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LOGROLLING, VOTE TRADING, AND THE PARADOX OF VOTING: A GAME-THEORETICAL OVERVIEW

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In a recent article, David Koehler (1975a) claimed that "vote trading" and "the voting paradox" are "logically equivalent," i.e., each phenomenon implies the other. In a following "Comment," Peter Bernholz (1975) denied both that "logrolling" implies "the paradox of voting" and also the converse. As readers of this journal are surely aware, the Koehler-Bernholz exchange is but one episode in an extended debate within the public choice literature concerning conceptual, logical, empirical, and normative issues pertaining to logrolling and vote trading.

Until quite recently, many political scientists condemned logrolling and vote trading (and other forms of "strategic" voting) as devious machinations tending to undermine the democratic political process and frustrate majority will. And this probably remains the popular view of the matter.

But this traditional disapproval was powerfully challenged by JamesBuchanan and Gordon Tullock (1962), who argued (in rough summary) that: (i) the right to vote on an issue is an economic good and should be so treated, (ii) vote trading is

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empirically prevalent, (iii) vote trading allows for a finer expression of individual preferences (in particular, by allowing voters to express their varying intensity of concern over issues), and (iv) vote trading generally leads to a determinate outcome more "desirable" socially than the outcome without vote trading (though still not "optimal," given majoritarian political institutions). And, most relevant for our purposes, Tullock in particular argued (1962, pp. 330, 332) that "if logrolling is the norm . . ., then the problem of the cyclical majority vanishes" and, more generally, then "the particular type of irrationality described by Arrow [1951] is impossible." Subsequently James Coleman (1966, p. 1115) also suggested that "vote exchanges" lead to "freedom from Arrow's impossibility theorem."

However, a number of more recent papers have pretty directly disputed the Tullock-Coleman suggestion. The earliest of these, written by R. E. Park (1967) in direct response to Coleman, apparently attracted little attention at the time. Then just a few years ago something like Park's basic result was independently and more or less simultaneously rediscovered by some half dozen scholars, who produced papers all getting at the same basic point – the reverse of Tullock's claim: if logrolling is the norm, then the problem of the cyclical majority is pretty well universal. In addition to Koehler (1975a), these papers include those by Joe Oppenheimer (1972), Peter Bernholz (1973 and 1974), and Thomas Schwartz (1975a), as well as my own note (1975), which derived from part of an earlier dissertation (1973, pp. 249-255, 382-391). And, as several of these authors noted, the logic underlying this basic result was certainly anticipated slightly earlier by Joseph B. Kadane (1972) and was perhaps anticipated considerably earlier by Anthony Downs (1957, pp. 55ff), though neither was explicitly concerned with logrolling or vote trading. But even on this relatively limited point – i.e., the relationship between "logrolling" or "vote trading" and "cyclical majorities" or "the paradox of voting" even the authors in this last group do not exactly agree with one another, as the Koehler-Bernholz exchange illustrates. Indeed, the exact relationship between these two phenomena remains somewhat unclear, primarily – I think – because the various treatments exhibit little consistency in terminology, concepts, and the framework within which the problem is analyzed.[1]

In this overview, I try to pull together part of the literature on logrolling and vote trading and specify the relationship with the paradox of voting once and for all. In order to accomplish this, I work explicitly within a game-theoretical framework adapted from Robin Farquharson (1969), and I restrict my attention to that portion of the literature which can be fitted within this framework. That portion includes, in particular, Park (1967), Kadane (1972), Oppenheimer (1972), William Riker and Steven Brams (1973), Ferejohn (1974, pp. 7-14), Miller (1975), Koehler (1975a), and Schwartz (1975a and 1976); the restriction excludes, in particular, Coleman (1966), who formulates the vote-exchanging process in terms of individual decision making under uncertainty, Robert Wilson (1969), who hypothesizes a price

[1]For a concise review of much of this literature, which highlights this inconsistency, see John A. Ferejohn (1974), pp. 1-7.

system for votes, Dennis C. Mueller, et. al. (1972), who assume that votes are literally exchanged (also see Mueller, 1967), and Arnold B. Urken (1976), who deals with "fungible" voting systems.

Any game-theoretical notion of logrolling or vote trading obviously entails some measure of collaboration among voters – between a pair of voters at the least. Thus, any game-theoretical model of logrolling or vote trading must be a partially or fully cooperative one. But such an analysis must deal with the non-cooperative case as well. For it is generally consistent with the cited literature to say that a "logrolling" or "vote trading" situation exists when the "individualistic" voting situation—that is, the situation we would expect in the absence of any kind of collaboration among voters—is in some sense "unstable"—a bit more specifically, when two or more voters can "manipulate" the situation to their common advantage.

This statement identifies two main points to be made in this article. The first is that the nature of the individualistic voting situation must be well specified. In part, confusion about whether logrolling or vote trading does or does not entail the paradox of voting stems from disagreement (or imprecision), not about the nature of logrolling or vote trading per se, but about the nature of individualistic voting, in particular about whether such voting is—to use the now familiar language of Farquharson—"sincere" or "sophisticated."

The second main point is that there is also disagreement (or imprecision) about the nature of logrolling and vote trading per se. Two conceptually distinct notions of "instability" have often (not always) been confused and thus also two conceptually distinct phenomena: on the one hand, the formation of a coalition including a majority of voters that engages in "decisive" strategic collaboration in order to impose on the voting body a voting decision different from, and preferred to, the individualistic voting decision; and on the other hand, concerted action among voters who may lack the power to impose a decision on the voting body but who engage in "marginal" strategic collaboration in order to bring about a voting decision different from, and preferred to, the individualistic voting decision. This important distinction can easily be overlooked, especially when analysis is limited (as often has been the case) to three-member voting bodies, in which any voter collaboration produces a "decisive" majority. We shall maintain the distinction between these two types of voter collaboration by labelling the first "logrolling" and the second "vote trading."[2] (Thus, to restate the previous point, only in a three-member voting body do two "vote traders" also constitute a "logrolling" coalition.) We find a close relationship between logrolling and the paradox of voting

[2]This usage seems natural—more so, in any case, than the reverse. Most authors tend initially to equate the two terms; see, for example, Buchanan and Tullock (1962), pp. 132, 155; Wilson (1969), pp. 331, 332; Pennock (1970), p. 714; Wade and Curry (1970), p. 45n; Mueller, et. al. (1972), p. 55; Riker and Brams (1973), pp.1235, 1241; Bernholz (1974), p. 49; Ferejohn (1974), p. 1; Schwartz (1975), p. 1; Koehler (1975a), pp. 954, 958; Uslaner and Davis (1975), p. 929. But those who fit better in the "decisive" category tend in fact to use the term "logrolling" preponderantly, and those who fit better in the "marginal" category tend in fact to use the term "vote trading" preponderantly. This usage is also consistent with Enelow (1976).

but not between vote trading and the paradox of voting.

I. THE STRUCTURE OF A VOTING BODY

We consider an n-member voting body that is considering some finite number m of *bills* (or *issues, motions,* etc.), A, B, . . . ,M. Each bill is a set of *positions* (or *alternatives*, etc.)–certainly including at least two positions, i.e., "passage" and "defeat," and perhaps more, e.g., "passage with a given set of amendments." A *dichotomous* bill has just two positions. In general, let the positions in bill A be labelled a_1, a_2, etc., and likewise for other bills. A *voting outcome* is a specified resolution of all bills, i.e., a collection of m positions, one from each bill.

