ELECTION INVERSIONS
UNDER PROPORTIONAL REPRESENTATION

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Abstract

Kurrild-Klitgaard (2013) has shown that proportional representation can produce "election inversions" such that a coalition of parties collectively supported by a majority of voters fails to win a majority of parliamentary seats, and he identifies several empirical examples under the Danish electoral system. However, Kurrild-Klitgaard's examples result from imperfections in its proportional representation system introduced to serve goals other than proportionality. Here I carry Kurrild-Klitgaard's analysis a step further by showing that election inversions can occur even under the purest type of proportional representation — namely, that with (i) a single national constituency, (ii) no explicit seat threshold, and (iii) a highly proportional electoral formula. Inversions result from the unavoidable "whole number problem." I examine recent election data from Israel and the Netherlands and find examples of inversions under their relatively pure PR systems. I also find inversions after recalculating seat allocations without a threshold and on the basis of the most proportional electoral formulas and when the analysis is restricted to seat-winning parties. I then reexamine the Kurrild-Klitgaard's Danish data in the same fashion, as well as the most recent U.S. apportionment of House seats, and find more examples of inversions.

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ELECTION INVERSIONS UNDER PROPORTIONAL REPRESENTATION

1. Introduction

For more than a hundred years political scientists and other commentators have observed that outcomes under "majoritarian" electoral systems — in particular, single-member district plurality systems — do not stack up well when evaluated by proportional criteria. Some twenty years ago, van Deemen (1993) turned this long-standing argument on its head by observing that proportional representation (PR) systems may not do well when evaluated by "majoritarian" (in particular, Condorcet) criteria. Using a combination of election and survey data, Van Deemen and Vurgunst (1998) and Kurrild-Klitgaard (2008) later provided empirical examples of "paradoxical" outcomes under the Dutch and Danish PR systems,

More recently, Kurrild-Klitgaard (2013) has shown that proportional systems do not always comply with a criterion that seems compelling by both proportional and majoritarian standards: more votes should mean no fewer seats. In particular, Kurrild-Klitgaard has shown that proportional representation systems can produce "election inversions" of the sort that awarded a majority of electoral votes (and the U.S. Presidency) to George W. Bush in 2000, despite the fact that more people had voted for Albert Gore. Since proportional representation systems typically produce parliaments in which no party wins a majority of either votes or seats, in this context an election inversion is best defined as a situation in which a coalition of parties collectively supported by a majority of voters fails to win a majority of seats and, conversely, the complementary coalition supported by only a minority of voters wins a majority of seats. Kurrild-Klitgaard identifies several examples of such inversions under the Danish proportional representation system.

However, Kurrild-Klitgaard's examples result from *avoidable* imperfections in the Danish (and many other) proportional representation systems deliberately introduced to serve goals other than proportionality (e.g., providing local representation, deterring excessive party proliferation). Here I carry Kurrild-Klitgaard's analysis a step further by showing that election inversions can and do occur even under the purest types of proportional representation — namely, those that (i) use a single national constituency, (ii) impose no explicit threshold for winning seats, and (iii) employ a highly proportional electoral formula. This is because even the purest proportional representation systems have *unavoidable* imperfections that result from the "whole number problem" — that is, the fact parties must be awarded seats in terms of relatively small whole numbers, while their vote support comes in terms of far larger numbers, making party vote shares essentially continuous quantities. As a result, perfect proportionality can be achieved only in extraordinarily exceptional cases.

Having established this theoretical point, I examine recent election data from Israel and the Netherlands to look for empirical examples of election inversions. Both countries are noted for having relatively pure proportional representation systems in that they use a single nationwide constituency combined with a low threshold. However, both countries use the less proportional D'Hondt formula. I therefore also look for inversions after recalculating seat allocations on the basis of the highly proportional Largest Remainder and Sainte-Laguë formulas and without an explicit seat threshold and when the analysis is restricted to seat-winning parties. I then reexamine Kurrild-Klittgaard's Danish data in these respects, as well as the most recent apportionment of U.S. House seats. Empirical examples of inversions are found in most circumstances.

2. Election Inversions

Following Miller (2012, p. 93), "an *election inversion* occurs when the candidate (or party) that wins the most votes from the nationwide electorate fails to win the most electoral votes (or parliamentary seats) and therefore loses the election." Discussions of the U.S. Electoral College have used such terms as "misfire" (Peirce and Longley, 1981), "wrong winner" (Abbot and Levine, 1991), and "reversal of winners" (Edwards, 2004) to refer to this phenomenon, while the more theoretical literature on voting and social choice has used such varied terms as "compound majority paradox" (Nurmi, 1999), "referendum paradox" (Laffond and Laine, 2000), "majority deficit" (Felsenthal and Machover, 1998), and "representative inconsistency" (Chambers, 2008). As noted above, the U.S. Electoral College produced an inversion in 2000; in like manner, the postwar Labour Government in the U.K. was turned out of office in 1951 by the Conservative Party even though Labour candidates won more votes nationwide than the Conservative candidates. Each of these instances occurred in essentially two-party contexts and entailed what we may call a *partywise* inversion, i.e., party A wins more votes than party B, while B wins more seats than A. Partywise inversions may also occur in a multi-candidate or multi-party context. For example, in the four-candidate 1860 U.S. Presidential election, Douglas (Northern Democrat) received more popular votes than either Brenkinridge (Southern Democrat) or Bell (Constitution Union), but Brenkinridge and Bell each won more electoral votes than Douglas. These inversions were inconsequential, since Lincoln (Republican) won a 39.65% plurality of the popular vote and a majority of electoral votes and the presidency.[1] Inconsequential partywise inversions have also occurred in British parliamentary elections; for example in 1997, the Ulster Unionist Party won 10 seats with less than half the electoral support of the Scottish National Party, which won only 6 seats.

