

---

# 1. Introduction: issues in social choice and voting

*Jac C. Heckelman and Nicholas R. Miller*

---

## 1.1 THE FIELD OF SOCIAL CHOICE

Individuals often make decisions as part of a group. While an individual acting alone can choose as he or she prefers, a collective decision must aggregate the preferences of multiple individuals. Collective decisions may involve as few as two people, such as a couple deciding where to eat dinner, or several members voting in a committee, hundreds of members voting in a legislature, or millions of people voting on a referendum or electing a parliament or president. Any such decision – except perhaps an informal decision in a small group – requires a clearly defined rule to aggregate diverse individual preferences and identify the social choice; for example, the proposal or candidate to be selected. Yet different rules have different properties, and they may produce different social choices even for the same individual preferences.

Broadly understood, social choice theory identifies, analyzes and evaluates rules that may be used to make collective decisions. So understood, social choice is a subfield within the social sciences (especially economics and political science) that examines institutions that may be called ‘voting rules’ of various sorts. More narrowly understood, social choice theory is a specialized branch of applied logic and mathematics that analyzes abstract objects called ‘preference aggregation functions’, ‘social welfare functions’ and ‘social choice functions’. While this *Handbook* includes several chapters that introduce the reader to social choice theory in its narrower sense, we included the word ‘voting’ in the title to signal that it covers the field in its broader sense.

The most familiar voting institutions are based on majority rule. Majority rule is straightforward in the event that a choice is to be made between just two alternatives, but it presents complications once the field of choice expands beyond two. Even in the two-alternative case, other voting rules (for example, supermajority rule and weighted voting) are available and sometimes used.

Discussion of voting rules dates back at least to classical times. But elections then were conducted largely by lot, and voting was restricted

## 2 *Handbook of social choice and voting*

almost entirely to the two-alternative case, for example, voting on conviction or acquittal following a trial. However, a famous letter from Pliny the Younger (discussed in Chapter 2) provides an early discussion of alternative voting rules to choose among more than two alternatives, and it illustrates the possibility of manipulating social choice by using one rule rather than another.

It was not until the latter part of the eighteenth century that the problem of social choice received systematic attention. During the period leading up to the French Revolution, Jean-Charles de Borda and the Marquis de Condorcet wrote papers examining voting rules to choose among multiple alternatives, and to this day their names are associated with rival approaches to the problem. Under Borda's preferred system, voters rank the alternatives, points are assigned to alternatives based on their ballot rankings, and the alternative with the greatest number of points is selected. In contrast, Condorcet focused on majority preference between pairs of alternatives; in so doing, he discovered what has come to be known as a 'Condorcet cycle'.

The pioneering work of Borda and Condorcet was largely forgotten until the mid-twentieth century, when the modern study of social choice was initiated by the work of two scholars. Kenneth Arrow's monograph on *Social Choice and Individual Values* (1951) presented his famous 'Impossibility Theorem' and thereby effectively founded (and named) the field of social choice theory in its narrower and more technical sense. At about the same time, Duncan Black (1948) introduced the concept of 'single-peaked preferences' and stated his 'Median Voter Theorem' and thereby initiated the 'spatial model' of voting. Thereafter, Kenneth May (1952), Amartya Sen (1970) and others used similar logical and mathematical tools to produce further landmark theorems. Since then social choice theory, in both its narrower and broader senses, has developed in a cumulative fashion and at an impressive pace.

Arrow's work demonstrated the value of a formal and axiomatic approach to social choice. Condorcet had previously demonstrated by example that pairwise majority rule can produce cyclical inconsistency. Arrow sought to determine what rules could avoid such inconsistency and at the same time meet other conditions that he thought any acceptable voting rule should obey. While an example is sufficient to demonstrate that a particular rule violates a particular condition, providing examples cannot demonstrate that every possible voting rule violates at least one of his conditions. Instead Arrow provided formal definitions of the conditions he thought an acceptable rule should obey and then, using the logic of sets and relations, provided a formal proof that these conditions were incompatible. In this way, he demonstrated that no rule

(whether already known or yet to be discovered) could meet all of his conditions.

