Lab 8 Worksheet

You are required to hand this worksheet in at the end of the lab.

Name:____________________ User Name:____________________

- All the example codes are at `/n/ursa/A288C/alien/python_template`.
- The places you need to put your answer in this worksheet are marked by "__________".

1. [**Poisson distribution**] Photon counting instruments, like the *Swift* BAT, XRT, and most of the high-energy telescopes, follows Poisson statistics. The Poisson distribution is expressed as the following,

$$P(x) = \frac{\mu^x e^{-\mu}}{x!} \tag{1}$$

The example code `Poisson_distribution.py` shows how you can use python to create an array of random numbers that follow the Poisson distribution, with average μ and sample size N. Modify from this example code and do the following:

(a) Create an array of randomly distributed numbers that follow a Poisson distribution with average $\mu = 5$ and a sample size $N = 100000$.

(b) Plot the histogram of this distribution (you can use the "hist" command in python, as shown in the example code).

(c) Calculate the average, variance, and the standard deviation of this sample.
Average = __________.
Variance = __________.
Standard deviation = __________.
What is the percentage of uncertainty at the mean value?
That is, (Standard deviation)/Average $\times$ 100 = __________ %.

(d) Repeat step (a) to (c) with different sample sizes of $N = 10$, $N = 100$, $N = 1000$, $N = 10000$, and fill out the following table:

Table 1: Poisson distribution

Sample size	Average	Variance	Standard deviation	Percentage of uncertainty

2. [**Gaussian distribution**] The Poisson distribution can be approximated by a Gaussian distribution in the high-count regime, which is applicable for BAT. The example code `Gaussian_distribution.py` shows how you can use python to create an array of random numbers that follow the Gaussian distribution, with average μ, standard deviation σ, and sample size N. Modify this example code and do the following:

(a) Create an array of randomly distributed numbers that follow a Gaussian distribution with average $\mu = 5$ and a sample size $N = 100000$.

(b) Plot the histogram of this distribution (you can use the "hist" command in python, as shown in the example code).

(c) Calculate the average, variance, and the standard deviation of this sample.
Average = __________.
Variance = __________.
Standard deviation = __________.
What is the percentage of uncertainty at the mean value?
That is, (Standard deviation)/Average $\times$ 100 = __________ %.

(d) Repeat step (a) to (c) with different sample sizes of $N = 10$, $N = 100$, $N = 1000$, $N = 10000$, and fill out the following table:

Table 2: Gaussian distribution

Sample size	Average	Variance	Standard deviation	Percentage of uncertainty

3. **[False-detection rate]** The normalized Gaussian distribution is expressed as the following

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{2}$$

Assuming = 100.0 and $\sigma = 10.0$.

(a) Write a code to integrate this Gaussian from x=$\mu - 10\sigma$ to x=$\mu + 10\sigma$.
The answer is __________.

(b) Modify your code and integrate from $x = -3\sigma$ to $x = 3\sigma$.
The $x \pm 3\sigma$ covers __________ % of the total area.

(c) If you have a detection that is 7 σ, what is the chance that this is detection is coming from the background?
That is, the chance of having a false detection (i.e., the false-detection rate) for $\geq 7\sigma$ is __________.

4. **[Chi-square (χ^2)]** The example code `Chi-square.py` adds some Gaussian noise to data points that follow a linear correlation $y = 2.0x + 20.0$. Follow this example code to

(a) Write a code that produces a set of 10 data points that is drawn from this linear relation, with Gaussian noise.

(b) Plot both the data point vs x, and the underlying model vs x.

(c) Modify your code to calculate the value of $\chi^2 = \sum \frac{(\text{data}-\text{y})^2}{\sigma^2}$ for your data set.
χ^2 is ________.
The degree of freedom is ________.
The reduced χ^2 is ________.

(d) Increase your data set to 100 samples, 1000 samples, 10000, and 100000 samples. Repeat (b) and (c) and fill out the following table.

Table 3: χ^2 Test

Sample size	Chi-square	Reduce Chi-square