If the number of seats is even (as is true of the present U.S. Electoral College), even a strictly two-party (or two-candidate) contest may produce a tie with respect to seats, even though almost certainly one party (or candidate) wins more votes than the other. Regardless of whether the number of seats is even or odd, if three or more parties win seats, two (or more) parties may win equal numbers of seats, even though almost certainly one wins more votes. We refer to such events as *partial* inversions.[2]

Every proportional representation formula is (weakly) monotonic with respect to parties, i.e., if party A wins more votes than party B, A receives no fewer seats than B. But, as previously noted, in the context of proportional representation it makes sense to focus (as Kurrild-Klitgaard does) on *coalitionwise* inversions, in which a coalition of parties collectively supported by a majority of voters fails to win as many seats as the complementary coalition of all other parties supported by a minority of voters. While partywise inversions cannot occur under an undistricted PR system, we shall show that coalitionwise inversions can occur under even the purest types of PR system.

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[1] However, as noted in Miller (2012), Lincoln would have won a majority of electoral votes even if supporters of the three other candidates had united behind a single ticket, which would have thereby won 60.35% of the popular vote, producing a massive election inversion.

[2] The opposite type of partial inversion — when two parties are tied with respect to votes but one wins more seats than the other — is logically possible but obviously far less likely.

3. Election Inversions Under the Danish Proportional Representation System

Kurrild-Klitgaard's (2013) empirical examples of election inversions under proportional representation occurred under the Danish electoral system, which provides for (i) 135 seats apportioned on the basis of population among about a dozen multi-member districts of varying magnitudes, (ii) 40 national adjustment seats, and (iii) two seats for each of two autonomous overseas territories (Greenland and the Faroe Islands). The latter four seats are guaranteed regardless of population and give the overseas territories disproportionate representation. The 40 national adjustment seats effectively create a single "continental" constituency with 175 seats allocated among political parties on the basis of the (modified) Sainte-Laguë formula with a 2% seat threshold.[3] While the regional districts provide a measure of geographical representation, their existence has almost no impact on the overall allocation of seats to parties.[4] Thus, the Danish system departs from pure PR primarily in two respects: the "federal" status (to use Kurrild-Klitgaard's terminology) of Greenland and the Faroe Islands and the 2% seat threshold. The instances of election inversions that Kurrild-Klitgaard identifies are due to what he calls "federal effects" and "threshold effects," which result from these two imperfections in the Danish PR system.

"Federal effects" are a special case of what Miller (2012, pp. 108ff) calls *apportionment effects* in an electoral system based on states, regions, or other geographically defined districts, and cannot occur in undistricted systems. A districted electoral system is *perfectly apportioned* if the number of votes cast in each district is precisely proportional to the number of seats (or electoral votes) awarded to each district. Given a system with *uniform* districts (each having the same number of seats), perfect apportionment requires that precisely the same number of votes be cast in each district. In practice, perfect apportionment cannot hold because districts inevitably are not precisely equal in population (or eligible voters) and voting turnout inevitably varies somewhat across districts. Given a system with *non-uniform* districts (such as states in the U.S. Electoral College), the same considerations lead to imperfect apportionment. Moreover, non-uniform districts cannot be allocated seats precisely proportional to their population (or eligible voters) for the same reason that parties cannot be allocated seats precisely proportional to their votes, i.e., the "whole number problem." In addition, a districted system may be "federal" in nature in that it deliberately favors some (typically smaller) districts in apportioning seats, e.g., small states in the U.S. Electoral College and the overseas territories in the Danish electoral system. If different parties have differing strengths in different (e.g., smaller vs. larger, low-turnout vs. high-turnout) districts, imperfect apportionment may create partisan bias in the electoral system that in turn can produce election inversions.

Kurrild-Klitgaard (2013, Table 3) provides a hypothetical example of federal effects causing a partywise inversion in a proportional representation system with two districts, the smaller of which has disproportionate representation. He also tracks down a possible empirical example of a Danish

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[3] See Kurrild-Klitgaard (2013) and Elklit (1993). Our concern here is only how seats are allocated among parties, not how individual candidates fill these seats.

[4] However, a party retains whatever seats it wins at the district level, even if this gives it more seats than it is entitled to on the basis of its national vote (Elklit, 1993, pp. 42, 44).

election inversion due to federal effects (Table 4). In 1971, the coalition of all center-right parties gained a bare majority of both votes and seats in "continental" Denmark. But a left-of-center coalition secured the support of three of the four overseas representatives, thereby securing a bare majority of seats in parliament without also securing a majority of the vote nationwide. However, due to peculiarities in the party systems in the two territories, this may not represent a true coalitionwise inversion.

Kurrild-Klitgaard's more compelling example of an election inversion in Denmark was produced by "threshold effects," which constitute a special case of what Miller (2012: pp. 112ff) calls *distribution effects*. In non-PR systems, distribution effects result when one party's vote is more "efficiently" distributed over districts than another's. For example, in a two-party system, party A may win a minority of districts (or states with a minority of the electoral votes) by large margins, while party B wins a majority of districts (or states with a majority of the electoral votes) by small margins, with the result that party B wins a majority of the seats (or electoral votes) with a minority of the votes.

Under proportional representation, the principal source of "inefficiently" distributed votes occurs when several small parties fail to win any seats, whereas a single larger party with the same total vote would win one or more seats.[5] Kurrild-Klitgaard (2013, Table 2) first provides a hypothetical example of such an inversion. A less hypothetical example of an election inversion under PR due to threshold effects might occur in the context of contemporary German politics, where recent elections have typically been contested between rival center-right (CDU/CSU+FDP) and center-left (SPD+Greens) prospective governing coalitions. If the smaller party in one coalition but not the other were to fall below the 5% threshold of the German electoral system, an inversion might occur, as is illustrated in Table 1.[6]

Kurrild-Klitgaard's (2013, Table 5) empirical example pertains to the 1990 election in which, with respect to "continental" Denmark only, the parties of the left collectively won a bare majority

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[5] Even in the absence of an explicit threshold, small parties may fail to win seats by virtue of the normal operation of the electoral formula applied to the numbers of seats available. But such an "implicit threshold" cannot be specified as a particular percent of the vote, because under every PR formula the number of seats a party wins (and whether a small party wins even one seat) depends not only on its vote share but also on how the rest of the vote is distributed among the other parties. It is precisely this fact that implies that coalitionwise, but not pairwise, inversions can occur under pure PR; it also leads to the distinction between "thresholds of representation" and "thresholds of exclusion" (Rae et al., 1971).

