

PROBLEM SET #2: VOTING RULES

Write your answers directly on this page. Unless otherwise specified, assume all voters vote *sincerely*, i.e., in accordance with their preferences.

I. *Preference Profile 1*

40%	35%	25%
A	B	C
C	C	B
B	A	A

1. Does Preference Profile 1 exhibit *single-peakedness*? _____
2. Which candidate (A, B, C) wins under *Simple Plurality Voting*? _____
3. Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
4. Which candidate wins under *Borda Point Voting*? _____
5. Which candidate (if any) the *Condorcet Winner*? _____
6. Which candidate (if any) is *Condorcet Loser*? _____
7. Which candidates (if any) are *clones*? _____
8. Given Simple Plurality Voting, which candidates (if any) are potential spoilers? (Hint: consider the outcome of each possible straight fight in relation to the outcome of the three-candidate election under Simple Plurality) _____
9. Given Simple Plurality Voting, which voters (if any) have an incentive to vote *strategically*, i.e., not in accordance with their preferences? _____
10. Under Plurality Runoff or IRV, which candidates (if any) are potential spoilers? (Hint: consider the outcome of each possible straight fight in relation to the outcome of the three-candidate election under Plurality Runoff/IRV.) _____
11. Given Instant Runoff Voting, which voters (if any) have an incentive to engage in strategic voting? _____

II.

Preference Profile 2

24%	16%	33%	27%
A	A	B	C
C	B	C	B
B	C	A	A

- Does Preference Profile 2 exhibit *single-peakedness*? _____
- Which candidate (if any) the *Condorcet Winner*? _____
- Which candidate wins under *Simple Plurality Voting*? _____
- Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
- Now suppose candidate B gets more popular — in particular, revise Preference Profile 2 so that the 16% of voters in the second column move B to the top of their preference orderings (and thereby drop A to second) and while all other preferences remain constant. Which candidate now wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____

III. Here is a preference profile that mimics the 2000 Bush-Gore-Nader election in Florida.

Preference Profile 3

49%	48%	3%
A	B	C
B	A	B
C	C	A

- Fill in the blanks with Bush, Gore, and Nader:
A = _____ B = _____ C = _____
- Does Preference Profile 3 exhibit *single-peakedness*? _____
- Which candidate (if any) the *Condorcet Winner*? _____

3. Which candidate wins under *Simple Plurality Voting*? _____
4. Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
5. Given Simple Plurality Voting, which candidates (if any) are potential spoilers?

6. Given Simple Plurality Voting, which voters (if any) have an incentive to engage in strategic voting?

7. Under Plurality Runoff (or IRV), which candidates (if any) are potential spoilers?

8. Given Instant Runoff (or IRV), which voters (if any) have an incentive to engage in strategic voting?

IV. ***Preference Profile 4***

49%	25%	26%
A	B	C
B	A	B
C	C	A

1. Does Preference Profile 3 exhibit *single-peakedness*? _____
2. Which candidate (if any) the *Condorcet Winner*? _____
3. Which candidate wins under *Simple Plurality Voting*? _____
4. Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
5. Given Simple Plurality Voting, which candidates (if any) are potential spoilers?

6. Given Simple Plurality Voting, which voters (if any) have an incentive to engage in strategic voting? _____

7. Under Plurality Runoff (or IRV), which candidates (if any) are potential spoilers? _____

8. Given Instant Runoff (or IRV), which voters (if any) have an incentive to engage in strategic voting? _____

V. ***Preference Profile 5***

# of voters	<u>4</u>	<u>4</u>	<u>2</u>	<u>9</u>
1st pref.	<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>
2nd pref.	<i>B</i>	<i>A</i>	<i>D</i>	<i>D</i>
3rd pref.	<i>D</i>	<i>D</i>	<i>A</i>	<i>A</i>
4th pref.	<i>C</i>	<i>C</i>	<i>C</i>	<i>B</i>

1. Does Preference Profile 5 exhibit *single-peakedness*? _____
2. Which candidate wins under *Simple Plurality Voting*? _____
3. Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
4. Which candidate wins under *Borda Point Voting*? _____
5. Which candidate (if any) the *Condorcet Winner*? _____
6. Which candidate (if any) is *Condorcet Loser*? _____

Parliamentary-style (or “knockout”) voting works as follows: pair two candidates in a straight fight and eliminate the loser; pair the winner with a third candidate and eliminate the loser; proceed until every candidate but one has been eliminated; the last candidate standing is the winner.

7. If the voting order (i.e., the order in which the candidates enter the straight fights) is alphabetical order, which candidate wins under “knockout” voting? _____
8. Does the “knockout” voting winner change when the voting order changes? _____

-
9. Which candidate(s) (if any) cannot win under any voting order? _____

VI. ***Preference Profile 6***

# of voters	<u>4</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>4</u>	<u>1</u>
1st pref.	<i>A</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>C</i>
2nd pref.	<i>B</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>B</i>	<i>A</i>
3rd pref.	<i>C</i>	<i>B</i>	<i>C</i>	<i>A</i>	<i>A</i>	<i>B</i>

1. Does Preference Profile 5 exhibit *single-peakedness*? _____
2. Which candidate wins under *Simple Plurality Voting*? _____
3. Which candidate wins under *Plurality Runoff* (or *Instant Runoff*) *Voting*? _____
4. Which candidate wins under *Borda Point Voting*? _____
5. Which candidate (if any) the *Condorcet Winner*? _____
6. Which candidate (if any) is *Condorcet Loser*? _____
7. If the voting order (i.e., the order in which the candidates enter the straight fights) is alphabetical order, which candidate wins under “knockout” voting? _____
8. Does the “knockout” voting winner change when the voting order changes? _____
9. Which candidate(s) (if any) cannot win under any voting order? _____

OVER =>

- VII. The 15 members of a Congressional committee are choosing among five different proposed funding levels for a government program. Here are the proposals and the first preferences of all committee members.

<i>Proposal 1</i>	\$10 million	which is the first preference of 5 members
<i>Proposal 2</i>	\$12 million	which is the first preference of 2 members
<i>Proposal 3</i>	\$18 million	which is the first preference of 1 member
<i>Proposal 4</i>	\$19 million	which is the first preference of 4 members
<i>Proposal 5</i>	\$21 million	which is the first preference of 3 members

With respect to second and lower preferences, each committee member ranks proposals according to their proximity (or “closeness”) to his or her first preference. For example, the preference ordering of the member who most prefers Proposal 3 is this:

	<u>Proposal</u>	<u>Proximity to first pref.</u>
first pref.	\$18	\$0
second pref.	\$19	\$1
third pref.	\$21	\$3
fourth pref.	\$12	\$6
fifth pref.	\$10	\$8

The Committee will adopt one proposal on the basis of parliamentary-style voting (which is more or less how legislative committees actually take votes). Can you determine which proposal will win in committee? Does the winning proposal depend on the order in which proposals are paired for votes?

Hint: Is there a Condorcet winner among the five proposals? If so, is there some way by which we can quickly determine which proposal it is?