#### HANDBOOK OF SOCIAL CHOICE AND VOTING

Jac C. Heckelman and Nicholas R. Miller, editors.

# 1. Introduction: Issues in Social Choice and Voting (Jac C. Heckelman and Nicholas R. Miller)

#### 2. Perspectives on Social Choice

#### 2.1 The Strange History of Social Choice (Iain McLean)

This chapter reviews the strange history of social choice, elements of which have been understood for perhaps two thousand years. Isolated thinkers in the 13th, 15th, 18th and 19th centuries made substantial contributions to the field but almost always in ignorance of the work of their predecessors. Social choice was not established as a cumulative field of study until the middle of the 20th century. The chapter concludes with some reflections on this strange state of affairs.

#### 2.2 Unanimous Consent and Constitutional Economics (Randall G. Holcombe)

Constitutional economics makes heavy use of the normative benchmark of unanimous consent. The idea appears explicitly in the analysis of optimal constitutional rules in Buchanan and Tullock's *The Calculus of Consent*, and it continues through Buchanan's work that develops constitutional economics as a distinct area of inquiry within public choice. This chapter reviews some of the work that links unanimous consent closely with constitutional economics, and continues with a critical analysis of the implications and applicability of the benchmark of unanimous consent to real-world political decision making.

## 2.3 Rational Choice and the Calculus of Voting (André Blais)

According to the rational choice model of voting, a person should vote only if the expected benefits outweigh the costs. But in a large electorate, the expected benefits are bound to be extremely small since the probability of one's vote being decisive is close to nil, so the rational decision appears to be abstention. Yet even in large elections most people do vote. This chapter reviews both the theoretical amendments that have been proposed to 'solve' this 'paradox of voting' and the empirical evidence that has been brought to bear on this question. The conclusion reached is that the paradox of voting is still very much alive and the decision to vote or not is driven primarily by alternative considerations such as social norms and pressures.

## 2.4 Computational Social Choice (Robi Ragan)

This chapter provides an introduction to computational models of social choice. It discusses two major classes of computational models and provides examples of each. The first pertains to computational simulation of traditional analytical models to explore their robustness against changes in model parameters and functional forms. The second pertains to agent-based techniques for modeling social choice phenomena as complex adaptive systems in a way that is less restrictive than traditional analytical modeling and allows the researcher to build models with greater verisimilitude. The chapter concludes with a discussion of the tradeoffs and limitations in using simulation and agent-based models.

## 3. Pairwise Social Choice

## **3.1 Majority Rule and Tournament Solutions** (Scott Moser)

This chapter reviews majority rule and its role in social choice. Respecting the preferences of majorities is considered by many to be fundamental to democratic social choice. However, pairwise majority preference may be cyclical, so that choosing a 'best' alternative becomes a complex matter. The chapter introduces a structure called a 'tournament' for representing majority preference, and it surveys various 'tournament solutions' that propose ways to use pairwise majority rule as a basis for choosing among alternatives.

## **3.2** Supermajority Rules (Keith L. Dougherty)

Supermajority rules are widely used to alter basic procedures, pass constitutional amendments, enact referenda, and make other decisions in which the status quo should be favored. This chapter explores supermajority rules as special cases of k-majority rules. Given a fixed number of voters, a k-majority rule specifies the threshold of affirmative votes needed to pass a proposal, ranging from the rule of one to unanimity rule, with majority rule and various supermajority rules in between. The chapter introduces examples of k-majority rules that have been used in practice; shows how they might be used in cases with multiple alternatives; explains why a k-majority may be preferable to majority rule; and reviews previous research on which k-majority rule is optimal.