[6] In the September 2013 election, the FDP did fall below the 5% threshold, with the result that the prospective CDU/CSU+FDP coalition won more votes but fewer seats than a (hypothetical) SPD+Greens+Left coalition. (However, the Left Party was considered an unacceptable coalition partner, so the election produced a CDU/CSU+SPD "grand coalition.") A provision of the German Federal Elections Act guarantees that a party (but not a coalition) that wins a majority of the votes must receive a majority of seats (Friedrich Pukelsheim, personal communication). A similar provision is found in the Dutch Election Act, but neither law addresses the converse case in which a party that did not win a majority of votes might be allocated a majority of seats, though the Israeli Election Law limits such a party to no more than half the seats (Dan Felsenthal, personal communication).

of the votes but the parties of the center-right won a decisive majority of the seats, while no plausible accounting for votes in the overseas territories could give the center-right party an overall majority of the vote. The inversion resulted because a number of small leftist parties (but only one rightist one) fell below the 2% threshold and failed to win any seats, despite collectively winning 4.4% of the total vote. Like supporters of minor parties under a plurality system, supporters of these parties "wasted" their votes; if they had voted "tactically," they could have concentrated their votes on a single leftist party that would have met the 2% threshold and won an (approximately) proportional share of the seats and given the left coalition a parliamentary majority.[7]

4. Election Inversions under Pure Proportional Representation

We now show that neither federal (or other apportionment) effects nor threshold effects are necessary for election inversions under proportional representation. The claim is that, *provided that there are three or more parties, coalitionwise election inversions can occur under even the purest type of proportional representation, i.e., that with a single nationwide district, no seat threshold, and a highly proportional electoral formula*. The possibility of coalitionwise inversions results from the unavoidable "whole number problem."

At this point, it is in order to note that the word "coalition" is used in two distinct ways in formal political theory. In the more general sense, a *coalition* refers merely to *any set* of players (e.g., voters or parties); this is the standard terminology in the theory of simple games (e.g., Shapley, 1962) and in voting power theory (e.g., Felsenthal and Machover, 1998). In the narrower sense, a *coalition* is a *particular set* of players who enter into an agreement, e.g., to form a coalition government. We use the term here in the more general sense but should note that Kurrild-Klitgaard's examples of coalitionwise inversions in Danish politics pertain to sets of ideologically compatible parties that became (governing) coalitions in the narrower sense.

We now briefly discuss the specifics of several PR formulas.[8] Let us first define party K's *quota* Q_k of seats as the "quantity" (as opposed to "number") of seats that is precisely proportional to its vote share, i.e., $Q_k = S \times V_k / \Sigma V$, where S is the number of seats in parliament, V_k is the number of votes for party K, and ΣV is the total vote for all parties.

In the extraordinarily unlikely event that the quota for every party is a whole number, every party can be awarded seats equal to its quota and a perfectly proportional allocation of seats is possible (i.e., the "whole number problem" does not arise); in this event, all PR formulas produce this

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[7] Thus, while the parties of the center-right failed to win a majority of the vote cast for all parties, they did win a majority of the vote cast for seat-winning parties. Monroe and Rose (2002) show how threshold effects in conjunction with non-uniform districts can produce partisan bias (and therefore potential election inversions) due to the "variance effect" resulting from non-uniform district magnitudes (even if the non-uniform districts are perfectly apportioned). "Reinforced" PR (as in Italy and Greece), which gives the party winning the plurality of votes a seat bonus, can straightforwardly produce election inversions.

[8] This following discussion relies heavily on Balinski and Young (1982), who discuss PR formulas primarily in the context of apportioning House seats among the U.S. states.

perfect apportionment.[9] Otherwise, parties cannot be awarded their precise quota of seats, and different formulas may produce different apportionments.

The most common PR formulas are the Largest Remainder-Hare (LR-H), Sainte-Laguë, and D'Hondt systems. LR-H is a *quota* method: it gives each party its quota rounded down to the nearest whole number of seats and allocates any remaining seats to the parties with the "largest remainders," where a party's *remainder* is the difference between its quota and its quota rounded down. Sainte-Laguë and D'Hondt are *divisor* methods: they divide each party's vote by a common divisor approximately equal to the ratio $\Sigma V / S$ but adjusted so that, when the resulting *quotients* are rounded according to some rule, they add up to the number of seats available. Divisor methods differ according to the rounding rule used: under D'Hondt, quotients are always rounded down; under Sainte-Laguë, quotients are rounded in the normal manner, i.e., up or down to the nearest whole number. The D'Hondt formula favors large parties and often violates "upper quota" by giving some (large) parties more seats than their quotas rounded up; while it never violates "lower quota" by giving any parties fewer seats than their quotas rounded down, the quotas of small parties are very likely to be rounded down. In contrast, LR-H and Sainte-Laguë formulas exhibit no tendency to favor either large or small parties (Schuster et al., 2003). By design, LR-H "stays in quota," i.e., gives every party its quota rounded up or down. Like every divisor method, Sainte-Laguë may violate quota, but it is less likely to do so than any other divisor method and in fact almost always stays in quota. (A number of nations, including Denmark, use a modified Sainte-Laguë formula that is slightly more favorable to large parties and slightly more likely to violate quota.) LR-H and (unmodified) Sainte-Laguë have been reckoned to be the "most proportional" PR formulas, taking account of a variety of proportionality criteria simultaneously (Pennisi, 1998), and they usually allocate seats the same way.