Each of the n voters has a preference ordering of these outcomes. We assume that n is odd and all preference orderings are strong. Then majority preference over outcomes is complete (and, of course, asymmetric). The *Condorcet outcome* v^* is the one voting outcome, if any, preferred by majorties of voters to every other outcome. As is now well known, a Condorcet outcome may not exist, because of a *paradox of voting*, i.e., a cycle in majority preference, having the result that, for every outcome, there is another outcome that a majority of voters prefer to it. The *Condorcet set* V^* of outcomes is the minimal nonempty set of outcomes such that each outcome in V^* is preferred by majorities to every outcome not in V^*. Fairly obviously, such a set always exists, is unique, includes only v^* if such an outcome exists, and otherwise includes three or more outcomes over which there is a complete cycle of majority preference.[3]

We assume that the voting body is majoritarian and that bills are voted on sequentially. Dichotomous bills are passed or defeated by a single majority vote on each. In the general case, there must be some procedure for voting on the several positions in each bill. In practice, all parliamentary voting is *binary*, i.e., at each vote, a voter can vote in one of just two ways (e.g., "yea" and "nay"). (Cf. Farquharson, 1969, p. 9) We will give special attention to what Farquharson (1969, pp. 11-12) calls "amendment" procedure–two positions are put up for a majority vote, the defeated position is eliminated, and the surviving position is paired with a third, and so forth until every position but one has been eliminated.[4]

Finally, we assume that, at any given vote, each voter knows the results of every previous vote, and that each voter knows the preferences of all other voters (or at least knows how majority preference lies between each pair of outcomes).

[3] Thus a Condorcet set, in the absence of a Condorcet outcome, is often called a "top cycle." For a closer analysis, see Miller (1977).

[4] Black (1958) calls this "Procedure (α)" on p. 21 and "ordinary committee procedure" thereafter. Though seemingly quite different, this procedure approximates Anglo-American parliamentary practice in its logical structure. However, the latter does differ in a consequential way when two or more separate (vs. substitute) amendments are being considered.

II. SEPARABILITY OF PREFERENCES

We are particularly interested in one natural restriction on voters' preferences. A *complement* $\overline{a}_r$ of bill A is a collection of m-1 position, one from every bill other than A. Thus $a_h\overline{a}_r$ is an outcome. A voter's preferences are *separable* if, for any bill A, whenever he prefers $a_h\overline{a}_r$ to $a_k\overline{a}_r$ for *some* complement $\overline{a}_r$, he also prefers $a_h\overline{a}_q$ to $a_k\overline{a}_q$ for *any* other complement $\overline{a}_q$. In other words, he prefers a_h to a_k regardless of how other bills are resolved. (Otherwise, his preferences on A cannot be "separated" from the resolution of other bills.) Clearly if (and, in general, only if) all voters have separable preferences, we can speak of majority preference over the positions in each bill, and thus of a *Condorcet position* and the *Condorcet set* of positions in each bill. Let a* designate the Condorcet position in bill A, if it exists, and A* the Condorcet set of positions; and likewise for other bills.

Suppose all voters' preferences are separable. What then is the relationship between the Condorcet sets of positions in the several bills, i.e., A*, B*, . . . , M*, and the Condorcet set of outcomes, i.e., V*? The answer to this question is provided by what may be called the "fundamental theorem" of logrolling in a majoritarian voting body with separable preferences.

First, we label the positions in bill A so that a_1 is majority preferred to a_2, a_2 is majority preferred to a_3, and so forth. Such a labelling scheme is always possible because we assume preferences are separable and because (given an odd number of voters with strong preferences) majority preference is a "tournament" and every such structure has a "complete path" (cf. Harary, et. al., 1965, p. 295). Clearly, a_1 must belong to A*; further, since A* contains a complete cycle, any position in A* may be labelled a_1. Similar conventions and considerations hold for the other bills.

An outcome belongs to V* if and only if there is a "path" of majority preference from that outcome to every other outcome. By the labelling convention, it is clear that there is such a "path" from $a_1b_1 \ldots m_1$ to every other outcome. But any position in A* may be labelled a_1; and likewise for other bills. Thus we have the following "fundamental theorem."

SEPARABILITY THEOREM. Every outcome in A* x B* x . . . x M* belongs to V*.[5]

In words, every outcome including only positions in the Condorcet sets of positions belongs to the Condorcet set of outcomes.

This theorem has an immediate and important corollary.

SEPARABILITY COROLLARY. V* is a one-element set only if each A*, B*, . . . , M* is also a one-element set.

In words, a Condorcet outcome exists only if every bill has a Condorcet position, in which case (by the theorem) the Condorcet outcome is that outcome that includes the Condorcet position from every bill. Note that the reverse does not

[5]This is identical to Theorem 1 in Kadane (1972), p. 52, and the reader is referred there for a more complete statement of the proof. Also see Miller (1977), Theorem 7 and Proposition 14.

hold; even if every bill has a Condorcet position (e.g., if all bills are dichotomous), there may be no Condorcet outcome. More generally, V^* may include outcomes not in $A^* \times B^* \times \ldots \times M^*$.

III. INDIVIDUALISTIC VOTING

A *voting strategy* s_i of voter i prescribes a voting choice at every vote that may arise, where such prescriptions may be contingent upon the results of previous votes. A *voting situation* is a strategy n-tuple $s = (s_1, \ldots, s_n)$, one for each voter. Every voting situation belongs to some voting outcome – that is, when each voter selects a strategy, a voting outcome is determined. We call the outcome that is actually realized the *voting decision.*

"Sincere" voting has an obvious and natural meaning if preferences are separable and all bills are dichotomous or amendment procedure is in use – i.e., "vote for one's preferred position at every vote." More generally, a "sincere" voter acts as if the voting decision depended only on his own voting choices, and therefore, at every vote, his choice is aimed at the outcome he most prefers from among those that (i) remain as possible decisions and (ii) may be eliminated as possible decisions as a result of that vote (cf. Farquharson, 1969, p. 18). If a voter has strong preferences, only one of his voting strategies is in every way consistent with "sincere" voting; call this his *sincere voting strategy*, the n-tuple of such strategies the *sincere voting situation*, and the outcome to which that situation belongs the *sincere voting decision*.