## **3.3** The Measurement of A Priori Voting Power (Dan S. Felsenthal and Moshé Machover)

This chapter discusses the concept and measurement of a priori voting power. The a priori voting power of a voter under a given decision rule is, roughly speaking, the potential influence over the possible voting outcomes that a voter possesses by virtue of the decision rule. The qualification 'by virtue of the decision rule' serves to distinguish a priori from actual or a posteriori voting power. The discussion is largely confined to a priori voting power under decision rules with just two possible inputs and two possible outcomes: each voter has a choice between voting 'yes' or 'no' on a proposed bill (no abstentions); and the proposed bill is either adopted or rejected (no draws).

# **3.4 Condorcet Jury Theorems** (Bryan C. McCannon)

The Condorcet Jury Theorem states that, under specified conditions, a group of individuals making a collective decision using simple majority voting is more likely to make the correct decision than any individual member of the group. Moreover, as the size of the group increases, its probability of correctness increases and in the limit is correct with certainty. This powerful result has been used to justify democracy by highlighting how majority voting can aggregate information, and it has guided debates about institutional design in political science, economics, management and other fields. This chapter presents the original theorem, explores its robustness to changes in various assumptions, and reviews implications of the theorem and the boundaries of its applicability.

## 4. Spatial Models of Social Choice

# 4.1 The Spatial Model of Social Choice and Voting (Nicholas R. Miller)

This chapter presents the basic elements of the standard spatial model that is commonly used as a

framework for developing theories of legislative, electoral, and other forms of social choice and that is increasingly used to guide empirical research as well. It introduces concepts such as win sets, the core, the Condorcet winner, the top cycle set, and the uncovered set, and presents foundational results such as Black's Median Voter Theorem, Plott's Majority Rule Equilibrium Theorem, McKelvey's Global Cycling Theorem, and Greenberg's Core Existence Theorem.

# 4.2 A Unified Spatial Model of American Political Institutions (Thomas H. Hammond)

This chapter considers one- and two-dimensional spatial models of a bicameral voting body and a voting body that is 'checked' by an executive armed with a veto, but perhaps with the power to override such a veto under some k-majority rule. In each institutional setting, the chapter focuses on the existence, location and size of the 'core', i.e., the set of alternatives that are stable against attempts to upset them. In this way, it presents a spatial model of the US legislative system. It also makes some comparisons with parliamentary systems and considers implications for policy stability versus policy responsiveness.

# **4.3 Competing for Votes** (James F. Adams)

This chapter surveys spatial models of electoral competition. It begins with the assumptions found in the original formulation due to Anthony Downs: there are only two competitors, they are motivated solely by the desire to win office, and competition takes place on a single left-right ideological dimension. The chapter then considers multiparty competition, competition between policy-seeking parties, and multidimensional competition, as well as competition in which one party has a 'valence advantage' resulting from its reputation for competence and integrity. A key question throughout is the extent to which electoral competition produces centrist policies.

# 4.4 **Probabilistic Voting in Models of Electoral Competition** (Peter J. Coughlin)

The pioneering Hotelling-Downs model of electoral competition assumed that candidates embody policies and that voters' choices between them are fully determined by their policy preferences. But factors other than policy can affect voters' choices, causing candidates to be uncertain about how they will vote. This uncertainty can be modeled by a probabilistic description of the voters' choice behavior. This chapter provides a game-theoretical framework for presenting both the classical model and subsequent models that incorporate candidate uncertainty about voters' choices.

# 5. Social Choice from Multiple Alternatives

# 5.1 Arrow's Theorem and Its Descendants (Elizabeth Maggie Penn)

In the field of social choice theory, an 'impossibility theorem' takes as given a set of normatively appealing criteria and proves that it is logically impossible to satisfy all of them simultaneously. The most famous impossibility theorem was proved in 1950 by Kenneth Arrow, who showed that any voting system must fail to satisfy at least one of several appealing criteria. Arrow's Theorem has had a profound impact on the study of collective choice mechanisms and has spawned a large literature that axiomatically studies social choice. This chapter introduces the reader to Arrow's Theorem and its many descendants, including classic results by Black, Nakamura, Sen, Blau, Gibbard, Satterthwaite and Muller, along with newer work.