We are now in a position to demonstrate the central claim: given three or more parties, coalitionwise inversions can occur under even the purest types of PR. Since the theoretical claim is only that such inversions are possible, it is sufficient to provide a hypothetical example. Table 2 provides two closely related examples, both involving three parties and 35 seats. To implement the LR-H formula, quotas are calculated for each party. In the first example, the quotas rounded down sum to 34; party A has the largest remainder and gets the extra seat for a majority of 18 out of 35, despite having fewer votes than the coalition of B and C. In the second example, the quotas rounded down sum to 33; parties B and C have the two largest remainders and get the two extra seats for a collective majority of 18 out of 35, despite having fewer votes than party A. The Sainte-Laguë formula produces the same seat allocations and thus the same inversion examples.[10]

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[9] A formula that did not do this could hardly be deemed "proportional"; Balinski and Young (1982, p. 97) call this minimal property *weak proportionality*. But the only way to guarantee perfect proportionality is to provide that $S = \Sigma V$, i.e., implement direct democracy.

[10] To apply the Sainte-Laguë formula, suppose there are 1000 voters, so the ratio $\Sigma V/S \approx 28.571$. In the first example, any divisor between 28.1 and 28.4 gives quotients that equal the seats shown when rounded in the normal manner. In the second example, any divisor between about 28.743 and 28.952 will do. D'Hondt gives the same seat allocation in the first example but gives party A an extra seat at the expense of C in the second example and thus does not produce an inversion.

Once recognized, the logical possibility of coalitionwise election inversions under proportional representation may appear to be unsurprising, since the "whole number problem" is well known and frequently commented on. However, to the best of my (and, evidently, Kurrild-Klitgaard's) knowledge, this possibility has not before been explicitly recognized, with one almost incidental exception. In a footnote, van der Hout and McGann (2009: p. 744) give a hypothetical example (credited to Iain McLean) of an election inversion under PR (which they characterize as a "manufactured majority"). But their example includes a small party that fails to win a seat, and van der Hout and McGann appear to attribute the possibility of inversions exclusively to the presence of one or more small non-seat-winning parties. Table 2 shows that the presence of such parties is not necessary.

It is important to note that PR formulas treat "coalitions" of parties and "fusions" of parties differently, and therefore permit coalitionwise inversions but not partywise inversions. If parties B and C were fused into a single party, no inversions would occur. In the first example, adding the quotas of B and C gives the fused party the largest remainder and the extra seat. In the second example, adding the quotas of B and C gives the fused party 17 seats at the outset, but A now has the largest remainder and wins the extra seat. Sainte-Laguë (and D'Hondt) produce the same seat allocations and the same non-inversions. This illustrates the point asserted earlier that, if party A wins more votes than party B, every PR formula awards A at least as many seats as B. It also illustrates why the central theoretical claim requires three or more parties.

Clearly the vote profiles in Table 2 were carefully devised, by giving party A either slightly more or slightly less than half of the vote and "tuning" the division of the residual vote between parties B and C in relation to the total number of seats. Given only three parties, a coalitionwise inversion clearly is possible only if the leading party gets slightly more or less than 50% of the vote. However, we shall see that, if there are relatively many parties with varying levels of vote support, inversion possibilities may become more varied and probable.

While the logical possibility of election inversions under (pure) proportional representation may be deemed unsurprising, what is perhaps more surprising is that — contrary to my initial expectation was that it would be difficult if not impossible to find empirical examples — this logical possibility is quite frequently exemplified in actual election data. The next section considers how to search efficiently for the phenomenon in electoral data and the following section shows that the phenomenon does in fact appear quite regularly.

5. Finding Inversion Possibilities

In the almost certain event that parties cannot be awarded their precise quota of seats, even under the purest PR system some parties get a (fractionally) greater "quantity" of seats than their quota and other a lesser "quantity." Let us call the difference between the number of seats a party wins (under a given formula) and its quota the party's (positive or negative) *seat differential*. As an accounting identity, the seat differentials of all parties add up to zero.

Given a profile of party votes and seats, we can search for possible coalitionwise inversions by calculating seat differentials and then looking for coalitions that control a small majority of seats

and are composed of parties all or most of which have positive seat differentials, and which therefore are likely to have the support of less than a majority of voters. This may not be possible, because election inversions under proportional representation are close-run things involving complementary coalitions each with very close to half of the votes and seats, which many party configurations do not allow. Generally speaking, the likelihood of finding inversions increases as the number of parties and their variability of seat shares increase. Clearly if one party has a majority of both votes and seats, no inversion is possible. If no or few parties win just one or a few seats, it is less likely that we can find coalitions of just the right seat size. The effect of increasing the number of seats *S* is unclear. On the one hand, as *S* increases, PR becomes more proportional, the magnitudes of seat differentials become smaller relative to *S*, and generally the "whole number problem" becomes less significant, suggesting the likelihood of inversions may decrease. On the other hand, as *S* increases, the share of seats won by the largest party can be expected to decrease and number of small seat-winning parties can be expected to increase (Taagepera, 2007, Chapter 8), making it more likely that coalitions of just the right size exist.[11]

Finally, we may note that, as suggested by van der Hout and McGann (2009) and by Kurrild-Klittgaard's (2013) threshold effects example, there is one easy way to construct hypothetical examples of inversion possibilities. This is to allow the number of small non-seat-winning parties to proliferate to the extent that they collectively win a significant proportion of the total vote. It then will be easy to find coalitions of seat-winning parties that control a majority of seats with less than a majority of the total vote but nevertheless control a majority of the vote cast for seat-winning parties actually represented in parliament. Therefore, we further inquire whether we can find empirical examples of election inversions under PR when vote shares are calculated on the basis of seat-winning parties only.

6. Empirical Examples of Election Inversions

We first examine the most recent election data from Israel and the Netherlands to look for empirical examples of election inversions under proportional representation. Both countries are noted for having relatively pure PR systems, particularly by having a single nationwide constituency combined with relatively low seat thresholds.[12] However, both countries use the less proportional D'Hondt formula. We therefore also look for inversions after recalculating seat allocations without a threshold and on the basis of the more proportional LR-H and Sainte-Laguë formulas. We also examine whether inversions occur when vote shares are calculated on the basis of seat-winning parties only.