It is widely acknowledged that, given three or more voting outcomes, sincere voting may be inexpedient. Farquharson is primarily concerned with identifying one type of "most expedient" individualistic voting strategy–what he calls a *sophisticated voting strategy.*[6] Farquharson (1969, pp. 38-43, 74-75) defines sophisticated voting strategies as those that remain after successive elimination of "inadmissible" (or "dominated") strategies, and he shows that, if preferences are strong and the procedure is binary, this elimination continues until there is but one *sophisticated voting situation* or, failing that, all of the several sophisticated voting situations belong to the same outcome. In either case, under these conditions, there is therefore a unique *sophisticated voting decision.*

While Farquharson (1969, pp. 64-67) presents a complete table of sophisticated (as well as sincere) voting decisions under a variety of procedures for the case of three voters and three outcomes and remarks (1969, p. xii) that the tabulation "can readily be extended to cover any desired number of either," the manner in which this extension can "readily" be accomplished is not clear from his discussion. Certainly the "reduction method" that he uses in his text for identifying sophisticated strategies, and thus sophisticated decisions, is prohibitively tedious

[6]As defined by Farquharson (1969, p. 50) "sophisticated" voting *is* strictly individualistic, and we shall maintain this usage. Occasionally, however, the term has been used to include *any* strategic deviation from sincere voting; for example, see Riker and Brams (1973), p. 1237, and Uslaner and Davis (1975), pp. 930, 935.

given voting games of any magnitude. Fortunately, there is an alternative "tree method"—independently developed by Richard McKelvey and Richard Niemi (see Niemi, et. al., 1974, and McKelvey and Niemi, 1976) and by myself (1973, pp. 345ff)—for indentifying sophisticated voting decisions under binary procedure.[7] The discussion in the next section is based on this method.

IV. INDIVIDUALISTIC VOTING DECISIONS

We argued in the introductory section that the existence of incentives for logrolling, vote trading, or indeed any kind of strategic collaboration among voters depends in part on what happens in the absence of such collaboration, i.e., on the nature of individualistic voting. Thus it is in order briefly to summarize certain propositions concerning sincere and sophisticated voting decisions.

We say that voting complies with the *Condorcet Criterion* if the voting decision always belongs to the Condorcet set. The criterion may be applied to voting on the whole set of bills and thus refer to the Condorcet set of outcomes (we abbreviate this CCO), or—provided that preferences are separable—to voting on individual bills and thus refer to the Condorcet set of positions (we abbreviate this CCP). By the Separability Theorem, if voting complies with CCP, it also complies with CCO.

We also say that voting complies with the *Weak Condorcet Criterion* if, given that a Condorcet outcome (or position) exists, it always is (or is included in) the voting decision. (We abbreviate this WCCO and WCCP, as applied to outcomes and positions respectively.) By the Separability Corollary, if voting complies with WCCP, it also complies with WCCO. And clearly if voting complies with CCP, it also complies with WCCP; and if voting complies with CCO, it also complies with WCCO.

We now state three separate sufficient conditions for individualistic voting to comply with CCO (and thus also WCCO[8]).

CONDITION I. Preferences are separable and all bills are dichotomous.

Ferejohn (1975, p. 6) demonstrates that, under the stated condition, the unique sophisticated voting strategy of each voter is his sincere strategy. Sincere voting obviously complies with CCP. By the Separability Theorem, it also complies with CCO.

CONDITION II. Preferences are separable and amendment procedure is in use.

[7]It can be shown (Niemi, et. al. (1974), pp. 10-12; McKelvey and Niemi (1976), pp. 24-25) that the sets of sophisticated voting situations identified by the two methods are not always identical. But, given strong preferences and binary procedure, both methods identify a unique sophisticated voting decision, and it is "clear" that they identify the same decision, though no satisfactory formal proof of this point yet exists. See Miller (1977) for extended use of the "tree method" in analyzing sophisticated voting decisions.

[8]Every "ordinary" voting process that complies with WCCO in fact complies with CCO as well. However, this is not logically required, and voting processes can be devised that comply with WCCO but not CCO.

By Proposition 1′ in Miller (1977), sincere voting complies with CCP[9]; by the Separability Theorem, it also complies with CCO. By Proposition 8′ in Miller (1977), or Corollary 2 in McKelvey and Niemi (1976), sophisticated voting complies with CCO.[10]

CONDITION III. Voting is sophisticated and binary procedure is in use.

Again this follows from Proposition 8′ in Miller (1977), or Corollary 2 in McKelvey and Niemi (1976).[11]

V. VOTER COLLABORATION

We said in the introductory section that it is generally consistent with the literature to say that a logrolling or vote trading situation exists when the individualistic voting situation is "unstable." We also said that this general formulation leaves two key points to clarify – the nature of an individualistic voting situation, and the sense in which it may be "unstable." We have dealt with the first point and now turn to the second.

Following Farquharson (1969, p. 51), a voting situation $s = (s_1, \ldots, s_n)$, belonging to outcome y, is *vulnerable* to a set S of voters if there is another situation $t = (t_1, \ldots, t_n)$, belonging to outcome x, such that (i) $t_j = s_j$ for all voters j not in S and (ii) all voters in S prefer x to y. In words, a situation belonging to outcome y is vulnerable to a set S of voters if (some of) the voters in S can change their strategy selections in such a way that, *provided that the voters not in S do not change their strategy selections*, the resulting situation belongs to an outcome x they all prefer to y. In this case we say that situation s is vulnerable with respect to outcome x.

Note that, in defining vulnerability, we require that $t_j = s_j$ for all voters j not in S, but we do not require that $t_i \neq s_i$ for all voters i in S. Thus we can partition the members of S into two subsets S_1 and S_2: those who are *active*, i.e., for whom $t_i \neq s_i$, and those who are *passive*, i.e., for whom $t_i = s_i$, respectively. Of course, the situation is vulnerable (with respect to x) to the set S_1 alone, and indeed it is vulnerable (with respect to x) to some proper subset S_1' of S_1 if some members of S_1 are *inessential*, i.e., if there is some situation $t' = (t_1', \ldots, t_n')$, also belonging to x, such that $t_j' = t_j$ for all j not in S_1'. And, to look in the other direction, the most inclusive set to which s is vulnerable with respect to x is the set of all voters who prefer outcome x to outcome y (to which s belongs). Henceforth, when we say a situation is vulnerable to a set S, it is to be understood that S is this most inclusive set.

[9] Black (1958), p. 43, shows that sincere voting complies with WCCP.

[10] It is also true, by virtue of Proposition 8 in Miller (1977) in conjunction with a fairly straightforward modification of Ferejohn's (1975, p. 6) argument to cover non-dichotomous bills and the Separability Theorem, that sophisticated voting complies with CCP.

[11] If in addition preferences are separable, the comments made in the previous footnote apply here as well.

In sum, "instability" may be interpreted in terms of vulnerability. Since our concern is with overt and consequential collaboration among two or more voters, our concern is with an individualistic voting situation vulnerable to some set S of voters including at least two active and essential members.

A situation that is not vulnerable to any one-voter set is an *individual* (or *Nash*) *equilibrium*. A sincere voting situation may fail to be an individual equilibrium; a sophisticated voting situation (under binary procedure) is always an individual equilibrium but may be vulnerable to larger sets of voters. A situation not vulnerable to any set of voters is a *collective* (or *strong*) *equilibrium*. (Cf. Farquharson, 1969, pp. 24-25, 51-53, 75.)

In considering the vulnerability of situations to sets of two or more voters, we are clearly envisaging some measure of cooperation among voters. But if full cooperation is possible—that is, if players can enter into binding agreements—equilibrium conditions are no longer critical and we can focus directly on voting outcomes. A *coalition* is a set of voters who, through "pre-play" communication and binding agreements, can concert their strategy selections. A coalition S is *decisive* for outcome x if it has the power to impose x as the voting decision—more precisely, if the members of S can concert their strategy selections in such a way that, *whatever the strategy selections of the voters not in S*, the resulting voting situation belongs to outcome x. Outcome x *dominates* outcome y if there is some coalition S that is decisive for x and all of whose members prefer x to y. In this case we say x dominates y through coalition S.

