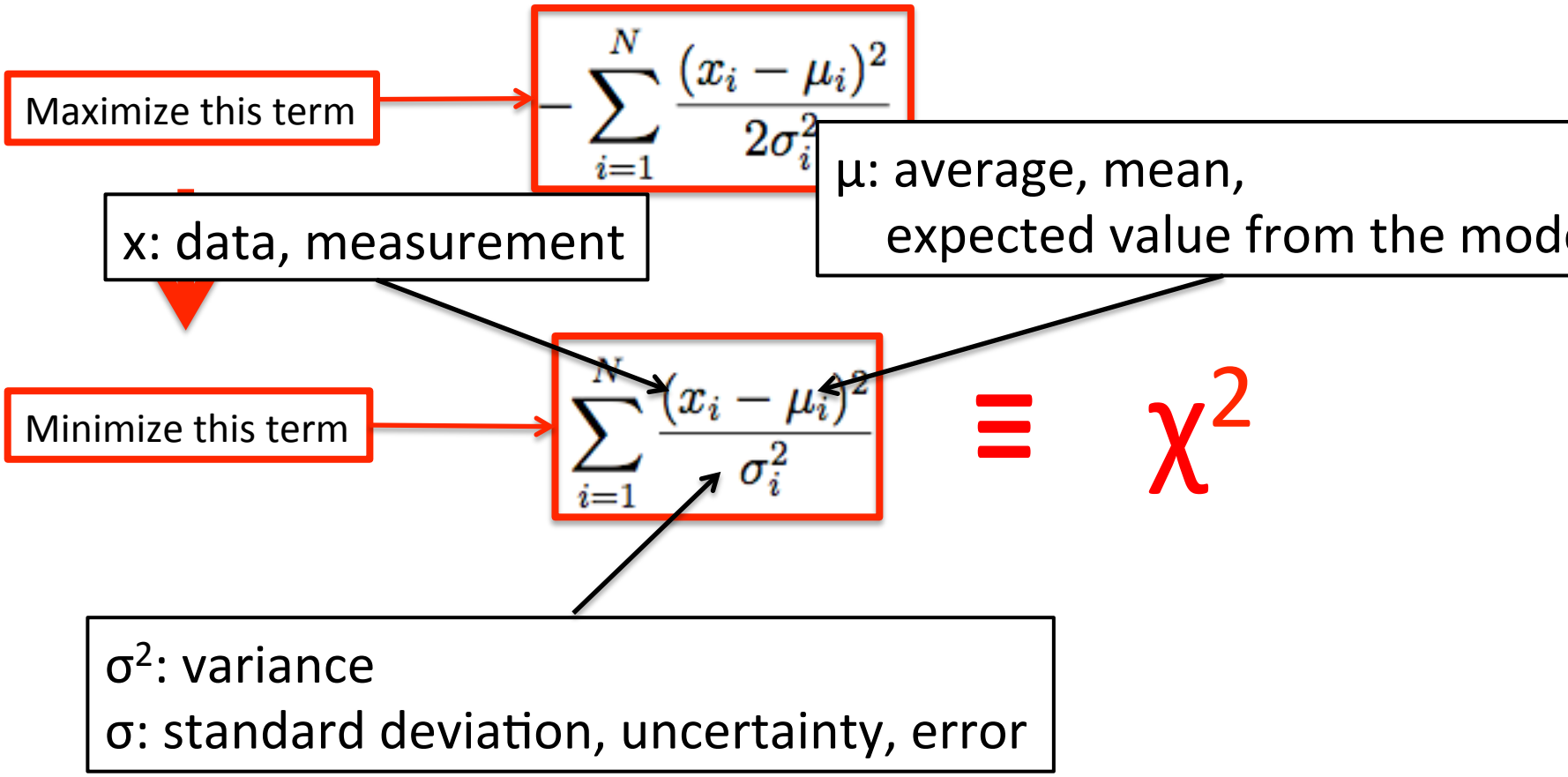