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[11] However, at the logical lower limit of a $S = 1$, every PR formula is equivalent to plurality rule, so every configuration in which no party is supported by a majority of votes entails a coalitionwise inversion.

[12] The Netherlands has districts, but they do not affect the overall allocation of seats to parties; its threshold is 0.67%. The seat threshold in Israel, originally 1%, was increased to 1.5% in 1992, to 2% in 2003, and will be 3.25% in future elections.

Table 3 shows the results of the 2013 election in Israel. To say Israel has a multiparty system is an understatement. However, only twelve parties met the 2% threshold and were awarded seats. Since the Israeli parliament has an even number of seats (120), we might not expect to find examples of (full) inversions, since a coalition with electoral support under 50% must control at least two more seats than its complement (61 vs. 59). However, it turns out to be easy enough to find examples of inversions, as shown in Table 3. Coalitions whose members are marked with an "X" control at least 61 (and as many as 65) seats but are nevertheless supported by less than 50% of the voters. When we restrict our attention to seat-winning parties, seat allocations and coalition possibilities remain as before, but the vote shares of seat-winning parties increase, as the votes cast for non-seat-winning parties are now excluded from the total vote. As a result, the vote support for every coalition also increases, but two of the coalitions still collected less than half of the total vote cast for seat-winning parties.

Next we "purify" the seat allocation by eliminating the seat threshold and replacing D'Hondt with LR-H or Sainte-Laguë (which produce identical seat allocation), as shown in Table 4. The effect is to increase the number of seat-winning parties to 18 and to reduce the average magnitude of the seat differentials (and eliminate, among seat-winning parties, their correlation with party size). Despite proportionality that is as good as possible, we can again identify a number of examples of inversions. Moreover, the effect of better proportionality is that every inversion holds up even when vote shares are calculated on the basis of seat-winning parties only.[13]

Table 5 shows the results of the 2012 election in the Netherlands. Like the Israeli parliament, the Dutch parliament has an even number of seats (150), making (full) inversions more difficult. However, it is again easy enough to find examples of inversions, as shown in Table 5, including one which holds up when the analysis is restricted to seat-winning parties. When we "purify" the seat allocation by eliminating the threshold requirement and replacing D'Hondt with LR-H or Sainte-Laguë (which in this case produce slightly different seat allocations), we can still identify a number of examples of inversions, as shown in Table 6, including one which holds up when vote shares are calculated on the basis of seat-winning parties only.[14]

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[13] None of the coalitions identified in Tables 3 and 4 actually formed the government, but at least one was a plausible governing coalition. While Israeli coalition governments are noted for ideologically odd combinations, clearly neither Hadash nor the National Democratic Assembly would be invited into a government, especially one led by Likud. Similar tables in an earlier version of this paper pertaining to the 2009 election identified a number of inversions based on the actual seat allocations, none of which held up when the analysis was restricted to seat-winning parties, and others based on LR-H/Sainte Laguë allocations, some of which held up when restricted to seat-winning parties. The question arises as to whether an actual Israeli governing coalition controlling a majority of seats has ever been supported by less than half of the total vote. Dan Felsenthal (personal communication) has examined all Israeli elections and discovered that the governing coalition (of Likud plus several small parties) following the 1981 election controlled 62 seats but gained only 48.2% of the total valid vote cast. However, this represented 50.8% of the votes cast for seat-winning parties.

[14] Similar tables in an earlier version of this paper pertaining to the 2009 election identified a number of inversions based on the actual seat allocations (one the actual governing coalition), one of which held up when the analysis was restricted to seat-winning parties, and others based on LR-H/Sainte Laguë allocations, none of which held up when the analysis was restricted to seat-winning parties.

We now reexamine Kurrild-Klittgaard's 1990 Danish example of an inversion due to threshold effects but exclude complications due to possible "federal effects" by considering Denmark's 175 "continental" seats only. Table 7 reproduces his Table 5. The third coalition (that won 91 seats) — the ideologically coherent center-right coalition that actually formed the government — corresponds to Kurrild-Klitgaard's inversion based on threshold effects (though his calculation also includes the non-seat-winning right-of-center Justice Party). We see, however, that at least two other possible coalitionwise inversions exist, though none survives as an inversion with respect to seat-winning parties only. Table 8 shows the seat allocation with the seat threshold eliminated and modified Sainte-Laguë replaced by either LR-H or pure Sainte-Laguë (which produce identical seat allocations). While the previously identified coalitions, including the governing one, lose their seat majorities, we can identify new two examples of coalitionwise inversions. Moreover, removing the threshold requirement and using the most proportional formulas means that all parties are seat-winning except the vanishingly small Humanist Party, with the result that the same coalitions provide examples of inversions with respect to seat-winning parties.[15]

Finally, we examine the most recent apportionment (based on the 2010 census) of the 435 seats in the U.S. House of Representatives among the 50 states. The apportionment of House seats is based on the population of each state and since 1940 has used the Hill-Huntington formula, which is a divisor method that uses "geometric" rounding. As such, it is very similar to Sainte-Laguë but slightly more favorable to small states, though this slight bias effectively disappears with this many seats (Schuster et al., 2003, p. 675).[16] This data is useful for getting a sense of inversion possibilities when the number of seats increases somewhat and the number of (seat-winning) "parties" (i.e., states) increases substantially. State "coalition" possibilities are extraordinarily numerous and cannot be exhaustively searched, but what we can straightforwardly do is to rank states in order of their seat differentials and cumulate their seat and population shares until we approach a majority of 218 House seats. The most favored (with respect to seat differentials) states with 207 seats have 46.25% of the population. Adding the next most favored state (Michigan) produces a "coalition" with 221 seats and 49.46% of the population. We can replace Michigan with the slightly less favored but also less populous Tennessee to create a "coalition" with a bare majority of 218 seats and only 48.31% percent of the population. It is possible but unlikely that there is another even more "efficient" coalition, and certainly there are many additional inversion possibilities.