In a majoritarian voting body, a coalition is decisive for any outcome if and only if it includes a majority of voters. The relationship of domination is then equivalent to majority preference, a Condorcet outcome is an undominated outcome, and the Condorcet set is the minimal non-empty set of outcomes that dominate every outcome outside of the set.

In the fully cooperative sense, therefore, the interpretation of "instability" is very simple. In this sense, our concern is with an individualistic voting situation that belongs to a dominated outcome, i.e., any outcome other than a Condorcet outcome.

What is the formal relationship between domination and vulnerability? First, note that domination pertains to outcomes and vulnerability to situations. It is clear that, if outcome x dominates outcome y through coalition S, any situation belonging to y is vulnerable to the same set S with respect to x; as a corollary, a situation is a collective equilibrium only if it belongs to an undominated outcome. However, if a situation belonging to y is vulnerable to a set S of voters with respect to x, x dominates y only if S happens also to be a decisive coalition. In sum, "instability" in the domination sense implies "instability" in the vulnerability sense but not vice versa.

What is the substantive difference between these two notions of "instability"? The critical difference is that the definition of domination (through a coalition S) makes no assumptions concerning the actions (i.e., strategy selections) of the voters not in S, while the definition of vulnerability (to a set S) is based on the assumption that the voters not in S are inert (i.e., maintain their present strategy selections). In other words, domination results from the kind of "decisive" strategic collaboration we previously identified with "logrolling," while vulnerability results from the kind of "marginal" strategic collaboration we previously identified with "vote trading." We are now in a position, therefore, to offer formal characterizations of these two phenomena.

VI. LOGROLLING AND VOTE TRADING SITUATIONS

We say that a *logrolling situation* exists when the individualistic voting situation is subject to "decisive" strategic collaboration–formally, when the individualistic voting situation belongs to a dominated outcome.

We say that a *vote trading situation* exists when the individualistic voting situation is subject to "marginal" strategic collaboration–formally, when the individualistic voting situation is vulnerable to a set of voters at least two of whom are active and essential.

It is useful to make further distinctions concerning vote trading situations. We say that a *pairwise vote trading situation* exists when the individualistic voting situation is vulnerable to a set of voters exactly two of whom are active and essential. We say that a *majority supported vote trading situation* exists when the individualistic voting situation is vulnerable to a set including a majority of voters at least two of whom are active and essential. And we say a *minority supported vote trading situation* exists when a vote trading situation exists that is not majority supported.[12]

Note that a majority supported vote trading situation is defined in terms of the size of the whole set S (i.e., all voters who benefit from the strategic collaboration), not the size of the set S_1 (i.e., those voters who actively participate in the strategic collaboration), provided the latter includes at least two voters who are essential. Thus, for example, a pairwise vote trading situation may be either majority or minority supported. There is, of course, one exception to this last statement: in a three-member voting body, any vote trading situation must be majority supported; clearly a minority supported vote trading situation can exist only in a voting body with five or more members.

Let us relate to these definitions the plausible and precise "conditions for

[12]The definitions of logrolling and vote trading situations offered here imply that such situations may exist even if the voting body is considering only a single (non-dichotomous) bill. They likewise imply, in the general case, that logrolling or vote trading may take place within (non-dichotomous) bills, and not just across bills. Some authors exclude such possibilities from their definitions; see, for example, Bernholz (1975), pp. 961-962. Since the definition of a bill (or issue) is theoretically rather arbitrary, I prefer to accept this possibility. Of course, if all bills (issues) are dichotomous (as many assume), logrolling or vote trading must take place across bills, if it takes place at all.

vote trading" stated by Riker and Brams (1973, pp. 1237-1239).[13] As summarized by Koehler (1975a, p. 954), there must be two "traders" (voters) and at least two "motions" (dichotomous bills) and:

(1) ". . . the traders must be on opposite sides on two motions."
(2) Each trader must be in a majority on one motion and a minority on the other.
(3) Each trader's salience must be higher on his minority motion than his majority motion.[14]
(4) Each trader must be pivotal in his majority.

This statement of conditions entails two important assumptions: first (as noted) the bills ("motions") are dichotomous and, second, that preferences are separable (for otherwise one cannot speak of majority and minority preference on particular issues[15]). Thus the individualistic voting decision includes the Condorcet position on every "motion" and is clear that, given the stated conditions and relative to the individualistic voting situation, if both "traders" change their strategy selections so that each votes against his "majority motion," and provided the remaining voters do not change their strategy selections, the resulting situation belongs to and outcome they both prefer, i.e., the individualistic voting situation is vulnerable to the pair of "traders." It is also clear that both "traders" must act in order to accomplish this change of decisions, i.e., both are active and essential. Accordingly, the Riker-Brams-Koehler conditions for vote trading imply a pairwise vote trading situation.

But a vote trading situation may exist in the absence of the Riker-Brams-Koehler vote trading conditions, for the former still exists if the individualistic voting situation is vulnerable but only to a set of voters including more than two active and essential members, either because individual voters are not "pivotal" in their majorities (see Example 1 in the Appendix) or because only a "trade" encompassing three or more votes (e.g., dichotomous bills) can generate the necessary support (see, for example, Schwartz, 1975b, p. 103). And, if preferences are not separable, even a pairwise vote trading situation can exist in the absence of the Riker-Brams-Koehler conditions for vote trading. Thus, in Example 2 in the Appendix, a pairwise vote trading situation exists even though the active "traders" are on the same side on both bills in the individualistic (sincere) voting situation.[16]

Finally, we should note that the Riker-Brams-Koehler conditions leave entirely open the important matter of whether the pairwise "trade" is majority or minority supported.

[13] Rather similar conditions are suggested by Haefele (1970), p. 78, and (1971), p. 356, and by Ferejohn (1974), p. 9.

[14] That is, each "trader" must prefer to get his way on his "minority motion" only rather than on his "majority motion" only. Cf. Enelow (1976), pp.8-9.

[15] More precisely, one cannot define the sets S(x) and -S(x); see Riker and Brams (1973), p. 1237.

[16] One might, I suppose, use this example instead to support the assumption (implicit in many discussions) that the concept of "vote trading" is appropriate only when preferences are separable.

What now is the relationship between a logrolling situation and the various kinds of vote trading situations? First, a vote trading situation does not in general imply a logrolling situation. This follows from the previous discussion concerning the relationship between vulnerability and domination, and is illustrated by Example 3 in the Appendix. However, a majority supported voting trading situation clearly does imply a logrolling situation, for the set S to which the individualistic voting situation is vulnerable is also a decisive coalition.

A logrolling situation "almost implies" a vote trading situation–precisely, a logrolling situation does imply that the individualistic voting situation is vulnerable but, if that situation fails to be an individual equilibrium, it may fail to be vulnerable to any set of voters at least two of whom are active and essential. (See Example 4 in the Appendix.) But, given conditions that assure that the individualistic voting situation is an individual equilibrium (e.g., Conditions I or III), a logrolling situation does imply a vote trading situation–and specifically a majority supported vote trading situation. A logrolling situation, however, does not imply a pairwise vote trading situation (or the Riker-Brams-Koehler conditions for vote trading). This fairly obvious point is illustrated by Example 1 in thyAppendix.