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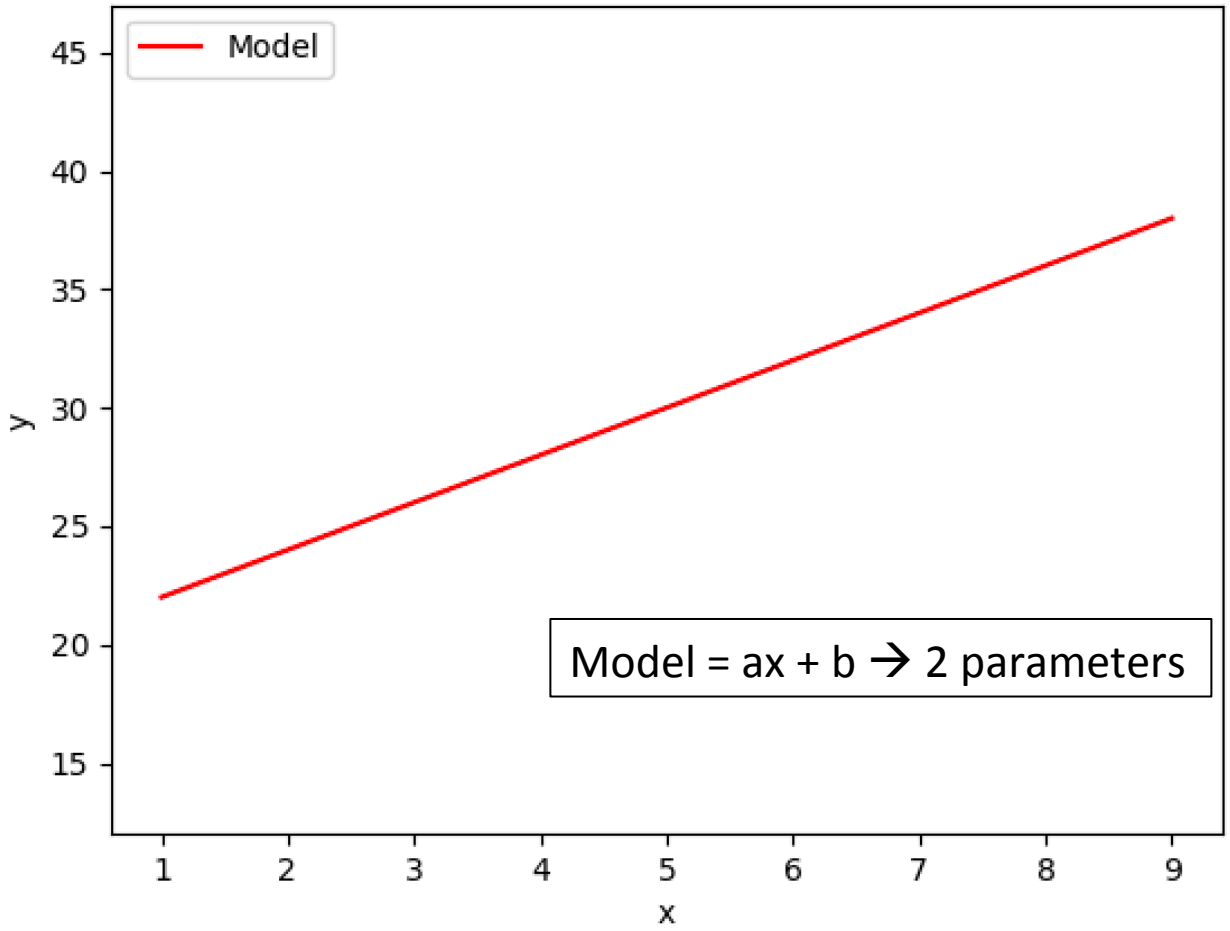

