POLI 388 due 03/01/10

PROBLEM SET #1

NAME ______________________________ **Please put all your answers in this booklet.**

The first questions focus on payoff matrices for (one-player) *games against nature*. Remember the basic set-up.

The player chooses a *strategy* (the a row in the matrix). Nature chooses a *contingency* (the column in the matrix). The player has complete information concerning the payoff matrix but must choose his strategy without knowing what contingency nature will choose. The player's chosen strategy and nature's chosen contingency define the outcome of the game, i.e., the *cell* in the matrix at the intersection of the chosen row and column. The numbers in the cells give the *payoffs* to the player — i.e., they indicate what outcomes are "good" and what are "bad" (and in what degree) for the player. The player aims to get the largest payoff possible. But nature is indifferent over all outcomes and gets no payoff (which is why there is only one number in each cell).

Here is a quick review of *decision principles* the player might follow (and answer the Yes/No questions):

Maximax principle ("aim for the best"):

(1) find the maximum payoff in each row, and

(2) choose the row with the maximum of the maximums.

Does a player always have a maximax strategy? Yes No

Is it always unique? Yes No

2+ strategies may have same security level

Maximin Principle ("avoid the worst'):

(1) find the minimum payoff in each row (its security level), and then

(2) chose the row with the maximum of the minimums (the highest security level).

Does a player always have a maximin strategy? Yes No

Is it always unique? Yes No

2+ strategies may lead to same maximum payoff

Maximize Average Payoff ("don't focus on the best outcome or the worst outcome but on the average outcome"):

(1) add up all the payoffs in the row,
(2) divide by the number of contingencies, and
(3) choose the row with the highest average.

Does a player always have a average payoff maximizing strategy? Yes No

Is it always unique? Yes No

2+ may tie with same maximum average

Maximize Expected Payoff (i.e., take account of the differing probabilities of contingencies):

(1) determine (or make a subjective estimate of) the probability p_k of each contingency c_k (where $p_k > 0$ and $\sum p = 1$),

(2) multiply each payoff by its probability and add these products together to get the expected payoff of each row, and

(3) choose the row with the highest expected payoff.

Does a player always have an expected payoff maximizing strategy? Yes No

Is it always unique? Yes No

Note: average and expected payoff are the same in the event each contingency has equal probability.

Dominance (or "Sure Thing") ***Principle***:

Definition: strategy s_k *dominates* strategy s_h if

(1) s_k gives at least as high payoff as s_h in every contingency and a higher payoff in at least one contingency.

Basic Principle: "Don't choose a dominated strategy" (or, "Always choose an undominated strategy").

Does a player always have an undominated strategy? Yes No

Is it always unique? Yes No

Definition: a strategy is *dominant* if it dominates every other strategy.

Corollary Principle: "Always choose a dominant strategy, if you have one."

Does a player always have a dominant strategy? Yes No

Is it always unique? Yes No

I. Answer the following questions pertaining to each of the three payoff matrices.

	c_1	c_2	min	Av.
s_1	4	5	4	9/2
s_2	2	8	2	10/2

1. What is the player's maximax strategy? S_2 (which can give 8 payoff

2. What is the player's maximin strategy? S_1 (highest minimum)

3 What strategy maximizes the player's average payoff? S_2

4. If p_1 (i.e., the probability of c_1) is .25 and p_2 is .75, what strategy maximizes the player's expected payoff? S_1 (6.8 vs. 4.75)

5. If p_1 is .9 and p_2 is .1, what strategy maximizes the player's expected payoff? S_2 (4.1 vs. 2.6)

6. Does the player have a dominated strategy? No

7. Does the player have a dominant strategy? No

II.

	c_1	c_2	c_3	min	Av
s_1	3	5	4	3	12/3
s_2	4	2	10	2	16/3
s_3	5	5	5	5	15/3

1. What is the player's maximax strategy? ~~S_3~~ S_2

2. What is the player's maximin strategy? S_3

3 What strategy maximizes the player's average payoff? S_2

4. Does the player have a dominated strategy? No

5. Does the player have a dominant strategy? No

III.

	c_1	c_2	c_3	c_4	min	Av.
s_1	4	8	3	2	2	17/4
s_2	6	3	4	2	2	15/4
s_3	3	5	3	2	2	13/4

1. What is the player's maximax strategy? S_1 S_1, S_2, S_3 all tied

2. What is the player's maximin strategy?

3 What strategy maximizes the player's average payoff? S_1

4. Does the player have a dominated strategy? yes S_3
5. Does the player have a dominant strategy? No $S_1 + S_2$ both undominated

Now "transpose" each matrix, i.e., suppose the player chooses columns and nature chooses rows and answer the same set of questions for each.

Ia.