The LR-H formula (which as the "Hamilton method" has been used to apportion House seats in the past) produces the same apportionment of seats as Hill-Huntington. The Sainte-Laguë formula (which as the "Webster method" has also been used in the past) produces an apportionment that

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[15] It is fair to report that I also examined the results of the most recent 2011 Danish election and found no inversion examples given either the actual or "purified" seat distributions, because all coalitions composed primarily of parties with positive seat differential had substantially more than 88 seats.

[16] In addition, the U.S. Constitution guarantees every state at least one House seat, i.e., the "electoral formula" has a "floor" rather than a "threshold." However, this guarantee has no present effect, as every state is entitled to at least one seat based on its population, present House size, and the normal operation of the Hill-Huntington formula.

differs from Hill-Huntington with respect to only two states. The most favored (with respect to seat differentials) states with 217 seats have 48.56% of the population. Adding the next most favored state (Hawaii) produces a "coalition" with 219 seats and 49.01% of the population. We can replace Hawaii with the slightly less favored but also less populous North Dakota to create a "coalition" with a bare majority of 218 seats and only 48.78% percent of the population. Again, it is possible but unlikely that there is another even more "efficient" coalition, and certainly there are many additional inversion possibilities.

We can likewise examine inversions based on electoral votes, rather than House seats. As mentioned earlier, the apportionment of electoral votes deliberately favors (as a "federal effect") small states by giving each state electoral votes equal to its total representation in Congress, i.e., its House seats plus two (for its equal two seats in the Senate). In addition, the District of Columbia has three electoral votes, for a total of 538. The most favored (39 smallest) states with 255 electoral votes have 40.56% of the population. Adding the next most favored state of Georgia gives a majority of 271 electoral votes with only 43.70% of the population. We can replace Georgia with the slightly less favored but also slightly less populous North Carolina to create a "coalition" with a bare majority of 270 electoral votes and only 43.65% percent of the population. Again, it is possible but unlikely that there is another even more "efficient" coalition, and certainly there are a great many additional inversion possibilities. The unsurprising lesson here is that, while election inversions under more or less pure proportional apportionment are always close-run things, inversions become both more probable and more substantial given major "federal effects."[17]

7. Conclusion

It is well known that plurality systems based on districts can produce election inversions, in which one party or candidate wins a majority of seats or electoral votes even though another party wins more votes. While it might seem that proportional representation systems cannot produce inversions, Kurrild-Klitgaard (2013) shows that Denmark's proportional representation system has produced inversions in which a coalition of parties supported by a minority of votes won a majority of seats. However, Kurrild-Klitgaard's examples rest on imperfections in Denmark's PR system, introduced to serve goals other than proportionality, which can produce inversions through "federal" or "threshold" effects. Moreover, Kurrild-Klitgaard's inversion examples do not hold when vote shares are calculated on the basis of seat-winning parties only.

Here we have shown that coalitionwise election inversions can occur even under the purest type of proportional representation and even with respect to votes cast for seat-winning parties only. They result from the unavoidable "whole-number" effect found in any practical PR system. Moreover, we have identified a number of empirical examples in elections in several countries. Indeed, almost every case examined included some inversion possibilities, suggesting that they are typical, not exceptional, when PR systems produce a sufficiently fragmented party system that a

[17] The U.S. Senate provides the most notorious example. The "coalition" of the 26 smallest states controls a majority of 52 out of 100 seats with only 17.83% of the 2010 census population.

number of barely winning coalitions exist — in practice, this means systems with a sufficiently low effective nationwide threshold (Taagepera, 2007, pp. 247-250).

Of course, many or most barely winning coalitions, in the general sense of sets of parties, are not plausible coalitions, in the narrower sense of sets of ideologically compatible parties that might form a government and, even among coalitions of the latter type, only one can actually form. Thus, realized coalitionwise inversions under proportional representation are undoubtedly rather rare and, like almost all inversions in majoritarian systems, are close-run things and are probably not significant threats to political legitimacy. But proportional representation systems deal only with numbers of votes and seats, not ideological relationships among parties and coalition formation, so it is worth recognizing that they also are unavoidably subject to the inversion phenomenon. It may still be worth considering what institutional mechanisms might be devised to avoid inversions, but that is a subject for future research.

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Table 1. A Hypothetical Election Inversion in Germany Due to the 5% Threshold

Party	Party Votes	Coalition Votes	Party Seats	Coalition Seats
CDU/CSU	46.0%	50.5%	289	289
FDP	4.5%		0	
SPD	42.0%	49.5%	264	311
Greens	7.5%		47	

Table 2. Hypothetical Coalitionwise Election Inversions under Pure Proportional Representation (LR-H or Sainte-Laguë) with 35 Seats

Party	Party Votes	Coalition Votes	Party Quota	Party Seats	Coalition Seats
A	49.7%	49.7%	17.395	18	18
B	29.4%	50.3%	10.290	10	17
C	20.9%		7.315	7	

Party	Party Votes	Coalition Votes	Party Quota	Party Seats	Coalition Seats
A	50.3%	50.3%	17.605	17	17
B	30.4%	49.7%	10.640	11	18
C	19.3%		6.755	7	