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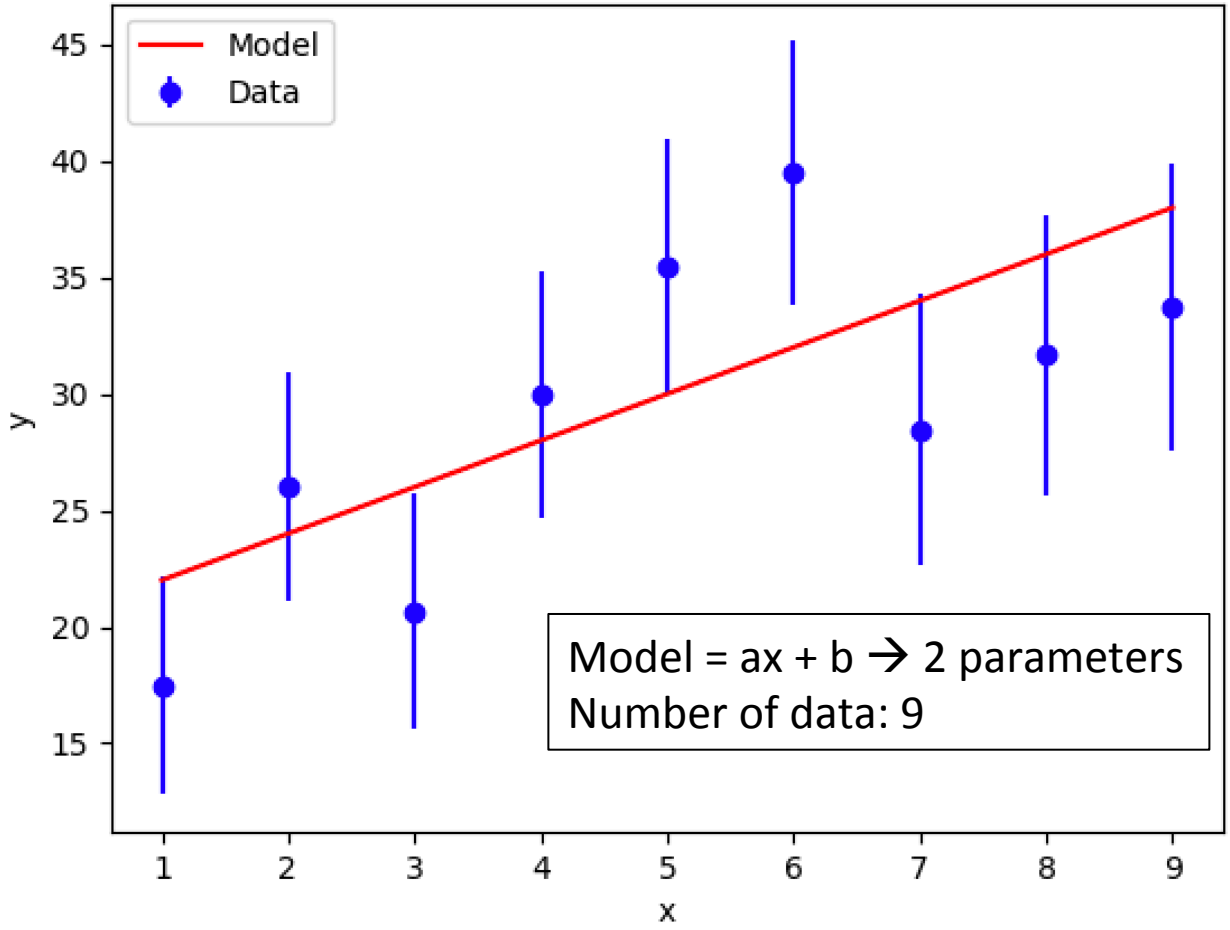