1. What is the player's maximax strategy? C_2
2. What is the player's maximin strategy? C_2
3 What strategy maximizes the player's average payoff? C_2
4. Is there any probability distribution such that c_1 has greater expected utility than c_2? No, since C_2 dominates C_1
5. Does the player have a dominated strategy? yes C_1
6. Does the player have a dominant strategy? yes C_2

IIa.

1. What is the player's maximax strategy? C_3
2. What is the player's maximin strategy? C_3
3 What strategy maximizes the player's average payoff? C_3
4. Does the player have a dominated strategy? yes C_1 dominated by C_3
5. Does the player have a dominant strategy? no $C_2 + C_3$ both undominated

IIIa.

1. What is the player's maximax strategy? C_2
2. What is the player's maximin strategy? C_1, C_2, + C_3
3 What strategy maximizes the player's average payoff? C_2
4. Does the player have a dominated strategy? yes C_4 dominated by C_1, C_2, + C_3
5. Does the player have a dominant strategy? No

$C_1 + C_2$ both undominated + C_3 dominated by C_1

IV. In class, we discussed the following example of a game against nature:

You are following a car into to the lower level of the Administration Drive parking structure. The lot is either full or almost full (no empty spaces are in sight.) Your goal is to get a parking space. You must either to follow the car in front of you or go around the other way.

Construct a payoff matrix for this game and explain why you have a dominant strategy.

Nature chooses contingencies:
C_1 = no spaces left
C_2 = one space left
C_3 = two or more spaces left

P	C_1	C_2	C_3
Follow	0	0	1
Otherway	0	1/2	1

payoffs is chance of getting parking space

V. For each 2 × 2 payoff matrix below:

(a) circle each cell that is a Nash Equilibrium

(b) identify what type of game (Matching Pennies, Prisoner's Dilemma, etc.) each payoff matrix exemplifies. (*Note*: the numbers in the matrix may be different from those used in class, but the nature of the game is determined by the *relative*, not *absolute*, magnitudes of the payoff.)

(a)

3, 3	4, 4
1, 1	2, 2

zero-conflict game with no coordination problem

(b)

3, 3	1, 1
1, 1	3, 3

zero-conflict coordination game

(c)

2, 3	1, 1
1, 1	3, 2

coordination game with conflict of interest (Battle of Sexes)

(d)

−2, 2	−3, 3
−1, 1	−4, 4

strictly determined zero-sum game

(e)

1, 3	2, 2
3, 1	0, 4

non-strictly determixed zero-sum game; no pure strategy Nash equilibrium. (There is a mixed strategy Nash equilibrium

(f)

3, 3	1, 4
4, 1	2, 2

Prisoner's Dilemma

POLI 388 Spring 2010

PROBLEM SET #1: REVIEW OF DECISION PRINCIPLES

Maximax Principle ("aim for the best — and ignore everything else"):

(1) find the maximum payoff in each row, and
(2) choose the row with the maximum of the maximums.

Every row has a maximum payoff, and one of these maximum payoffs must be the overall maximum payoff, so a player always has a maximax strategy. However, several rows may be tied with the same overall maximum payoff, so a player may not have a unique maximax strategy. (The player's several maximax strategies probably are not equally good with respect to other decision principles.)

Maximin Principle ("avoid the worst — and ignore everything else"):

(1) find the minimum payoff in each row (its security level), and then
(2) chose the row with the maximum of the minimums (the highest security level).

Every row has a minimum payoff, and one of these minimums payoffs must be the maximum of the minimum payoffs, so a player always has a maximin strategy. However, several rows may be tied with the same maximim payoff, so a player may not a unique maximin strategy. (The player's several maximin strategies probably are not equally good with respect to other decision principles.)

Maximize Average Payoff ("don't focus on the best outcome or the worst outcome but on the average outcome"):

(1) add up all the payoffs in the row,
(2) divide by the number of contingencies, and
(3) choose the row with the highest average.

Every row has an average payoff, and one of these average payoffs must be the maximum average payoff, so a player always has a strategy that maximizes average payoff. However, several rows may be tied with the same maximum average, so a player may have several strategies that maximizes average payoff.

Maximize Expected Payoff (i.e., take account of the differing probabilities of contingencies):

(1) determine (or make a subjective estimate of) the probability p_k of each contingency c_k (where $p_k > 0$ and $\sum p = 1$),
(2) multiply each payoff by its probability and add these products together to get the expected payoff of each row, and
(3) choose the row with the highest expected payoff.

Note: average and expected payoff are the same in the event each contingency has equal probability.

For any probability distribution over contingencies, every row has an expected payoff, and one of these average payoffs must be the maximum expected payoff, so a player always has a strategy that maximizes expected payoff. However, several rows may be tied with the same maximum expectation, so a player may have several strategies that maximizes expected payoff.