# 5.2 **Properties and Paradoxes of Common Voting Rules** (Jac C. Heckelman)

Strict majority rule presents significant limitations when there are more than two alternatives from which to choose, since no alternative may be supported by a majority of first preferences.

This chapter examines various voting rules used to choose among multiple alternatives, including runoff systems, Hare, plurality, Borda, Condorcet, approval, range, sequential binary procedures, and proportional lotteries, and it compares them with respect to their normative properties and provides examples that illustrate seemingly 'paradoxical' violations of such properties.

## 5.3 Voting Mysteries: A Picture is Worth a Thousand Words (Donald G. Saari)

While 'voting' appears to be conceptually very simple, it can be accompanied with disturbing paradoxical outcomes. Beyond providing surprises, paradoxical outcomes must be taken seriously because they could indicate ways in which the 'election winner' is not who or what the voters really want. Thus it is necessary to explain and understand these various mysteries by discovering how and why these puzzles arise. This chapter follows the adage that 'a picture is worth a thousand words' by using nothing more complicated than the geometry of an equilateral triangle or an ordinary cube. The resulting 'pictures' provide insights and answers to a wide variety of puzzles while offering new, more global perspective about why these paradoxes arise.

## 5.4 Multiple-Winner Voting Rules (Nicolaus Tideman)

When more than one candidate is to be elected, the goal could be to elect the best candidates or the ones who would be most effective as a team, but usually the purpose is to elect the candidates who would best represent the electorate. This chapter assumes that representation is the goal when selecting a voting rule, and it evaluates seven rules in terms of ease of understanding the rule, ease of voting, ease of counting the election, representativeness of the elected candidates, and minimization of strategy. The rules that are examined can be placed in a sequence in which each one overcomes a limitation of the previous one, highlighting another limitation in the process.

#### 6. Empirical Social Choice

## 6.1 Measuring Ideology in Congress (Christopher Hare and Keith T. Poole)

This chapter surveys methods for measuring the ideological preferences of members of Congress and other legislative bodies. It first details the theoretical underpinnings of these methods — the spatial voting and random utility models — and then turns to a discussion of the ideal point estimation procedures currently in use. The pros and cons of each method are presented and empirical results from the French Fourth Republic and the US Congress are examined. Although these methods often produce very similar measures of ideology, an instance in which their results diverge is detailed. Finally, the chapter illustrates the way in which the spatial maps produced by these methods can be used to better understand the nature of ideological differences among legislators.

# **6.2** The Uncovered Set and its Applications (William T. Bianco, Christopher Kam, Itai Sened and Regina A. Smyth)

Since it was first proposed, the 'uncovered set' has appeared to have desirable attributes as a solution concept for the spatial theory of electoral competition and legislative behavior. But for many years, a major drawback was that, except in very special cases, very little was known about its location, size and shape in the context of a spatial model of two or more dimensions.

However, recent work overcame this problem in an important class of cases by allowing quite precise computation of the uncovered set. A subsequent series of theoretical, empirical and experimental papers using this computational technique has reinforced the empirical relevance of the spatial theory of politics. This chapter summarizes this line of work.

#### 6.3 Empirical Examples of Voting Paradoxes (Marek M. Kaminski)

This chapter reviews empirical occurrences of voting paradoxes Referring to some of the normative properties discussed in earlier chapters and introducing several new ones, it shows that violations of such properties are not merely of theoretical interest but have actually occurred in practice. The discussion covers election inversions, the Condorcet paradox, failure to elect a Condorcet winner, apportionment problems, violations of independence, monotonicity failure, and House size effects in the US Electoral College. Examples are drawn from Chilean and US presidential elections, Polish parliamentary elections, professional society elections, and elsewhere.

#### **Glossary of Terms Pertaining to Social Choice and Voting**