VII. LOGROLLING, VOTE TRADING, AND THE PARADOX OF VOTING

We are now prepared to address the specific question on which this article is focused: the logical relationship between logrolling or vote trading and the paradox of voting. Actually, with the preliminaries out of the way, the answers become quite obvious. In the first place, it is clear that we must consider logrolling and vote trading spearately, as they are distinct phenomena.

By definition, a logrolling situation exists if the individualistic voting decision is dominated. Thus, if individualistic voting complies with CCO (or WCCO), a logrolling situation implies that there is no Condorcet outcome and thus that there is a paradox of voting–specifically what David Klahr (1966, p. 385) calls a Type 2 paradox. (That is, several outcomes belong to the Condorcet set V^*; given CCO, the individualistic decision belongs to V^*, but given only WCCO, it may not.) Putting the matter the other way, if there is no paradox, i.e., if a Condorcet outcome v^* exists, CCO (or WCCO) guarantees that v^* is the individualistic decision; thus that decision is not dominated, and a logrolling situation does not exist.

Three separate, and jointly quite broad, conditions for individualistic voting to comply with CCO (and thus WCCO) were presented in Section 4. Under these conditions, a logrolling situation can exist only when preferences over outcomes are such as to produce a cyclical majority. This is the basic result that contradicts the earlier Tullock-Coleman suggestion.

However, if individualistic voting does not comply with CCO or WCCO–for example, if preferences are not separable and voting is sincere–a logrolling situation can exist even if majority preference is fully transitive over outcomes. This is illustrated by Examples 2 and 4 in the Appendix.

Does a paradox of voting imply a logrolling situation? Most generally: no, if the paradox is—again following Klahr (1966, p. 385)—Type 1, i.e, if a Condorcet outcome exists but there is a cycle including other outcomes. (See Example 5 in the Appendix.) However, a Type 2 paradox implies (obviously) a logrolling situation.

In sum, under rather broad (but not all) conditions a logrolling situation and a Type 2 paradox of voting are equivalent.

What then of the relationship between a vote trading situation and the paradox of voting? A majority supported vote trading situation, of course, implies a logrolling situation and thus, if individualistic voting complies with CCO (or WCCO), a paradox of voting. But, under the same conditions, a (minority supported) vote trading situation can exist in the absence of a logrolling situation and thus also in the absence of a paradox of voting. This is illustrated by Example 3 in the Appendix.

Conversely, a Type 2 paradox of voting can exist in the absence of a pairwise (or other minority supported) vote trading situation, and thus also in the absence of the Riker-Brams-Koehler voting trading conditions, as Example 1 in the Appendix illustrates. However, if a Type 2 paradox of voting exists, so that the individualistic voting situation must be dominated, that voting situation cannot be a collective equilibrium; then, at least under conditions that assure that the individualistic voting situation is an individual equilibrium, some kind of vote trading situation exists.

In sum, the logical connections between vote trading situations and the paradox of voting are generally tenuous andindirect.

VIII. DISCUSSION

It is probably useful to do two specific things in concluding this article. The first is simply to summarize the lines of reasoning that have been presented. This can be accomplished most conveniently and concisely by means of the implication digraph displayed in Figure 1. The digraph should be self-explanatory. It needs only to be remarked that, in order to enhance readability, the digraph has been pared down, so that only "direct" implications are shown and the "indirect" implications that follow from the transitivity of the implication relation are not shown.

The second useful thing to do is briefly to review selected other works on logrolling and vote trading to indicate how they relate to each other and to the present discussion. The summaries that follow treat only those portions of the works relevant to our present concerns and, for the most part, have been translated into the terminology introduced here.

Dahl (1956, p. 128). Dahl assumes (implicitly) separable preferences and considers dichotomous "issues" (bills). By example, he demonstrates that the "platform" (outcome) a*b* . . . m* may be dominated, i.e., that a logrolling (he calls it a "minorities rule") situation may exist. Though aware (pp. 42-44n) of the phenomenon of the paradox of voting, Dahl does not recognize that majority preference over platforms must be cyclical if a*b* . . . m* is dominated.

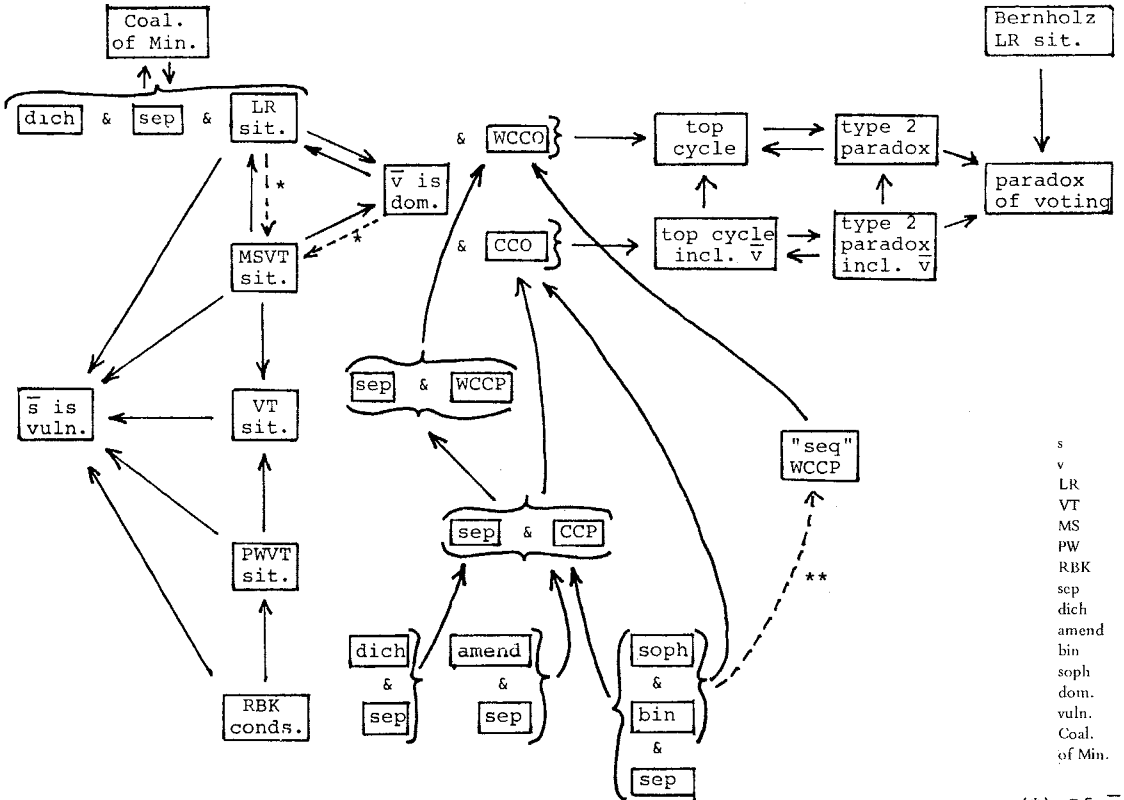


FIGURE 1

KEY TO FIGURE 1

s	individualistic voting situating
v	individualistic voting decision
LR	logrolling
VT	vote trading
MS	majority supported
PW	pairwise
RBK	Riker-Brams-Koehler
sep	separable preferences
dich	dichotomous bills
amend	amendment procedure
bin	binary procedure
soph	sophisticated voting
dom.	dominated
vuln.	vulnerable
Coal. of Min.	coalition of minorities

