POLI 388 due 03/22/10

PROBLEM SET #2 – STRATEGIC CHOICE IN TWO-PLAYER GAMES

1. Answer the following questions pertaining to the two-player *zero-sum* game depicted in the payoff matrix below. *Briefly explain each of your answers.* (The row Player 1 has four strategies; the column Player 2 has three strategies. The number in each cell is the payoff to Player 1; the payoff to Player 2 is the negative of the number — that is, P1 wants to maximize ands P2 wants to minimize, the payoff.)

max of mins

mins ↓

Player 2

Player 1	c_1	c_2	c_3	
s_1	4	2	3	2
s_2	2	1	3	1
s_3	4	3	3	3
s_4	3	2	4	2
maximums	4	3	4	

s_1 dom by s_3

s_2 dom by s_1, s_3, s_4

Nash equilibria (saddle points)

undominated

"sequentially dominated"

dominant

minimum of maximum

(1) Does this payoff matrix have a Nash equilibrium?

yes (s_3, c_2) [+ s_3, c_3] = have same payoffs

(2) Is this zero-sum game *strictly determined*?

P1 P2 yes

maximin = minimax = 3

(3) Would either player choose to use a mixed strategy?

No, because game is strictly determined

(4) Would the outcome be different if the game were played sequentially, with P1 making the first move?

"Look ahead & reason back" No – because is strictly det. – as first mover, P1 can expect to get only his security level from each strategy, because P2 will always choose his best reply so P1 chooses his maximin strategy s_3 (same as in simultaneous choice game)

(5) Would the outcome be different if the game were played sequentially, with P2 making the first move?

No – same considerations as for (4)

However, if P2 can somehow persuade P1 that P2 is committed to C_3, this will induce P1 to choose S_4 + if P2 then reneges + choose C_2, P2 benefits

(6) Supposing that the players make their strategic choices *sequentially* with Player 1 moving first, could Player 2 *communicate any pre-play message* to Player 1 that might improve the outcome for Player 2.

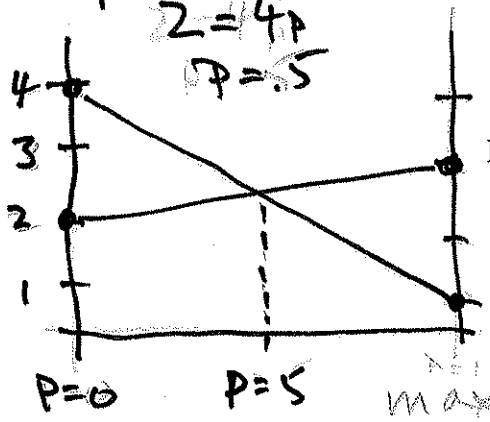
Q is: can P2 benefit by committing in advance to any strategy other than C_2? No, because C_3 is dominant

(7) Supposing that the players make their strategic choices *sequentially* with Player 2 moving first, could Player 1 *communicate any pre-play message* to Player 2 that might improve the outcome for Player 1.

No

3. Answer the following questions pertaining to the two-player *zero-sum* game depicted in the payoff matrix below. *Briefly explain each of your answers.* (The row Player 1 has four strategies; the column Player 2 has three strategies. The number in each cell is the payoff to Player 1; the payoff to Player 2 is the negative of the number — that is, P1 wants to maximize ands P2 wants to minimize, the payoff.)

$4-3p = 2+p$
$2 = 4p$
$p = .5$



dominated

	Player 2			
Player 1	c_1	c_2	c_3	
s_1	6	~~3~~ 0		0
s_2	5	~~0~~ 2		0
s_3	3	2	3	2
s_4	4	4	1	1
maximums	6	4	3	

mins — maximum of minimum

"sequentially dominated"

min of maximums

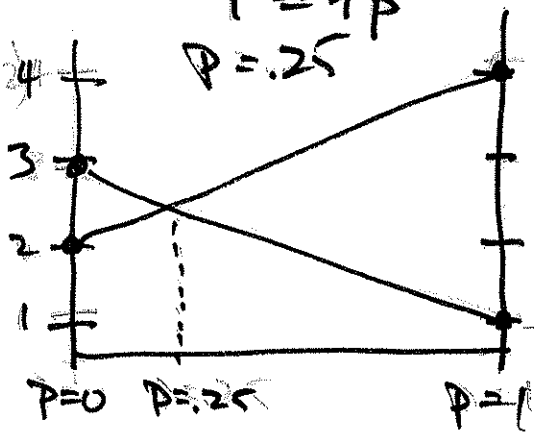
(1) Does this payoff matrix have a Nash equilibrium?

No (not in pure strategies)

$3-2p = 2+2p$
$1 = 4p$
$p = .25$

(2) Is this zero-sum game *strictly determined*?

No maximin (to P1) < minimax (to P2)
$2 < 3$



(3) Would either player choose to use a mixed strategy?