Since then theorists have used rigorously defined terminology and logical derivation to develop powerful insights into processes of social choice. The downside is that this formal apparatus has limited the ability to fully understand and appreciate this line of research to those well versed in the methodology. This *Handbook* has been developed to address this problem. Each chapter aims to present an expository primer on a particular topic or theme within the field of social choice. Notation, terminology and technical details have been kept to a minimum in order to make the material fully accessible to an academic but non-specialist audience; in particular, to scholars in economics, political science, mathematics, philosophy, law and other fields who are not specialists in social choice, as well as to graduate students and advanced undergraduates in the same disciplines. While some chapters (in particular, Chapters 6, 9, 10, 11, 13, 14 and 16) are slightly more challenging, even novice readers should be able to gain a basic understanding of the topics they cover.

Readers of this *Handbook* may wonder how it differs from other volumes with similar titles, in particular the two editions of *The Elgar Companion to Public Choice* (Shugart and Razzolini 2001; Reksulak et al. 2013) and the two-volume *Handbook of Social Choice and Welfare* (Arrow et al. 2002, 2011). While the terms ‘social choice’ and ‘public choice’ may seem interchangeable, they have in fact acquired distinct meanings. ‘Public choice’ is usually defined as the application of economic modes of analysis – in particular methodological individualism and rational choice – to political problems. As such, social choice can be viewed as a subfield of public choice. Thus only a few of the chapters in the *Companion to Public Choice* would be suitable for this *Handbook*, as most do not deal directly with voting rules. Given our narrower focus, this *Handbook* is able to cover a wider variety of issues related to social choice, and to do so in greater detail, than does the *Companion*. However, most of the *Companion* chapters are written in the same introductory and accessible style that we aim for in our volume. In contrast, the chapters in the *Handbook of Social Choice and Welfare* summarize advanced research within particular areas of social choice theory in its narrower and more technical sense, and they are aimed at a specialist audience of social choice theorists. While many of its chapters deal with topics that are covered, or at least alluded to, in our volume, they are presented there in the kind of formal theorem-proving style that we have aimed to avoid.

## 1.2 OVERVIEW OF SOCIAL CHOICE AND THE *HANDBOOK*

The remainder of the *Handbook* is divided into five parts: ‘Perspectives on Social Choice’, ‘Pairwise Social Choice’, ‘Spatial Models of Social Choice’, ‘Social Choice from Multiple Alternatives’ and ‘Empirical Social Choice’. Chapters in each part focus on particular topics within these broad categories. For the novice reader, at the end of the *Handbook* is a convenient glossary of social choice terminology used in various chapters.

### 1.2.1 Perspectives on Social Choice

The history of social choice can be traced back many centuries, as documented in Iain McLean and Arnold Urken’s *Classics of Social Choice* (1995). But, as McLean shows in his opening Chapter 2, this has been a ‘strange’ history in that, until the middle of the twentieth century, individual scholars repeatedly made important discoveries in social choice that were then forgotten until independently rediscovered decades or centuries later. Even the flowering of quite sophisticated social choice arguments in France in the late eighteenth century – notably the sometimes acrimonious debates on voting methods between Condorcet and Borda – dropped almost entirely out of sight until their rediscovery by Duncan Black (1958) 150 years later. McLean lays out ‘The strange history of social choice’ by detailing its fits and starts, going back to the time of the Roman Empire, picking up again in Medieval times, with breakthroughs during the Enlightenment period and in Victorian England, up through its relatively recent revival, initiated by Arrow and Black, and its establishment as a scholarly field.

In the following Chapter 3, Randall Holcombe lays out the connection between ‘Unanimous consent and constitutional economics’. Constitutional political economy, with roots going back to the social contract theories of Hobbes and Locke, places a premium on the normative criterion of unanimous consent to the establishment of political institutions. Holcombe elaborates this concept as central to the analysis of optimal constitutional rules pioneered by James Buchanan, who not only was a founding father of the field of public choice but also developed constitutional economics as a distinct area of inquiry. Holcombe’s chapter reviews work that links constitutional economics closely with unanimous consent; this link tends to imply strict limits on the scope of activities that a polity can legitimately undertake. The chapter also provides a critical analysis of the implications and applicability of the benchmark of unanimous consent to real-world political decision-making.