(*) If $\bar{s}$ is an individual equilibrium

(**) Conjecture

Downs (1957, pp. 55ff). Downs assumes additive cardinal utility, which implies separability, and initially considers dichotomous issues. First, like Dahl, he demonstrates that the platform $a^*b^* \ldots m^*$ may be dominated through a "coalition of minorities" (a common and, under the stated conditions, appropriate term—I think more appropriate than Dahl's "minorities rule"). Second, he drops the requirement that issues be dichotomous and demonstrates that if even one issue fails to have a Condorcet position (is subject to the "Arrow problem") there is no Condorcet platform, i.e., he demonstrates the Separability Corollary. As I read Downs (some read him differently), he nowhere explicitly states that an effective coalition of minorities always implies that the set of platforms is subject to the "Arrow problem." However, he implicitly recognizes this relationship by recognizing (pp. 57-58) its strategic consequence, i.e., if the platform $a^*b^* \ldots m^*$ can be defeated, whatever platform the incumbant party commits itself to, the opposition can always win (a consequence of the Separability Theorem).

Park (1967). Park assumes additive "payoffs," which implies separability, and considers dichotomous "measures." He speaks of "vote trading," but it is clear from the statements of his Definition 3 and Theorem 3 that his concern is with the kind of "decisive" strategic collaboration that we call logrolling. He demonstrates (in effect) that, under the stated conditions, individualistic voting complies with CCP (his Theorem 1), and thus CCO (his Theorem 2), in which case a logrolling situation implies a Type 2 paradox (his Theorem 3). Though brief, his presentation appears to be more complicated than necessary.

Hillinger (1971). Hillinger assumes (implicitly) separable preferences and constructs two examples involving dichotomous issues. By the first example, he shows that platform a_1b_1 (= a^*b^*) may be dominated by platform a_2b_2, and he notes that this example "incidentally illustrates the voting paradox. . . ." By the second example, he shows that the platform $a_1b_1c_1$ (= $a^*b^*c^*$) may be unanimously dispreferred to (and thus dominated by) $a_2b_2c_2$. He incorrectly notes that "this is an example in which social ½i.e., majority] preferences are transitive. . ." (if this were true, the Separability Theorem would be contradicted); he falls into this error because he fails to examine majority preference over the whole set of platforms.

Kadane (1972). Kadane assumes separable preferences. In the major part of his discussion, but not in the formal proofs in his appendix, he assumes that the separable preferences are single-peaked over positions in each issue, assuring a Condorcet position for each. As stated and proved in his Appendix, his Theorem 1 is precisely the Separability Theorem; as stated in the text, his Theorem 1 states that $a^*b^* \ldots m^*$ belongs to V*--a special case of the Separability Theorem.

Oppenheimer (1972). Oppenheimer assumes separable ("independent") preferences and considers dichotomous ("two-sided") issues. He argues that an effective "coalition of minorities" (an appropriate substitute term for a logrolling situation under the stated conditions) implies a paradox of voting, and vice versa. Under the stated conditions, the forward implication holds; the reverse implication holds only if a paradox of voting is understood to be Type 2 (see Example 5 in the

Appendix). Oppenheimer's discussion focuses on sets of voters, not on the structure of majority preference, and accordingly is long and rather tortuous.

Bernholz (1973). Bernholz assumes separable preferences and considers just two dichotomous issues. He defines a "logrolling situation" exactly as I have,[17] and shows that it implies, under the stated conditions, an "Arrow paradox." His discussion, like Oppenheimer's, focuses on sets of voters, not the structure of majority preference.

Miller (1975). In this response to Bernholz (1973), I focused directly on the structure of majority preference and (in effect) proved the Separability Corollary.

Bernholz (1974). At first glance, this appears simply to be a generalization of Bernholz (1973) to cover any number of non-dichotomous issues. But in fact Bernholz here defines a logrolling situation somewhat differently—simply as a condition on preferences, independent of the individualistic (or any other) voting situation.[18] The definition applies only when preferences are separable. Roughly, Bernholz says that a logrolling situation exists if there are a pair of positions a_g and b_j and another pair a_h and b_k such that (i) a_g is majority preferred to a_h, (ii) b_j is majority preferred to b_k, and (iii) any outcome including both a_h and b_k is majority preferred to the otherwise equivalent outcome including both a_g and b_j. A logrolling situation in this sense can exist in the absence of a logrolling situation as defined here, and also in the absence of a Type 2 paradox of voting. (See Example 6 in the Appendix.) However, as Bernholz demonstrates, a logrolling situation in this sense implies a paradox of some type.[19] Since a logrolling situation as defined here can exist in the absence of any tpe of paradox (if individualistic voting fails to comply with CCO or WCCO), it follows that the two types of logrolling situations are in general entirely independent.

Ferejohn (1974, pp. 7ff). Ferejohn shows that a Type 2 paradox of voting does not imply a pairwise vote trading situation (his Theorem 1, demonstrated by the example from which Example 1 in the Appendix is adapted).

Koehler (1975a). Koehler shows that, in a three-member voting body considering two dichotomous bills, the Riker-Brams conditions for vote trading (which entail separability) imply a paradox of voting, and vice versa. The strong suggestion in Koehler's discussion (and in the title of the article) that this equivalence is general can, of course, be readily refuted—the forward implication by Example 3,

[17] "To make logrolling profitable to participating members, a complex alternative [i.e., outcome] dominating . . ." the outcome ". . . if issues were voted on separately and without binding agreements among voters" must exist. Bernholz (1973), p. 89.

[18] I am indebted to James Enelow for pointing this out to me and also for constructing an example similar to Example 6 in the Appendix, referred to just below.

[19] A proof may be readily sketched out. Suppose for the moment that there are just two bills, A and B, and that the logrolling situation described in the text exists. It is clear that we may follow the labeling convention presented in Section 2 in such a way that $g < h$ and $j < k$. Accordingly, it is also clear that there is a "path" of majority preference from $a_g b_j$ to $a_h b_k$. But $a_h b_k$ is majority preferred to $a_g b_j$, so there is a cycle. If there are additional bills, repeat the argument with any fixed complement of the pair A and B.

the reverse implication by Example 1, in the Appendix.

