

**IS 709/809: Computational Methods for IS Research**

Homework 1

(Handed Out: February 10, 2016 (Wednesday), Due: February 17, 2016 (Wednesday) Before Class)

*General Instructions:* Please use typewriting paper for your answer sheets. Use blue or black ink only. Number each page and write down the total number of pages on the right-hand corner of the first page. Thanks.

- (10 points) Compute the following using your calculator:  $\log_2 351$  and  $\log_5 93$ . Limit your answers to 4 decimal places.
- (10 points) Prove that  $\log_C A^B = B \log_C A$ .
- (10 points) Prove  $\sum_{j=1}^N (2j - 1) = n^2$ .
- (10 points) Prove: If  $n \geq 2$ , then  $n^3 - n$  is always divisible by 3, where  $n$  is a natural number.
- (10 points) Prove: There is no greatest even integer.
- (10 points) Prove or disprove:  $2n^2 + 5n + 10 < 1000n$ ,  $n$  is an integer and  $n > 0$ .
- (10 points) Evaluate  $\sum_{i=0}^{\infty} \frac{1}{4^i}$ .
- (10 points) Fill in the blanks:
  - \_\_\_\_\_  $\equiv 16 \pmod{7}$
  - $35 \equiv$  \_\_\_\_\_  $\pmod{4}$
  - $123 \equiv 73 \equiv 3 \pmod{\text{_____}}$
- (10 points) Given an array of  $N = 10,000$  integers, how many comparisons or checks does the binary search algorithm need to perform to determine if an arbitrary integer  $k$  is in the array or not? Give the number of comparisons as a whole number using the ceiling operator.
- (10 points) Write a recursive C function that computes the greatest common divisor of two integers. The greatest common divisor (gcd) of two integers  $a$  and  $b$  is given by:

$$\text{gcd}(a, b) = \begin{cases} a & \text{if } b = 0; \\ \text{gcd}(b, a \% b) & \text{if } a \geq b \text{ and } b > 0, \end{cases}$$

where the symbol % is the modulo operator in C/C++ (that is,  $a \% b$  gives the remainder when  $a$  is divided by  $b$ ).

Give the sequence of recursive calls when  $\text{gcd}(333, 185)$  is invoked. What is the result of  $\text{gcd}(333, 185)$ ?