While Anthony Downs's *An Economic Theory of Democracy* (1957) is cited several times in this *Handbook* for popularizing the 'spatial model' of electoral competition, it made another major theoretical contribution by identifying the tension between rational choice and participation in mass elections. Because there are so many potential voters in a democracy, and because the activities of any individual can have only the slightest or most improbable impact on political outcomes, individuals have essentially no instrumental incentive either to become informed about political matters or to vote in elections. From a rational choice point of view, the decision whether to vote in elections has proved particularly vexing. This is formalized by what has become known as the 'calculus of voting'. According to standard economic theory, an individual should be willing to take action only when the benefits outweigh the costs. While an individual may have a substantial interest in the outcome of an election and probably faces only minimal costs of voting, the benefit the individual receives from his or her preferred candidate or party winning must be discounted by the probability that the outcome of the election depends on whether and how the individual votes. Given a large electorate, the expected benefits are essentially zero because the probability that a single vote determines the outcome is essentially zero. Thus, while many observers wonder why so many citizens fail to vote, social choice scholars wonder why so many people do vote. Indeed, this anomaly has been called the 'paradox of voting' (though this term is also applied to a quite different phenomenon), and it has spawned a vast theoretical and empirical literature concerned with the probability of tie elections, the motivations of voters, and the nature of voting costs. This issue, together with much of the literature it has generated, is surveyed by André Blais in Chapter 4 on 'Rational choice and the calculus of voting'. Blais reviews the theoretical arguments, together with the related empirical findings to which he has made important contributions (Blais 2000). He concludes that the paradox remains unsolved and suggests that the decision to vote or not to vote is driven in large part by social norms and pressures.

Recent advances in the availability of computer technology have transformed many fields of investigation, including social choice theory. In Chapter 5, Robi Ragan provides an introduction to the relatively new field of 'Computational social choice'. Ragan discusses two major classes of computational models in social choice: computer simulations of analytical models and agent-based models of social choice phenomena as complex adaptive systems. Computer simulations are used to explore properties of, and extensions to, analytic models (such as the spatial model discussed below) that would be difficult to do by formal derivation and proof. Agent-based models do not require tractable mathematical solutions,

so they can more easily incorporate high levels of complexity. Ragan concludes by discussing the trade-offs in using simulation or agent-based models and the limitations of both approaches.

### 1.2.2 Pairwise Social Choice

While other voting rules are often employed in democratic settings, majority rule is probably the most fundamental voting rule and provides a basis for comparison with other rules. The operation of majority rule is straightforward in the event that social choice involves just two alternatives. Each individual votes for one or the other alternative, and the alternative with the most votes is selected. In fact, in the two-alternative case, Kenneth May (1952) proved that majority rule is the only voting rule that satisfies four conditions that are appealing in many (though not all) contexts. (Similar characterizations have subsequently been developed for several other voting rules.) In Chapter 6 on 'Majority rule and tournament solutions', Scott Moser begins by presenting May's Theorem, and then shows how majority rule can be extended to the multi-alternative case by applying it – as Condorcet advocated – to all pairs of alternatives. (This extension is referred to throughout the *Handbook* as 'pairwise majority rule'.) But majority rule so extended also presents the problem first identified by Condorcet: it can produce inconsistent social preference. For example, alternative  $x$  may beat  $y$  in a majority vote,  $y$  may beat  $z$ , and yet  $z$  may beat  $x$ , thereby producing a so-called 'Condorcet cycle' such that, whichever alternative is selected, some majority prefers a different choice. (This is the second phenomenon to which the term 'paradox of voting' is applied.) Moser shows how a mathematical structure called a 'tournament' may conveniently be used to represent majority preference over pairs of multiple alternatives, and he then discusses various 'tournament solutions' that can identify what may be deemed 'best' alternatives for social choice even in the face of Condorcet cycles.

Despite the prevalence of majority rule, supermajority rules are quite often used in special cases; for example, to establish constitutions or enact constitutional amendments, to change voting or other procedural rules, to remove public officials from office, to override vetoes, and in other circumstances in which there may be reason to favor (but not absolutely entrench) some status quo (thereby violating one of May's conditions that requires equal treatment of alternatives). In Chapter 7, Keith Dougherty explores 'Supermajority rules' as special cases of  $k$ -majority rules, where  $k$  is the number of votes (often called the 'quota') out of a total of  $n$  voters required to pass a proposal. Unanimity rule sets  $k$  equal to  $n$ ; simple majority rule sets  $k$  equal to the smallest integer greater than  $n/2$ ; and a

range of supermajority rules lie between these extremes. Dougherty takes account of May's Theorem and explains why more demanding  $k$ -majority rules may be reasonable in some circumstances. He also considers various practical examples of supermajority rules and shows how they may be extended to cover multiple alternatives.