Bernholz (1975). Bernholz presents several counter-examples to Koehler's implied claim of general equivalence—surprisingly, none similar to Examples 1 and 3 in the Appendix. His first example is intended to show that "... certain logrolling situations do not imply the paradox of voting." It is reproduced here as Example 4 in the Appendix. (Note that a logrolling situation in the sense of Bernholz (1974) does not exist in this example and cannot, since preferences are not separable. Note also that no logrolling situation as defined here exists in this example if individualistic voting is sophisticated; therefore, Bernholz is not quite right in saying that "... to prove that logrolling implies the paradox, it has been necessary to assume the absence of a certain kind of complementarity among issues," i.e., to assume separable preferences, for Condition III will suffice.) In his subsequent discussion, Bernholz speaks of an "exchange of votes" between voters 2 and 3. But in fact 3 is an inessential participant; the sincere voting situation is vulnerable to {2} alone. (When 2 changes from his sincere strategy, and 1 and 3 continue with theirs, 1's voting choice on issue B—as prescribed by his sincere strategy—changes from b_2 to b_1.[20]) Bernholz's second example simply shows that more than three outcomes can be included in a cycle. His third example supports the argument that, given a single non-dichotomous issue, a (Type 2) paradox may exist yet "... substantive logrolling is obviously not possible..."; cf. footnote 12.

Koehler (1975b). In this "Rejoinder," Koehler cites the counter-example (Example 1 in the Appendix) to his implied claim of general equivalence due to Ferejohn (1974, p. 10) and suggests that, "if the restriction to pairwise trades is lifted to allow all mutually beneficial exchanges...," general equivalence is restored, i.e., that a (not necessarily pairwise) vote trading situation implies a (Type 2) paradox, and vice versa. But the forward implication at least of this revised claim still does not hold (Example 3 in the Appendix).

Enelow (1976). In somewhat the way I have done, Enelow distinguishes between "vote trading" and "logrolling" situations. However, he defines a vote trading situation in terms of the Riker-Brams-Koehler conditions and accepts the Bernholz (1974) definition of a logrolling situation. He employs examples identical to Examples 1 and 3 in the Appendix to show that there is no necessary connection between a (Riker-Brams-Koehler) vote trading situation and the paradox. In so doing, he shows that Koehler's claim of equivalence holds only in a three-member voting body. He also proves that, if there is an outcome $a^*b^* \ldots m^*$ but no outcome v^*, there is a logrolling situation (in the sense of Bernholz, 1974) involving $a^*b^* \ldots m^*$.

Schwarts (1975a). Schwartz assumes separable preferences (his Assumption I) and (in effect) that individualistic voting complies with CCP (his Assumption II). He then proves (in effect) the Separability Corollary (his Consequence 1) and then the Separability Theorem (his Consequence 2), using his Assumption II to relate these underlying results to "the outcome chosen in the absence of vote trading,"

[20] The comments made previously in footnote 16 may apply here as well.

i.e., the individualistic voting decision. Thus his Consequence 1 states that every outcome other than "the outcome chosen in the absence of vote trading" is dominated. Along the way, he points out that the critical characteristic of "vote trading" is whether it is majority or minority supported; and he demonstrates that, in the former case, it is logically related to a (Type 2) paradox of voting, while in the latter case it is not.

Schwartz (1976). Schwartz first restates his previous Assumption II, now using (in effect) WCCP instead of CCP. With this slightly weaker assumption, he then reproves and interprets his previous Consequence 1. He then introduces a new assumption (his statement (6))—a "Condorcet type" of condition on individualistic voting on positions that is applicable even when preferences are not separable. This condition rests on the fact that bills are voted on sequentially in some definite order; speaking very loosely, we might call it "sequential WCCP." I would conjecture that sophisticated voting under any binary procedure satisfies this condition. In any case, Schwartz shows that the new assumption implies WCCO, and thus suffices to prove a result corresponding to his previous Consequence 1.

A couple of general comments are in order before bringing this review to a close. First, as noted, a number of works assume additive cardinal utility. This is a stronger assumption that is required for the results (cf. Enelow, 1976, pp. 5ff)—a fact recognized by some of the authors themselves (e.g., see Downs, 1957, pp. 65-66n). On the other hand, some other authors use the separability assumption but do not introduce it explicitly, or fail to give it sufficient emphasis, regarding it perhaps as a natural property of preferences. (Actually, it is quite restrictive and in many cases quite implausible.)

Second—and this has not been noted—several of the items in the literature are more general than the present discussion or than my summaries indicate. In particular, Oppenheimer (1972) and Bernholz (1973 and 1974) allow for "decision rules" other than simple majority rule. Schwartz (1975 and 1976) is the most general of all; his discussions are based on the existence of a "collective preference" relation (which reflects, in an unspecified way, individual preferences, voting procedure, the "decision rule," etc.) assumed to have certain properties. (In my summaries I assumed the "collective preference" relation was simple majority preference.) It is for this reason that Schwartz uses "formal" assumptions such as WCCP rather than such "substantive" assumptions, as Conditions I-III, from which other works explicitly or implicitly begin.

I hope that the relationships among logrolling, vote trading, and the paradox of voting—and also among various items in the literature—have been clarified. It may be asked, of course, whether this exercise has been worthwhile, since the paradox of voting has sometimes been dismissed as a mathematical curiousity of little or no political significance. While I have not addressed the question of significance here, Oppenheimer (1972 and 1975) has done so, and I would generally endorse his argument that the relationship does have important substantive political implications. At the same time, however, it is clear the relationship with the paradox of voting is not the only, or necessarily the most important, question bound up with logrolling and vote trading, and that more theoretical attention ought now to turn to some of these other issues.[21] It is my further hope that the game-theoretical framework outlined here may be helpful in addressing and resolving some of these other questions as well.

APPENDIX

Example 1: Five voters, two dichotomous bills, separable preferences.[22]

Preference Orderings

1	2	3	4	5
a_1b_2	a_1b_2	a_2b_1	a_1b_1	a_1b_1
a_2b_2	a_2b_2	a_2b_2	a_1b_2	a_2b_1
a_1b_1	a_1b_1	a_1b_1	a_2b_1	a_1b_2
a_2b_1	a_2b_1	a_1b_2	a_2b_2	a_2b_2

Majority Preference[23]

$$\begin{tikzcd} a_1b_1 \arrow[r] \arrow[dr] & a_1b_2 \arrow[d] \arrow[dl] \\ a_2b_2 \arrow[u] & a_2b_1 \arrow[l] \end{tikzcd}$$

1. The individualistic (sincere or sophisticated) voting decision is a_1b_1.
2. The individualistic voting situation is not vulnerable to any pair of voters.
3. The individualistic voting situation is vulnerable to $\{1,2,3\}$.
4. The individualistic voting decision is dominated by a_2b_2.
5. There is a Type 2 paradox of voting.

Thus:

A. A vote trading situation does not imply the Riker-Brams-Koehler conditions for vote trading.
B. A logrolling situation does not imply a pairwise vote trading situation.
C. A Type 2 paradox of voting does not imply a pairwise vote trading situation (or the Riker-Brams-Koehler conditions).

[21] One of these issues, for example, pertains to the welfare implications of logrolling and vote trading, which in fact is the primary focus of several of the cited works, in particular Buchanan and Tullock (1962), Coleman (1966), Haefele (1970 and 1971), Riker and Brams (1973), and Uslaner and Davis (1975). Conclusions are mixed—again, I think, because of inconsistency in terminology, concepts, and framework. Another task is to examine the dynamics of vote trading, rather than merely identifying conditions under which vote trading "situations" exist. Enelow and Koehler (1976) have made progress here, working generally within the Farquharson game-theoretical framework.