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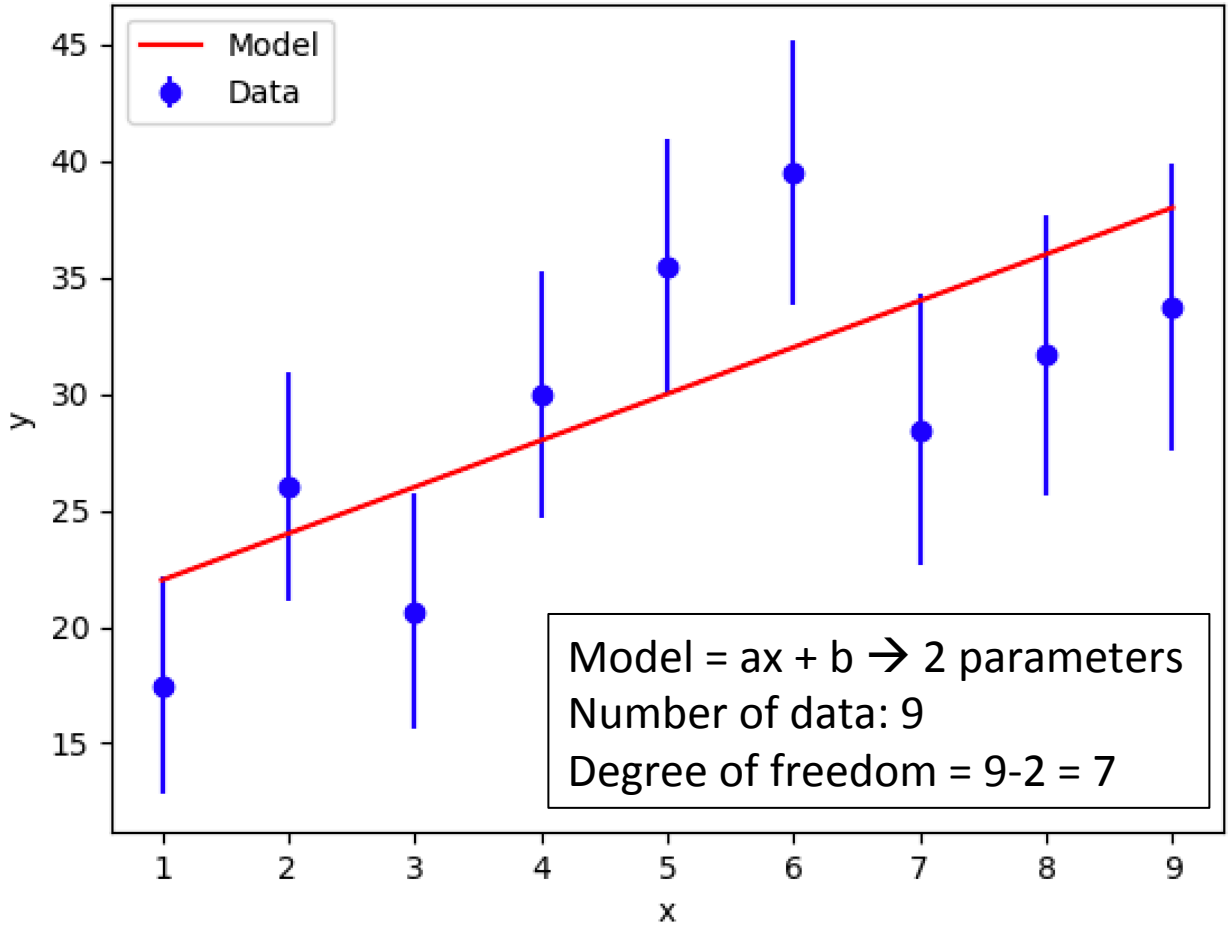
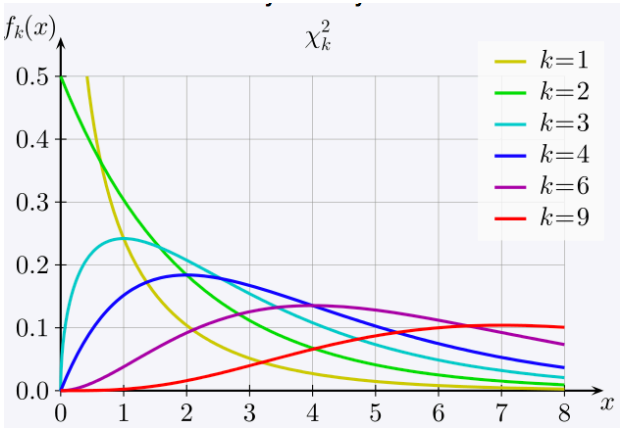


Chi-square distribution

- If χ2 is a sum of the squares of independent normal/Gaussian random variables.

$$f(x;\,k) = \begin{cases} \dfrac{x^{\frac{k}{2}-1}e^{-\frac{x}{2}}}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}, & x > 0; \\ 0, & \text{otherwise.} \end{cases}$$



x: value of χ^2

k: degree of freedom

$$\langle x \rangle = k$$

→ IF you have enough data points, the reduced $\chi^2 = \chi^2/k \sim 1$