University of Maryland Baltimore County
Department of Information Systems
Spring 2016

IS 709/809: Computational Methods for IS Research
Homework 2
(Handed Out: February 17, 2016 (Wednesday), Due: March 2, 2016 (Wednesday) Before Class)

General Instructions: Use printer paper for your answer sheets. Use blue or black ink. Number each page and write down the total number of pages on the upper right-hand corner of the first page. Thanks.

1. **(10 points)** Order the following functions by growth rate: N, $\sqrt{N}$, $N^{1.5}$, N^2, $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, 2^N, $2^{N/2}$, 37, $N^2 \log N$, N^3. Indicate which functions grow at the same rate.

2. **(10 points)** Compute the running time $T(n)$ of the program fragment below and provide an analysis of the running time (Big-Oh notation will do). For convenience, assume that operations inside for loops take constant time, i.e. $\Theta(1)$.

```
sum = 0;
for (i = 0; i < n; i++)
  for (j = 0; j < i * i; j++)
    for (k = 0; k < j; k++)
      sum++;
```

3. **(10 points)** An algorithm takes $0.5ms$ for input size 100. How long will it take for input size 500 if the running time is the following (assume low-order terms are negligible): linear, $O(n \log n)$, quadratic, and cubic?

4. **(10 points)** Give an $O(\log N)$ algorithm to determine if there exists an integer i such that $A_i = i$ in an array of integers $A_1 < A_2 < A_3 < ... < A_N$. For example, in $\{-10, -3, 3, 5, 7\}$, $A_3 = 3$. In $\{2, 3, 4, 5, 6, 7\}$, there is no such i.

5. **(10 points)** Show that X^{62} can be computed with only eight multiplications.

6. **(10 points)** Given two functions $T_1(n) = 20n$ and $T_2(n) = n^2 + 10n - 200$ and assuming integer $n > 0$, what is the minimum value of n such that $T_1(n) < T_2(n)$?

7. **(10 points)** Is a linear-time algorithm always faster than a quadratic-time algorithm? Why or why not?