[22] Cf. Ferejohn (1974), p. 10.

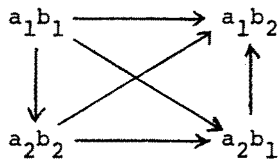
[23] Where "$x \rightarrow y$" means "a majority of voters prefers x to y."

Example 2: Nine voters (divided into three groups of three), two dichotomous bills (A is voted on first), nonseparable preferences.

Preference Orderings

1st group	2nd group	3rd group
a_1b_1	a_2b_1	a_2b_2
a_2b_2	a_1b_1	a_1b_1
a_2b_1	a_2b_2	a_1b_2
a_1b_2	a_1b_2	a_2b_1

Majority Preference



1. The sincere voting decision is a_2b_2.
2. The sincere voting situation is an individual equilibrium vulnerable to any pair of voters in the 2nd group (even though the voters are on the same side on both bills), the resulting situation belonging to a_1b_1.[24]
3. The sincere voting decision is dominated by a_1b_1.
4. There is no paradox of voting (of any type).
5. The sophisticated voting decision is a_1b_1, the undominated outcome.

Thus:

A. If individualistic voting is sincere, a pairwise vote trading situation does not imply the Riker-Brams-Koehler conditions for vote trading.

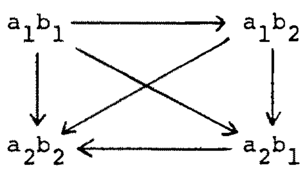
B. If individualistic voting is sincere, a logrolling situation does not imply a paradox of voting (of any type).

Example 3: Five voters, two dichotomous bills, separable preferences.[25]

Preference Orderings

1	2	3	4	5
a_1b_2	a_2b_1	a_1b_2	a_2b_1	a_1b_1
a_2b_2	a_2b_2	a_1b_1	a_1b_1	a_1b_2
a_1b_1	a_1b_1	a_2b_2	a_2b_2	a_2b_1
a_2b_1	a_1b_2	a_2b_1	a_1b_2	a_2b_2

Majority Preference



[24] This is a majority supported vote trading situation. The three voters in the first group and the remaining voter in the second group all benefit from the collaboration, even though they do not (and, in the case of the voters in the first group, cannot) actively participate in the collaboration.

[25] Cf. Schwartz (1975), pp. 12, 14-15.

1. The individualistic (sincere or sophisticated) voting decision is a_1b_1.
2. The individualistic voting situation is vulnerable to the pair $\{1,2\}$, the resulting situation belonging to a_2b_2.[26]
3. The individualistic voting decision is undominated.
4. There is no paradox of voting (of any type).

Thus:

A. Neither a vote trading situation nor the Riker-Brams-Koehler conditions for vote trading imply a logrolling situation.

B. Neither a vote trading situation nor the Riker-Brams-Koehler conditions for vote trading imply a paradox of voting (of any type).

Example 4: Three voters, two dichotomous bills, nonseparable preferences.[27]

Preference Orderings

1	2	3
a_2b_2	a_2b_1	a_1b_2
a_1b_1	a_1b_1	a_1b_1
a_1b_2	a_2b_2	a_2b_2
a_2b_1	a_1b_2	a_2b_1

Majority Preference

$$
\begin{tikzcd}
a_1b_1 \arrow[r] \arrow[d] \arrow[dr] & a_1b_2 \arrow[d] \\
a_2b_2 \arrow[r] \arrow[ur] & a_2b_1
\end{tikzcd}
$$

1. The sincere voting decision is a_2b_2.
2. The sincere voting decision is dominated by a_1b_1.
3. The sincere voting situation fails to be an individual equilibrium (it is vulnerable to $\{2\}$).
4. The sincere voting situation is not vulnerable to any set of voters at least two of whom are active and essential.
5. The sophisticated voting decision is a_1b_1.
6. There is no paradox of voting (of any type).

Thus:

A. If individualistic voting is sincere, a logrolling situation does not imply a vote trading situation.

B. If individualistic voting is sincere, a logrolling situation does not imply a paradox of voting (of any type).

[26] No other voter benefits from this collaboration; thus this is a minority supported vote trading situation.

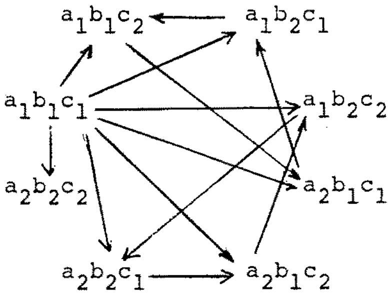
[27] Cf. Bernholz (1975), p. 961.

Example 5: Three voters, three dichotomous bills, separable preferences.[28]

Preference Orderings

1	2	3
$a_2b_1c_1$	$a_1b_1c_2$	$a_1b_2c_1$
$a_1b_1c_1$	$a_1b_1c_1$	$a_1b_1c_1$
$a_2b_2c_1$	$a_2b_1c_2$	$a_1b_2c_2$
$a_1b_2c_1$	$a_2b_1c_1$	$a_1b_1c_2$
$a_2b_1c_2$	$a_1b_2c_2$	$a_2b_2c_1$
$a_1b_1c_2$	$a_1b_2c_1$	$a_2b_1c_1$
$a_2b_2c_2$	$a_2b_2c_2$	$a_2b_2c_2$
$a_1b_2c_2$	$a_2b_2c_1$	$a_2b_1c_2$

(Partial) Majority Preference



1. The individualistic (sincere or sophisticated) voting decision is $a_1b_1c_1$.
2. The individualistic voting decision is undominated.
3. There is a paradox of voting (Type 1).

Thus:

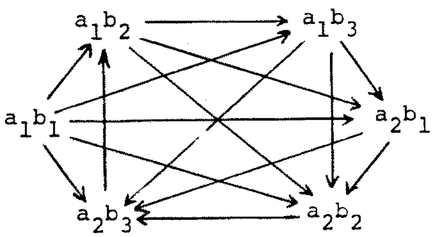
A. A paradox of voting (Type 1) does not imply a logrolling situation.

Example 6: Three voters, two bills (one non-dichotomous), amendment procedure, separable preferences.

Preference Orderings

1	2	3
a_2b_1	a_1b_2	a_1b_3
a_1b_1	a_1b_1	a_1b_1
a_2b_2	a_1b_3	a_2b_3
a_2b_3	a_2b_2	a_1b_2
a_1b_2	a_2b_1	a_2b_1
a_1b_3	a_2b_3	a_2b_2

Majority Preference



1. The individualistic (sincere or sophisticated) voting decision is a_1b_1.
2. The individualistic voting decision is undominated.
3. There is a logrolling situation in the sense of Bernholz (1974), since $a_2b_3 \rightarrow a_1b_2$.
4. There is no Type 2 paradox of voting.
5. There is a Type 1 paradox of voting.

[28]Adapted from Enelow (1976), p. 15.

Thus:

A. A logrolling situation in the sense of Bernholz (1974) does not imply a logrolling situation (as defined in the text).

B. A logrolling situation in the sense of Bernholz (1974) does not imply a Type 2 paradox of voting (even when individualistic voting complies with CCO or WCCO).

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