Although the principle of 'one person, one vote' (as formalized by another of May's conditions that requires equal treatment of voters) is generally a guiding principle within a democratic setting, there are cases in which it may not be appropriate. For example, shareholders' votes are weighted by the number of shares owned; national votes in the International Monetary Fund are weighted by financial contribution; the European Council of Ministers operates under 'qualified weighted majority voting'; and the US Electoral College in effect creates a weighted voting system among states. A measure of 'a priori voting power' reflects each voter's degree of potential control over the voting outcome when account is taken only of the nature of the voting rule – in this case, the distribution of weights and the quota required for action – and not preferences within the voting body. Such a measure may be either relative, indicating the share of power held by each voter; or absolute, indicating the probability that the outcome of a pairwise vote depends on the way that a member votes. Perhaps the most fundamental conclusion of voting power theory is that voting power may not be equal, or even proportional, to voting weight. To take the most obvious example, if a shareholders' meeting uses weighted majority rule, anyone who owns more than half of the outstanding stock has dictatorial power in decision-making. Some years ago Dan Felsenthal and Moshé Machover (1998) produced a treatise that examined the conceptual underpinnings, properties and applications of the various measures of a priori voting power that had been developed over the previous 70 years. In Chapter 8 on 'The measurement of a priori voting power', Felsenthal and Machover summarize the major points in their book and take note of several more recent theoretical developments.

The dominant line of research in social choice examines the process of aggregating individual preferences through voting. However, the second major contribution of Condorcet to social choice, commonly referred to as the 'Condorcet Jury Theorem', concerns the aggregation of individual beliefs into a social choice concerning a question that in principle has one 'correct' and one 'incorrect' answer (such as the guilt or innocence of a criminal defendant). The theorem states that if individual beliefs are more likely to be correct than incorrect, and certain other conditions hold, a group using majority rule is more likely to make the correct decision than is the average individual member acting alone. Moreover, the probability that the majority choice is correct increases as the size of the group

increases and, in the limit, approaches one. In Chapter 9 on ‘Condorcet jury theorems’, Bryan McCannon presents Condorcet’s powerful result, examines how its rather restrictive assumptions can be relaxed in various ways, and shows how it has guided institutional design debates in the fields of political science, economics, philosophy and management.

### 1.2.3 Spatial Models of Social Choice

Social choice theory most typically assumes a finite set of alternatives over which voter preferences are unrestricted. However, a ‘spatial model’ of social choice assumes that all points in a space of one or more dimensions represent possible alternatives and that voters have preferences that are plausibly shaped by this spatial structure. In the simplest one-dimensional case, voters have most preferred points (called ‘ideal points’) along a continuum that represents alternatives with respect to a particular issue (or ideology in some more general sense) and their preferences for alternatives decline with distance from these ideal points (forming a pattern that Black 1948 dubbed ‘single-peaked’). Spatial models are commonly used as a framework for developing theories of committee, legislative, and electoral forms of social choice and voting, and they are increasingly used to guide empirical research as well. In Chapter 10 on ‘The spatial model of social choice and voting’, Nicholas Miller outlines the basic elements of the standard spatial model and presents four foundational theorems pertaining to social choice in a space of one, two, or more dimensions. A common theme concerning the spatial model is that in a one-dimensional setting majority rule is well behaved, but in a higher-dimensional setting its operation becomes ‘chaotic’. Miller explains in what sense this is true but also introduces more recently developed concepts (including the ‘uncovered set’) to suggest why the ‘chaotic’ characterization may be overdrawn.

Although the spatial model is typically used to analyze voting within a sovereign unicameral voting body, in Chapter 11 Thomas Hammond extends it to consider voting in a stylized bicameral voting body, which may also be ‘checked’ by an executive veto, but perhaps with the power to override such a veto under some  $k$ -majority rule. In so doing, Hammond summarizes and extends earlier work (for example, Hammond and Miller 1987) to present ‘A unified spatial model of American political institutions’, with some comparisons to parliamentary systems. In each institutional setting, Hammond focuses on the existence, location and size of the ‘core’ (that is, the set of alternatives that are stable against attempts to upset them), and he considers the implication of this analysis for policy stability versus policy responsiveness in political systems.

Probably the most familiar application of the spatial model, going back to Downs (1957), is to the theory of electoral competition. In Chapter 12 on ‘Competing for votes’, James Adams surveys spatial models of competition between candidates or parties for the support of electoral majorities (or, if there are more than two competitors, pluralities). Adams begins with the traditional assumptions based on Downs’s original formulation: there are only two competitors, they are motivated solely by the desire to win office, and competition takes place on a single left–right ideological dimension. Adams then proceeds to consider multi-party competition, competition between policy-seeking parties, and multidimensional competition, as well as competition in which one party has a ‘valence advantage’. A key question throughout is whether electoral competition produces centrist policies.