Table 3. Israeli 2013 Election: Actual Seat Allocation

Party	*Vote*	*Seats*	*Vote %*	*Quota*	*Diff*	*Coalitions*						
Likud YB	885163	31	23.338	28.006	2.994	X	X	X	X			
Yesh Atid	543458	19	14.329	17.195	1.805	X	X	X		X	X	
Yisrael Labor Party	432118	15	11.393	13.672	1.328	X				X		X
Habayit Hayehudi	345985	12	9.122	10.947	1.053		X		X	X	X	X
Shas	331868	11	8.750	10.500	.500			X	X	X	X	X
United Tora Judaism	195892	7	5.165	6.198	.802				X		X	X
Hatenua	189167	6	4.988	5.985	.015						X	X
Meretz	172403	6	4.546	5.455	.545							X
United Arab List	138450	4	3.650	4.380	-.380							
Hadash	113439	4	2.991	3.589	.411					X	X	X
Nat. Dem. Assembly	97030	3	2.558	3.070	-.070							
Kadima	78974	2	2.082	2.499	-.499						X	
Otzma Leyisrael	66775	0	1.761	2.113	-2.113							
Am Shalem	45690	0	1.205	1.446	-1.446							
Green/Liberal List	43734	0	1.153	1.384	-1.384							
Eretz Hadasha	28080	0	.740	.888	-.888							
Koach Lehaspia	28049	0	.740	.887	-.887							
Hayisraelim	18939	0	.499	.599	-.599							
Green and Young	8117	0	.214	.257	-.257							
Dor Bonei Haaretz	5975	0	.158	.189	-.189							
Chaim Bekavod	3640	0	.096	.115	-.115							
Da-am/Workers Party	3546	0	.094	.112	-.112							
We Are Brothers	2899	0	.076	.092	-.092							
Tzedek Hevrati	2877	0	.076	.091	-.091							
Kulanu Haverim	2176	0	.057	.069	-.069							
The Pirates	2076	0	.055	.066	-.066							
Economics Party	1972	0	.052	.062	-.062							
Mitkademet Lib. Dem.	1352	0	.036	.043	-.043							
Light	1027	0	.027	.032	-.032							
Brit Olam	761	0	.020	.024	-.024							
Hatikva Leshinui	649	0	.017	.021	-.021							
Moreshet Avot	461	0	.012	.015	-.015							
TOTAL	3792742	120	100.000	120.000	.000							
Coalition Seats						**65**	**62**	**61**	**61**	**61**	**61**	**61**
Coalition Vote % (based on total vote)						**49.06**	**46.79**	**46.42**	**46.38**	**46.59**	**47.43**	**46.95**
Coalition Vote % (base on seat-winning parties only)						**52.80**	**50.36**	**49.96**	**49.91**	**50.14**	**51.04**	**50.54**

Source: http://www.bechirot.gov.il/elections19/eng/list/results_eng.aspx

Table 4. Israeli 2009 Election:
Seat Allocations under LR-H/Sainte-Laguë without a Threshold

Party	*Vote*	*Seats*	*Vote %*	*Quota*	*Diff*	*Coalitions*			
Likud Yisrael Beitenu	885163	28	23.338	28.006	-.006	X	X	X	
Yesh Atid	543458	17	14.329	17.195	-.195				
Yisrael Labor Party	432118	14	11.393	13.672	.328	X	X	X	X
Habayit Hayehudi	345985	11	9.122	10.947	.053				X
Shas	331868	11	8.750	10.500	.500	X	X	X	X
United Tora Judaism	195892	6	5.165	6.198	-.198				
Hatenua	189167	6	4.988	5.985	.015				X
Meretz	172403	6	4.546	5.455	.545	X	X		X
United Arab List	138450	4	3.650	4.380	-.380				
Hadash	113439	4	2.991	3.589	.411			X	X
National Dem. Assembly	97030	3	2.558	3.070	-.070				X
Kadima	78974	3	2.082	2.499	.501			X	X
Otzma Leyisrael	66775	2	1.761	2.113	-.113				
Am Shalem	45690	1	1.205	1.446	-.446				
Green Leaf – Liberal List	43734	1	1.153	1.384	-.384				
Eretz Hadasha	28080	1	.740	.888	.112		X		X
Koach Lehaspia	28049	1	.740	.887	.113	X			X
Hayisraelim	18939	1	.499	.599	.401	X	X	X	X
The Green and Young	8117	0	.214	.257	-.257				
Dor Bonei Haaretz	5975	0	.158	.189	-.189				
Chaim Bekavod	3640	0	.096	.115	-.115				
Da-am – Workers Party	3546	0	.093	.112	-.112				
We Are Brothers	2899	0	.076	.092	-.092				
Tzedek Hevrati	2877	0	.076	.091	-.091				
Kulanu Haverim	2176	0	.057	.069	-.069				
The Pirates	2076	0	.055	.066	-.066				
The Economics Party	1972	0	.052	.062	-.062				
Mitkademet Lib. Dem.	1352	0	.036	.043	-.043				
Light	1027	0	.027	.032	-.032				
Brit Olam	761	0	.020	.024	-.024				
Hativa Leshinui	649	0	.017	.021	-.021				
Moreshet Avot	461	0	.012	.015	-.015				
TOTAL	3792742	120	100.000	120.000	.000				
Coalition Seats						**61**	**61**	**61**	**61**
Coalition Vote % (based on total vote)						**49.27**	**49.27**	**49.05**	**48.41**
Coalition Vote % (base on seat-winning parties only)						**49.76**	**49.76**	**49.54**	**48.89**

Table 5: Dutch 2012 Election: Actual Seat Allocation

Party	*Votes*	*Seats*	*Vote%*	*Quota*	*Diff*	*Coalitions*			
People's Party (VVD)	2504948	41	26.580	39.870	1.130	X	X		
Labour Party (PvdA)	2340750	38	24.838	37.256	.744			X	X
Party for Freedom (PVV)	950263	15	10.083	15.125	-.125				X
Socialist Party (SP)	909853	15	9.654	14.482	.518	X	X	X	X
Christian Dem. Appeal (CDA)	801620	13	8.506	12.759	.241	X	X		
Democrats 66	757091	12	8.033	12.050	-.050			X	
Christian Union (CU)	294586	5	3.126	4.689	.311	X		X	X
Green Left	219896	4	2.333	3.500	.500		X	X	
Reformed Pol. Party (SGP)	196780	3	2.088	3.132	-.132		X		X
Party for the Animal (PvdD)	182162	2	1.933	2.899	-.899				
50PLUS	177631	2	1.885	2.827	-.827	X		X	
Pirate Party	30600	0	.325	.487	-.487				
Party for Men and Spirit	18310	0	.194	.291	-.291				
Sovereign Ind. Pioneers (SOPN)	12982	0	.138	.207	-.207				
Party of the Future (PvdT)	8194	0	.087	.130	-.130				
Dem. Pol. Turning Point (DPK)	7363	0	.078	.117	-.117				
Libertarian Party (LP)	4163	0	.044	.066	-.066				
Netherlands Local (NedLok)	2842	0	.030	.045	-.045				
Liberal Dem. Party (LibDem)	2126	0	.023	.034	-.034				
Anti-Europe Party (AEP)	2013	0	.021	.032	-.032				
Political Party NXD	62	0	.001	.001	-.001				
TOTAL	9424235	150	100.000	150.000	.000				
Coalition Seats						**76**	**76**	**76**	**76**
Coalition Vote % (based on total vote)						**49.75**	**49.16**	**49.87**	**49.79**
Coalition Vote % (based on seat-winning parties only)						**50.22**	**49.63**	**50.34**	**50.26**