Yes some mixture of S_3 + S_4 for P1 and some mixture of C_2 + C_3 for P2

note 2nd mover advantage in this non-strictly determined zero-sum game

no mixed strategies

(4) Would the outcome be different if the game were played sequentially, with P1 making the first move? Yes, given sequential choices with perfect information, P1 can expect P2 to use his best reply to whatever P1 does, so P1 will choose his maximin strategy (S_3, C_2)

(5) Would the outcome be different if the game were played sequentially, with P2 making the first move?

Yes, by similar considerations (S_3, C_3)

(6) Supposing that the players make their strategic choices *sequentially* with Player 1 moving first, could Player 2 *communicate any pre-play message* to Player 1 that might improve the outcome for Player 2.

Can P2 commit to any strategy to which P1's best reply would be beneficial to P2? No

However, if P2 can persuade P1 that P2 will choose C_2 & then P2 reneges on this "promise" P2 would benefit.

(7) Supposing that the players make their strategic choices *sequentially* with Player 2 moving first, could Player 1 *communicate any pre-play message* to Player 2 that might improve the outcome for Player 1.

No - but P1 can try the same kind of "bait & switch" strategem as P2.

Note that P2's best choice varies between (4) & (5) but P1's does not

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Given that P1's optimal mixed strategy (over S_3 & S_4) is (.5, .5) & P2's optimal mixed strategy (over $C_2 + C_3$) is (.25, .75), we can calculate the (expected) payoff for each player

	C_2 (.25)	C_3 (.75)
S_3 (.5)	.125×2 = .25	.375×3 = 1.125
S_4 (.5)	.125×4 = .5	.375×1 = .375

exp. payoff = .25 + 1.125 + .5 + .375 = 2.25

3. Answer the following questions pertaining to the (variable-sum) game depicted in the payoff matrix. *Then briefly explain each of your answers.* (Each player has just two strategies. The number in lower-left corner of each cell is the payoff to Player 1; the number in the upper-right corner of each cell is the payoff to Player 2. Each player is trying to maximize his payoff.)

P1 & P2 have a common interest in avoiding this outcome - safest way to do this is for them to choose (s_2, c_1) but this is not an equilibrium

Player 1 \ Player 2	c_1	c_2
s_1	5, 3	2, 2 ←
s_2	3, 4	3, 5

(1) What do you expect the outcome of the game to be if the players must make their strategic choices *simultaneously* (not knowing what choice the other is making)?

?

(2) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 1 moving first* and Player 2 second?

second-moving player will make best response to whatever first moving player does, so (s_1, c_1)

(3) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 2 moving first* and Player 1 second?

Ditto so (s_2, c_2) ← second-mover advantage

(4) Supposing that the players make their strategic choices *sequentially* with Player 1 moving first, could Player 2 *communicate any pre-play message* to Player 1 that might improve the outcome for Player 2.

commit to c_2 basically

(5) Supposing that the players make their strategic choices *sequentially* with Player 2 moving first, could Player 1 *communicate any pre-play message* to Player 2 that might improve the outcome for Player 1.

like

commit to s_1 threat

4. Answer the following questions pertaining to the (variable-sum) game depicted in the payoff matrix. *Then briefly explain each of your answers.* (Each player has just two strategies. The number in lower-left corner of each cell is the payoff to Player 1; the number in the upper-right corner of each cell is the payoff to Player 2. Each player is trying to maximize his payoff.)

* C_2 is not dominated in the sequential choice game with P2 moving first

Player 1 \ Player 2	c_1	c_2
s_1	3 / 5	2 / 2
s_2	5 / 3	4 / 3

	C_1	C_2
S_1/S_1	3 / 5	2 / 2
S_2/S_2	5 / 3	4 / 3
S_1/S_2	3 / 5	4 / 3
S_2/S_1	5 / 3	2 / 2

(1) What do you expect the outcome of the game to be if the players must make their strategic choices *simultaneously* (not knowing what choice the other is making)?

C_1 is dominant + S_1 is best reply, so (S_1, C_1)

(2) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 1 moving first* and Player 2 second?

P1 looks ahead + reasons back: P1 expects P2 to choose best reply to whatever P1 does, so (S_1, C_1) {same as simultaneous choice}

(3) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 2 moving first* and Player 1 second?

P2 expects P_1 to choose best reply to whatever P2 does, so (C_2, S_2). Note that P2 chooses C_2, which is dominated in the simultaneous choice game.

(4) Supposing that the players make their strategic choices *sequentially* with Player 1 moving first, could Player 2 *communicate any pre-play message* to Player 1 that might improve the outcome for Player 2.

commit to C_2

(5) Supposing that the players make their strategic choices *sequentially* with Player 2 moving first, could Player 1 *communicate any pre-play message* to Player 2 that might improve the outcome for Player 1.

commit to S_1