In the following Chapter 13 on ‘Probabilistic voting in electoral competition’, Peter Coughlin covers some of the same ground as Adams but with two important differences. First, he presents his analysis in a formal game-theoretic framework. Second, he presents an important alternative to the standard assumption that individuals vote with certainty for the candidate or party that they prefer on policy grounds. In contrast, Coughlin allows factors other than policy to affect a voter’s decisions. Uncertainty by the candidates about how individuals will vote leads Coughlin to assume that, from a candidate’s perspective, each voter’s choice is probabilistic in nature. Drawing in part on his own earlier work (Coughlin 1992), he then considers how the assumption of probabilistic voting in models of electoral competition affects electoral outcomes.

#### **1.2.4 Social Choice with Multiple Alternatives**

While some social choice problems involve just two alternatives, social choice rules must in general deal with the case of multiple alternatives. We have already noted that majority rule runs into problems in this general case and also that Arrow’s Theorem shows that all rules suffer from one problem or another. More specifically, Arrow’s Theorem shows that, given three or more alternatives, it is impossible for a ‘preference aggregation rule’ to satisfy all of a small number of seemingly weak conditions of fairness and consistency. In Chapter 14 on ‘Arrow’s Theorem and its descendants’, Elizabeth Maggie Penn lays out Arrow’s conditions and presents a precise statement of his ‘Impossibility Theorem’. She then sketches out other major theorems that may be deemed ‘descendants’ of Arrow’s. These include theorems that: (1) show that Arrow’s Theorem is robust in that problems remain even when several of his conditions are weakened; (2) identify an incompatibility between personal rights and

several of Arrow's conditions; and (3) demonstrate that an incentive to misrepresent one's preferences is pervasive in social choice.

In the following Chapter 15 on 'Properties and paradoxes of common voting rules', Jac Heckelman describes common practical voting rules used to choose among multiple alternatives, including those that assign scores to each alternative based on a voter's ranking, those that require majority support and utilize run-offs if necessary, those that are based on pairwise majority rule, and those that involve proportional lotteries. Heckelman compares these rules with respect to their normative properties and provides examples that illustrate seemingly 'paradoxical' violations of such properties by particular voting rules.

In Chapter 16 on 'Voting mysteries: a picture is worth a thousand words', Donald Saari applies his pioneering work on 'the geometry of voting' (Saari 1995) to help explain why some of these paradoxical or 'mysterious' outcomes occur as they do. Saari shows how geometrical 'pictures' no more complicated than an equilateral triangle or a cube can provide insights and answers to a wide variety of puzzles while offering new and more general perspectives about why these paradoxes arise.

All voting rules discussed in Chapter 15 select a single winner: for example, an executive official, a representative from a single-member district, a particular version of a bill, or even the targeted interest rate decided on by the US Federal Open Market Committee. But voting rules are sometimes needed to select multiple winners: for example, to fill multiple seats on a school or corporate board, to elect several representatives from a multi-member district, or to identify finalists to be interviewed for a job opening. In Chapter 17, Nicolaus Tideman identifies and describes a wide variety of 'Multiple-winner voting rules', some of which are straightforward generalizations of single-winner rules but others of which – in particular, various forms of proportional representation – are based on quite different principles. Tideman evaluates the operation of these multiple-winner voting rules with respect to various normative and practical criteria.

### **1.2.5 Empirical Social Choice**

Empirical research increasingly draws on social choice concepts to formulate questions and guide analysis, especially pertaining to the legislative process. The spatial model has been especially productive in this respect, because it is based on relatively elaborate but also relatively plausible assumptions that often imply specific predictions about outcomes. To make the connection between social choice theory and empirical analysis, measures must be devised to connect empirical data with social choice

concepts. In their Chapter 18 on ‘Measuring ideology in Congress’, Christopher Hare and Keith Poole describe the theoretical underpinnings of several methods for estimating the spatial location of the ‘ideal points’ of members of the US Congress and other legislative bodies on the basis of readily available roll-call data. Drawing on Poole’s own extensive work in this area (for example, Poole 2005), they present the pros and cons of each method and present some empirical results for both the French National Assembly and the US Congress. Finally, Hare and Poole illustrate how the spatial maps produced from this class of methods can be used to better understand the nature of ideological differences among legislators. This chapter will be particularly useful to practitioners interested in better understanding how to measure and interpret estimates of legislator ideology.