Source: *Dutch Election Results Since 1918* (www.nlverkiezingen.com).

Table 6. Dutch 2012 Election:
Seat Allocation under LR-H and Sainte-Laguë without a Threshold

		Largest Remainder-Hare					*Sainte-Laguë*			
Party	*Quota*	*Seats*	*Diff*	*Coalitions*			*Seats*	*Diff*	*Coalitions*	
People's Party (VVD)	39.870	40	.130	X	X		40	.130	X	
Labour Party (PvdA)	37.256	37	-.256			X	37	-.256		X
Party for Freedom (PVV)	15.125	15	-.125	X			15	-.125		
Socialist Party (SP)	14.482	14	-.482				15	.518	X	X
Christian Dem. Appeal (CDA)	12.759	13	.241	X	X	X	13	.241	X	
Democrats 66	12.050	12	-.050		X	X	12	-.050		X
Christian Union (CU)	4.689	5	.311				5	.311	X	X
Green Left	3.500	4	.500	X	X	X	4	.500		X
Reformed Pol. Party (SGP)	3.132	3	-.132			X	3	-.132		
Party for the Animal (PvdD)	2.899	3	.101		X	X	3	.101		
50PLUS	2.827	3	.173	X	X	X	3	.173	X	X
Pirate Party	.487	1	.513	X	X	X	0	-.487		
Party for Men and Spirit	.291	0	-.291				0	-.291		
Sovereign Ind. Pioneers (SOPN)	.207	0	-.207				0	-.207		
Party of the Future (PvdT)	.130	0	-.130				0	-.130		
Dem. Pol. Turning Point (DPK)	.117	0	-.117				0	-.117		
Libertarian Party (LP)	.066	0	-.066				0	-.066		
Netherlands Local (NedLok)	.045	0	-.045				0	-.045		
Liberal Dem. Party (LibDem)	.034	0	-.034				0	-.034		
Anti-Europe Party (AEP)	.032	0	-.032				0	-.032		
Political Party NXD	.001	0	-.001				0	-.001		
TOTAL	150.000	150	.000				120	.000		
Coalition Seats				**76**	**76**	**76**			**76**	**76**
Coalition Vote % (total vote)				**49.71**	**49.60**	**49.94**			**48.75**	**49.87**
Coalition Vote % (seat-winning)				**50.02**	**49.90**	**50.25**			**50.22**	**50.34**

Table 7. Danish 1990 Election: Actual Seat Allocation
("Continental" Denmark Only)

Party	*Votes*	*Vote%*	*Quota*	*Seats*	*Diff*	*Coalitions*		
Social Democrats	1221121	37.695	65.967	69	3.033	X	X	
Conservatives	517293	15.969	27.945	30	2.055			X
Liberal Party	511643	15.794	27.640	29	1.360			X
Socialist People's Party	268759	8.296	14.519	15	.481		X	
Progress Party	208484	6.436	11.263	12	.737	X		X
Center-Democrats	165556	5.111	8.944	9	.056			X
Radicals	114888	3.547	6.206	7	.794	X		X
Chirstian People's Party	74174	2.290	4.007	4	-.007		X	X
Common Course	57896	1.787	3.128	0	-3.128			
Unity List	54038	1.668	2.919	0	-2.919			
Greens	27642	.853	1.493	0	-1.493			
Justice Party	17181	.530	.928	0	-.928			
Humanist Party	763	.024	.041	0	-.041			
TOTAL	3239438	100.000	175.000	175	.000			
Coalition Seats						**88**	**88**	**91***
Coalition Vote % (based on total vote)						**47.68**	**48.28**	**49.15**
Coalition Vote % (seat-winning parties only)						**50.11**	**50.75**	**51.66**

*Governing coalition

Source: Kurrild-Klitgaard (2013) and *Danish Parliament* (www.ft.dk).

Table 8. Danish 1990 Election: Seat Allocation under LR-H/Sainte-Laguë without a Threshold ("Continental" Denmark Only)

Party	*Votes*	*Vote%*	*Quota*	*Seats*	*Diff*	*Coalitions*	
Social Democrats	1221121	37.695	65.967	66	.033		X
Conservatives	517293	15.969	27.945	28	.055	X	
Liberal Party	511643	15.794	27.640	28	.360	X	
Socialist People's Party	268759	8.296	14.519	15	.481	X	X
Progress Party	208484	6.436	11.263	11	-.263		
Center-Democrats	165556	5.111	8.944	9	.056	X	
Radicals	114888	3.547	6.206	6	-.206		
Chirstian People's Party	74174	2.290	4.007	4	-.007	X	X
Common Course	57896	1.787	3.128	3	-.128		
Unity List	54038	1.668	2.919	3	.081	X	X
Greens	27642	.853	1.493	1	-.493		
Justice Party	17181	.530	.928	1	.072	X	
Humanist Party	763	.024	.041	0	-.041		
TOTAL	3239438	100.000	175.000	175	.000		
Coalition Seats						**88**	**88**
Coalition Vote % (based on total vote)						**49.66**	**49.95**
Coalition Vote % (seat-winning parties only)						**49.67**	**49.96**