

1. Second order linear homogeneous equations with constant coefficients.

$$ay'' + by' + cy = 0$$

The characteristic equation is $ar^2 + br + c = 0$.

1. Real and unequal roots $r_1 \neq r_2$. $y_1(x) = e^{r_1x}$, $y_2(x) = e^{r_2x}$.

2. Real and equal roots $r_1 = r_2 = r$. $y_1(x) = e^{rx}$, $y_2(x) = xe^{rx}$.

3. Complex conjugate roots $r_1 = \lambda + i\mu$, $r_2 = \lambda - i\mu$.

$$y_1(x) = e^{\lambda x} \cos \mu x, y_2(x) = e^{\lambda x} \sin \mu x.$$

2. Second order linear nonhomogeneous equations.

$$y'' + p(x)y' + q(x)y = g(x); \quad y(x) = y_p(x) + c_1y_1(x) + c_2y_2(x)$$

a. Method of Undetermined Coefficients.

| $g(x)$ | $y_p(x)$ |
|-----------------------------------|---|
| $P_n(x) = a_0x^n + \dots + a_n$ | $x^s(A_0x^n + A_1x^{n-1} + \dots + A_n)$ |
| $P_n(x)e^{\alpha x}$ | $x^s e^{\alpha x}(A_0x^n + A_1x^{n-1} + \dots + A_n)$ |
| $P_n(x)e^{\alpha x} \sin \beta x$ | $x^s e^{\alpha x}[(A_0x^n + A_1x^{n-1} + \dots + A_n) \cos \beta x + (B_0x^n + B_1x^{n-1} + \dots + B_n) \sin \beta x]$ |
| $P_n(x)e^{\alpha x} \cos \beta x$ | $x^s e^{\alpha x}[(A_0x^n + A_1x^{n-1} + \dots + A_n) \cos \beta x + (B_0x^n + B_1x^{n-1} + \dots + B_n) \sin \beta x]$ |

Here s is the number of times 0 , α , or $\alpha + i\beta$ respectively are roots of the characteristic equation.

b. Method of Variation of Parameters.

$$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x), \quad u_1(x) = \int^x \frac{-g(t)y_2(t)}{W(t)} dt, \quad u_2(x) = \int^x \frac{g(t)y_1(t)}{W(t)} dt$$