Chapters 6 and 14 describe how the ‘uncovered set’ may provide a solution concept for social choice under majority rule in the presence of pervasive Condorcet cycles. For many years, a major drawback had been that, except in very special cases, very little was known about its location, size and shape in the context of a spatial model of two or more dimensions. However, recent advances in computational social choice have overcome this problem in an important class of cases. In Chapter 19 on ‘The uncovered set and its applications’, William Bianco, Christopher Kam, Itai Sened and Regina Smyth discuss a ‘grid search algorithm’ (first described in Bianco et al. 2005) that allows them to compute the uncovered set for any configuration of ideal points. They then use it to reinterpret classic voting experiments, design and analyze new experiments, analyze actual instances of legislative maneuvering in the US Senate, and assess the effects of a change in electoral rules in Israel.

In the final Chapter 20, Marek Kaminski presents several ‘Empirical examples of voting paradoxes’. Referring to some of the normative properties discussed in earlier chapters, and introducing several new ones, Kaminski shows that violations of such properties are not just of theoretical interest but have actually occurred in practice. Examples are drawn from Chilean and United States presidential elections, Polish parliamentary elections, professional society elections, and other elections.

## ACKNOWLEDGEMENTS

We close this introduction by expressing our appreciation to all the chapter authors for their contributions and willingness to revise their chapters in accordance with our guidelines. We also thank a number of outside readers who assisted in our editorial duties by preparing comments

on initial drafts for certain chapters; they include John Dinan, Scott de Marchi, Michael Munger, Ashley Piggins, Amanda Ross, Alex Tabborok, Craig Tovey and John Wood. Finally, we acknowledge the Edward Elgar Publishing Executive Editor, Alan Sturmer, who never wavered in his enthusiastic support for this project despite repeated delays. We hope everyone involved concludes that their time and effort devoted to the *Handbook* was worthwhile.

## REFERENCES

- Arrow, Kenneth J. (1951), *Social Choice and Individual Values*, New York: John Wiley & Sons.
- Arrow, Kenneth J., Amartya K. Sen and Kotaro Suzumura (eds) (2002), *Handbook of Social Choice and Welfare, Volume 1*, Amsterdam: Elsevier.
- Arrow, Kenneth J., Amartya K. Sen and Kotaro Suzumura (eds) (2011), *Handbook of Social Choice and Welfare, Volume 2*, Amsterdam: Elsevier.
- Bianco, William T., Ivan Jeliazkov and Itai Sened (2005), 'The uncovered set and the limits of legislative action', *Political Analysis*, 12 (3), 256–276.
- Black, Duncan (1948), 'On the rationale of group decision-making', *Journal of Political Economy*, 56 (1), 23–34.
- Black, Duncan (1958), *The Theory of Committees and Elections*, Cambridge: Cambridge University Press.
- Blais, André (2000), *To Vote or Not to Vote*, Pittsburgh, PA: University of Pittsburgh Press.
- Coughlin, Peter J. (1992), *Probabilistic Voting Theory*, Cambridge: Cambridge University Press.
- Downs, Anthony (1957), *An Economic Theory of Democracy*, New York: Harper & Row.
- Felsenthal, Dan S. and Moshé Machover (1998), *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*, Cheltenham, UK and Northampton MA, USA: Edward Elgar.
- Hammond, Thomas H. and Gary J. Miller (1987), 'The core of the Constitution', *American Political Science Review*, 81 (4), 1155–1174.
- May, Kenneth O. (1952), 'A set of independent necessary and sufficient conditions for simple majority decisions', *Econometrica*, 20 (4), 680–684.
- McLean, Iain and Arnold B. Urken (eds) (1995), *Classics of Social Choice*, Ann Arbor, MI: University of Michigan Press.
- Poole, Keith T. (2005), *Spatial Models of Parliamentary Voting*, Cambridge: Cambridge University Press.
- Reksulak, Michael, Laura Razzolini and William F. Shugart II (eds) (2013), *The Elgar Companion to Public Choice, Second Edition*, Cheltenham, UK and Northampton MA, USA: Edward Elgar.
- Saari, Donald G. (1995), *Basic Geometry of Voting*, Berlin: Springer.
- Sen, Amartya K. (1970), *Collective Choice and Social Welfare*, San Francisco, CA: Holden-Day.
- Shugart, William F., II and Laura Razzolini (eds) (2001), *The Elgar Companion to Public Choice*, Cheltenham, UK and Northampton MA, USA: Edward Elgar.